

## ANSWERS TO EXERCISES

## Pages 6, 7

1.  $x = \frac{1}{2}, -2.$

2.  $x = 1, -5.$

3.  $x = \frac{-5 \pm \sqrt{13}}{6}.$

4.  $x = \frac{-1 \pm \sqrt{-3}}{2}.$

5.  $x = \pm 1, \pm \sqrt{2}.$

6.  $x = \pm 1.$

7.  $x = \pm \frac{1}{2} \sqrt{3}.$

8.  $x = \frac{3 \pm \sqrt{13}}{2}.$

9.  $x = \frac{-1 \pm \sqrt{17}}{4}.$

10.  $x = \frac{1 \pm \sqrt{-3}}{2}.$

11.  $x = \pm 1, 2.$

12.  $x = 1, \frac{-1 \pm \sqrt{-3}}{2}.$

13.  $x = \pm 1, \pm \sqrt{-1}.$

14.  $x = \frac{\pm \sqrt{2} \pm \sqrt{-2}}{2}.$

15.  $x = \frac{\pm \sqrt{3} \pm \sqrt{-5}}{2}.$

## Page 9

1.  $x = 1, -1, 2.$

2.  $x = 2, 2, -\frac{5}{3}.$

3.  $x = -3, \frac{5}{2}, \frac{5}{2}.$

4.  $x = \frac{1}{2}, \frac{3}{2}, \frac{3}{2}.$

5.  $x = 1, \frac{1}{2}(3 \pm \sqrt{29}).$

6.  $x = -3, \frac{1}{2}(-1 \pm \sqrt{-3}).$

7.  $x = -1, -1, \pm \frac{1}{2}.$

8.  $x = -2, 3, \frac{3}{2}, -\frac{2}{3}.$

9.  $x = 2, \frac{3}{2}, 1 \pm \sqrt{-2}.$

10.  $x = 4, -\frac{5}{2}, \frac{1}{2}(3 \pm \sqrt{5}).$

## Page 10

1.  $x = -0.53, 0.65, 2.88.$

2.  $x = 1.41, -0.7 \pm 2.1 \sqrt{-1}.$

3.  $x = 0.54, -0.8 \pm 1.1 \sqrt{-1}.$

4.  $x = 1.2, 2.9.$

5.  $x = 0.7, -1.2.$

## Page 12

1.  $x > 1$  or  $x < -2.$

2.  $x > 1$  or  $-1 < x < 0.$

3.  $x < 1.$

4.  $x < -1.53$ , or  $-0.35 < x < 1.88.$

## Page 14

1.  $x = 1, y = 2.$

2.  $x = 2, y = -1, z = 3.$

3.  $x = 1, y = -1, z = 1.$

4.  $x = 1, y = \frac{1}{2}, z = \frac{1}{2}.$

5.  $x = \frac{2}{3}, y = -2, z = \frac{2}{3}.$

6.  $x = -1, y = 1,$  and

$x = -\frac{2}{3}, y = -\frac{4}{3}.$

7.  $h = 1, k = 2, r = \pm 5,$  and  
 $h = \frac{9}{49}, k = -\frac{25}{49}, r = \pm \frac{295}{49}.$

8.  $x = 1, -1, 2, -2.$

$y = -1, 1, -\frac{1}{2}, \frac{1}{2}.$

9.  $x = y = z = \pm \sqrt{2}.$

10.  $x = 1, y = -1, z = 2,$  and  
 $x = \frac{17}{3}, y = -\frac{14}{3}, z = \frac{23}{3}.$

## Page 16

11.  $x:y:z = 3:1:2.$

12.  $x:y:z = 2:-3:4.$

13.  $x:y:z = 2:-1:1,$   
or  $-1:2:1.$

14.  $x:y:z = 1:\pm 1:1,$   
or  $-2:\pm \sqrt{-2}:1.$

## Page 29

1.  $(\frac{5}{3}, 0).$

2.  $(3, 8).$

3.  $(5, 11), (\frac{7}{3}, 3).$

4.  $AB:CD = 3:-2.$

5.  $(5, 3), (-1, -5).$

## Page 31

1.  $5 + 3\sqrt{5}.$

7.  $(2 \pm 2\sqrt{3}, 5).$

8.  $(-\frac{1}{3}, 0).$

9.  $(\frac{17}{16}, 3\frac{17}{16}).$

10.  $(\frac{9}{16}, \frac{1}{16}).$

11.  $(2\frac{1}{6}, 1\frac{2}{3}).$

## Page 34

7.  $(-2, 2).$

8.  $[-\frac{1}{2}, -\frac{7}{2}], [4, 1], [-\frac{7}{2}, \frac{3}{2}].$

9.  $(-8, 5).$

10.  $(5, -1).$

12.  $24\frac{1}{2}.$

## Page 38

2.  $(0, \frac{1}{3}), (-4, 7).$

3.  $P(14, -13), Q(6, -5).$

4.  $(-10, 31).$

5.  $D(15, -3).$

8.  $D(\frac{2}{3}, \frac{11}{3}).$

9.  $P(1, 2\frac{2}{3}).$

12.  $(2\frac{1}{6}, 4\frac{4}{6}).$

## Pages 42, 43

1.  $\frac{1}{2}, -7, -2.$   
 2.  $\tan^{-1}(-0.8) = 141^\circ 20'.$   
 9.  $46^\circ 51', 97^\circ 8', 36^\circ 2'.$   
 11.  $7 \pm 5\sqrt{2}.$   
 12.  $71^\circ 34'.$
13.  $\frac{5}{11}\sqrt{3} + \frac{8}{11}i.$   
 15.  $(4.933, 4.966), (-6.933, -0.966).$   
 16.  $(-2\frac{2}{7}, -3\frac{4}{7}).$

## Pages 48, 49

2.  $x - 3y = 5.$   
 4.  $y = 3x + 9.$   
 5.  $x + 5y = 2.$   
 6.  $y^2 = 8y - 6x - 25.$
7.  $3x - 4y + 6 = 0.$   
 9.  $x^2 + y^2 = x + y + 14.$   
 11.  $x^2 + y^2 - 4x - 6y = 12.$

## Page 56

1.  $y + 3 = \sqrt{3}(x + 1).$   
 2.  $y = 3x - 7.$   
 3.  $x = 2.$   
 4.  $y = 2.$   
 5.  $2x + 8y = 17.$   
 6.  $y - 5 = \pm\sqrt{3}(x - 3).$
7.  $6y - 9x + 2 = 0.$   
 8.  $x - y + 2 = 0.$   
 9.  $y = 5x - 3,$  length  $= 2\sqrt{26}.$   
 10.  $x - 2y + 3 = 0, x - 2y + 8 = 0,$   
 $2x + y - 4 = 0, 2x + y - 9 = 0.$   
 11.  $21x + 16y = 6, 11x + 24y = 9.$

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14.  $45^\circ, 56^\circ 19', 78^\circ 4'.$   
 15.  $3x - 2y = 6.$   
 16.  $y = 5x - 4.$   
 17.  $y = x - 4.$   
 18.  $8x + 9y = 7.$   
 19.  $(1\frac{8}{17}, -1\frac{2}{17}).$   
 20.  $5x + 3y = 18.$
21.  $5, \sqrt{10}, \sqrt{17}.$   
 22.  $x - 5y + 8 = 0.$   
 23.  $12x - 15y = 8.$   
 24.  $4x - y + 9 = 0.$   
 25. A straight line perpendicular to the line through their centers.

## Pages 63, 64

8.  $x + y > 0, 2x - 3y - 1 < 0, y - 2 < 0.$   
 9.  $y + 3x - 4 > 0, 3x - 2y - 1 > 0, y - 6x + 14 > 0.$   
 10.  $\frac{1}{17}\sqrt{17}.$   
 11.  $\frac{1}{13}\sqrt{13}.$   
 12.  $\frac{2}{3}.$   
 13.  $x(2\sqrt{2} - 1) - y(\sqrt{2} + 3) = \sqrt{2} - 2.$   
 14.  $(\frac{14}{17}, \frac{12}{17}).$   
 15.  $(y - 2x)^2 = 14x + 18y - 56.$

## Pages 68, 69

1.  $(0, 0), r = 5.$   
 2.  $(2, 0), r = 2.$   
 3.  $(0, \frac{3}{2}), r = \frac{1}{2}\sqrt{7}.$   
 4.  $(-\frac{3}{2}, 0), r = \frac{3}{2}\sqrt{7}.$   
 5.  $(\frac{1}{2}, \frac{1}{2}), r = \frac{1}{2}\sqrt{2}.$   
 6.  $(a, a), r = a\sqrt{2}.$   
 7.  $(1, -2), r = \sqrt{3}.$   
 8. The locus is the point  $(1, 0).$   
 11.  $x^2 + y^2 - 5x + 4y - 46 = 0.$   
 12.  $x^2 + y^2 - 2x - 2y = 11.$   
 13.  $x^2 + y^2 - 2x - 2y + 1 = 0,$   
 $x^2 + y^2 - 10x - 10y + 25 = 0.$   
 14.  $x^2 + y^2 = 9 \pm 2\sqrt{8}.$

## Pages 73, 74

1.  $(0, 0), a = \sqrt{6}, b = \sqrt{3}.$   
 2.  $(1, -2), a = \frac{1}{2}, b = 1.$   
 3.  $(\frac{1}{2}, \frac{1}{4}), a = \frac{1}{4}\sqrt{6}, b = \frac{1}{4}\sqrt{3}.$   
 4.  $(1, -2), a = 2, b = \sqrt{6}.$   
 5.  $(-3, 1), a = \sqrt{13}, b = \frac{1}{3}\sqrt{39}.$
6. The locus is the point  $(1, -1).$   
 7.  $9x^2 + 4y^2 - 18x + 24y + 9 = 0.$   
 8.  $4x^2 + y^2 + 16x - 8y + 16 = 0.$   
 9.  $(x + y - 2)^2 + 16(x - y + 2)^2 = 32.$

## Page 77.

1. Axis  $y = 0,$  vertex  $(\frac{1}{2}, 0).$   
 2.  $y = 0, (\frac{1}{2}, 0).$   
 3.  $x = 1, (1, 2).$   
 4.  $x = -\frac{1}{2}, (-\frac{1}{2}, -\frac{9}{4}).$   
 5.  $y = \frac{3}{2}, (-\frac{9}{4}, \frac{3}{2}).$
6.  $x = \frac{1}{2}, (1\frac{1}{2}, 3\frac{1}{8}).$   
 7.  $3y^2 = 16x.$   
 8.  $5x^2 + 20x + 9y + 2 = 0.$   
 9.  $\frac{3}{2}\sqrt{2}.$   
 10.  $35\frac{5}{8}$  ft.

## Page 83.

1. Center  $(0, 2), a = \sqrt{3}, b = 2.$  Asymptotes,  $y - 2 = \pm\frac{2}{3}\sqrt{3}x.$   
 2. Center  $(0, -1), a = \frac{1}{3}\sqrt{15}, b = \sqrt{3}.$  Asymptotes,  $y + 1 = \pm x\sqrt{5}.$   
 3. Center  $(1, -1), a = b = 2.$  Asymptotes,  $x = 1$  and  $y = -1.$   
 Axes,  $x + y = 0$  and  $x - y - 2 = 0.$   
 4. Center  $(-1, 2), a = \sqrt{3}, b = \sqrt{2}.$  Asymptotes,  $y - 2 = \pm\frac{2}{3}\sqrt{3}(x + 1).$   
 5. The locus is two lines  $x - y = 4$  and  $x + y + 2 = 0.$   
 6. Center  $(0, \frac{3}{2}), a = b = 2.$  Asymptotes,  $x = 0$  and  $y = \frac{3}{2}.$   
 Axes,  $y - \frac{3}{2} = \pm x.$   
 7.  $16(x - 2)^2 - (y + 1)^2 = \pm 16.$

8.  $24(x+2)^2 - 5(y-1)^2 = 91.$   
 9.  $4(3x+2y)^2 - 25(2x-3y)^2 = \pm 1300.$

## Page 89.

1. The circle circumscribed about the square.
2. Two parabolas having the fixed diameter as common chord and with vertices at the middle points of the perpendicular radii.
3. A rectangular hyperbola passing through  $A$  and  $B$ .
4. A circle with center at the center of the triangle.
5. The circle passing through the vertices of the base angles, and tangent to the equal sides of the triangle.
6. A hyperbola.
9. A rectangular hyperbola.
10. Two circles passing through  $A$  and  $B$  with centers at the ends of the diameter perpendicular to  $AB$ .

## Page 120.

1.  $x = -2.$
2.  $y = 3.$
3.  $x - y = \sqrt{2}.$
4.  $y = x\sqrt{3}.$
5.  $x^2 + y^2 = 3x.$
6.  $x^2 + y^2 = 4y.$
7.  $x^2 + y^2 = \sqrt{2}(y-x).$
8.  $x^2 + y^2 = x-y.$
9.  $x^2 + y^2 = 4.$
10.  $x^2 + y^2 - 2x - 2\sqrt{3}y + 3 = 0.$
11.  $x^2 + y^2 - 2x - 2y + 1 = 0.$
12.  $3x^2 + 4y^2 - 4x = 4.$
13.  $4y^2 - 5x^2 - 36x = 36.$
14.  $y^2 = 6x + 9.$
15.  $x^2 = 4y + 4.$
16.  $xy = 4x + 4y - 8.$
17.  $xy = 2y - 3x.$
18.  $x^2 - y^2 = y.$
19.  $r(2\cos\theta - \sin\theta) = 1.$
20.  $r = 4\cot\theta\csc\theta.$
21.  $r = 2\cos\theta.$
22.  $r^2 = 14\csc 2\theta.$
23.  $r^2 = \sec 2\theta.$
24.  $r = 4\sqrt{2}\cos\left(\theta - \frac{\pi}{4}\right).$
25.  $r^2 + 2ar(\pm\cos\theta \pm \sin\theta) + a^2 = 0.$
26.  $r\left[1 - \cos\left(\theta - \frac{\pi}{6}\right)\right] = 4.$
27.  $r(3 - 2\sin\theta) = 3 - \sqrt{3}.$
28.  $\sqrt{2}.$

## Pages 126, 127.

35.  $\theta = -\frac{\pi}{24}, r = 3.285.$
36.  $\left(a\sqrt{2}, \frac{\pi}{4}\right).$
37.  $(0, 0), \left(\pm a, \frac{\pi}{2}\right), \left(\pm 2^{-\frac{1}{4}}a, \frac{\pi}{4}\right), \left(\pm 2^{-\frac{1}{4}}a, \frac{3}{4}\pi\right).$
38.  $(0, 0), (.785a, \pm 25^\circ 52'), (.409a, \pm 102^\circ 4'), (.898a, \pm 148^\circ 3').$
39.  $\left(a, \pm \frac{\pi}{6}\right), \left(a, \pm \frac{5}{6}\pi\right).$

## Pages 128, 129.

1.  $r = a\cos\theta.$
2.  $r = a(\sec\theta + \tan\theta).$
3.  $r(r\cos\theta - a) = k.$   $O$  is the origin and  $LK$  is perpendicular to  $OX$  at  $(a, 0).$
4.  $r = a\sin 2\theta.$  The length of the segment is  $2a.$
5.  $r = a + b\sec\theta.$  The radius of the circle is  $2a$  and the distance from the center to the fixed line is  $2b.$
6.  $r = 2a\tan\theta\sin\theta.$   $OA$  is the initial line and  $a$  the radius of circle.
7.  $r = a(1 + \cos\theta),$  a cardioid.
8.  $r = a(\csc\theta - 1).$
9.  $r\sin\left(\frac{1}{2}\theta\right) = a.$
10.  $r = a\sec\theta + b.$  The distance from  $O$  to  $BC$  is  $a$  and the constant distance is  $b.$
11.  $r = 2a\cos\theta + b.$  The diameter through  $O$  is the initial line, the radius of the circle is  $a,$  and the constant distance is  $b.$
12.  $r = a\cos^2\theta,$   $a$  being the length of  $OA.$
13.  $r = a(1 - \tan^4\theta)\cos\theta.$
14.  $r = \frac{c\sin\left(\frac{a}{b}\theta\right)}{\sin\left(\frac{c}{b}\theta\right)}.$  The radii are  $a$  and  $b$  and the distance between centers is  $c.$  The origin is at the center of circle of radius  $a.$

## Pages 131, 132.

2.  $x = a(1 + \tan\phi), y = a\tan\phi, x - y = a.$
3.  $x = a(1 + 2\sin^2\phi), y = 2a\tan\phi\sin^2\phi, y^2(3a - x) = (x - a)^3.$
4.  $x = 1 + \frac{1}{2}\cot\phi, y = \frac{1}{2} + \tan\phi, 2xy = x + 2y.$
5.  $r = a\sqrt{1 + \phi^2}, \theta = \phi - \tan^{-1}\phi. \quad \theta = \frac{1}{a}\sqrt{r^2 - a^2} - \cos^{-1}\left(\frac{a}{r}\right).$

## Pages 137, 138.

3.  $x^2 - y^2 = 4.$
4.  $(y - x)^3 = 2(x + y)^2.$
5.  $xy + x = 3y - 1.$
6.  $4x^2 - 4\sqrt{3}xy + 4y^2 = 1.$
7.  $x^2 - y^2 = 1.$
8.  $\sqrt{x^2 + y^2} = \tan^{-1}\left(\frac{x}{y}\right).$
9.  $r = 1 - 2\theta^2.$
12.  $x = a\cos^{-1}\left(\frac{a-y}{a}\right) - \sqrt{2ay - y^2}.$
13.  $\theta^2(1 + r^2)^3 = r^2(3 - r^2)^2.$
14.  $x^{\frac{2}{3}} + y^{\frac{2}{3}} = (4a)^{\frac{2}{3}}.$
16.  $r = \theta = t.$
17.  $x = t^2\cos(1+t), y = t^2\sin(1+t).$
21.  $(4, 4).$

22.  $(3, 4), (-4, -3)$ .  
 23.  $(\pm 0.5404a, 0.8414a)$ .  
 24.  $(\frac{4}{3}a, \pm \frac{2}{3}a\sqrt{2})$ .  
 25.  $x = a(1 + \cos 2\theta)$ ,  $y = a \sin 2\theta$ .  
 26.  $x = \frac{4}{m^2}$ ,  $y = \frac{4}{m}$ .
27.  $x = a \cos \phi$ ,  $y = b \sin \phi$ .  
 28.  $x = a \sec \phi$ ,  $y = b \tan \phi$ .  
 29.  $x = a \sin^3 \phi$ ,  $y = a \cos^3 \phi$ .  
 30.  $x = m^2$ ,  $y = m^3 - m - 2$ .

## Pages 140, 141.

1.  $x = b \tan \phi \mp a \sin \phi$ ,  $y = \pm a \cos \phi$ , the fixed point on the  $y$ -axis being  $(0, b)$ .  
 2.  $x = k(1 + \cos^2 \phi)$ ,  $y = k(\tan \phi + \sin \phi \cos \phi)$ .  
 3.  $x = 2a \cot \phi$ ,  $y = 2a \sin^2 \phi$ .  $(x^2 + 4a^2)y = 8a^3$ .  
 4.  $x = \frac{a}{2}(1 + \cos \phi)$ ,  $y = \frac{a}{2}(\sin \phi + \tan \frac{1}{2}\phi)$ .  
 $4xy^2 = (a - x)(a + 2x)^2$ .  
 5.  $x = (a - c \tan \phi) \sin^2 \phi$ ,  $y = (a - c \tan \phi) \sin \phi \cos \phi$ .  
 $x(ay - cx) = y(x^2 + y^2)$ .  $r = (a - c \cot \theta) \cos \theta$ .  
 6.  $x = a \tan \phi$ ,  $y = a \cos 2\phi$ .  $y(a^2 + x^2) = a^3 - ax^2$ .  
 7.  $x = a(\tan \phi + \sin \phi \cos \phi)$ ,  $y = a(1 + \cos^2 \phi)$ .  
 $y(x^2 + y^2) = a(x^2 + 2y^2)$ .  $r = a(\csc \theta + \sin \theta)$ .  
 8.  $x = 2a \cos^2 \phi$ ,  $y = 2a \sec \phi$ .  $xy^2 = 8a^3$ ,  $x \geq 2a$ .  
 O is the origin, OA the  $x$ -axis, and  $\phi = AOC$ .  
 9.  $r = a \sec^3 \theta$ .  
 10.  $(x^2 + y^2 - 2a^2)^2 = a^3(5a \pm 4y)$ . The fixed diameter is  $x$ -axis and the center of circle is origin.  
 11.  $x = c \cos 2\phi - a \sin \phi + b \cos \phi$ ,  $y = c \sin 2\phi + a \cos \phi + b \sin \phi$ .  
 The radii are  $a$  and  $b$  and the distance between centers is  $2c$ .  
 The  $x$ -axis passes through the centers and the origin is midway between them.  
 12. A rectangular hyperbola.  
 13.  $x = -a \sin(\phi + B)$ ,  $y = b \sin(\phi - A)$ . The curve is an ellipse.  
 14.  $r = a \csc \phi$ ,  $\theta = \csc \phi + \cot \phi + \phi - \frac{\pi}{2} - 1$ .

The origin is at the center of circle, the initial line passes through the intersection of curve and circle, and  $\phi$  is the angle formed at the pencil by the string.

15.  $x = a\phi - b \sin \phi$ ,  $y = a - b \cos \phi$ .  
 16.  $x = \frac{a}{4}(3 \cos \phi + \cos 3\phi)$ ,  $y = \frac{a}{4}(3 \sin \phi - \sin 3\phi)$ .  $x^{\frac{4}{3}} + y^{\frac{4}{3}} = a^{\frac{4}{3}}$ .  
 The radius of the fixed circle is  $a$ .  
 17.  $y + 1 = 0$ .

## Pages 147-149.

3.  $x^2 + 4y^2 = 4$ .  
 4.  $y^3 + 3x^2 + 16 = 0$ .  
 5.  $r = 2$ .  
 6.  $r = 4p \cot \theta \csc \theta$ .  
 7.  $2x - 3y = 0$ .  
 8.  $\frac{(x-a)^2}{a^2} + \frac{y^2}{b^2} = 1$ .  
 9.  $(x^2 + 4a^2)y + 2ax^2 = 0$ .  
 10.  $xy^2 = (x-a)^2(2a-x)$ .  
 11.  $x = a(\phi' + \sin \phi')$ ,  
 $y = a(\cos \phi' - 1)$ , where  
 $\phi' = \phi - \pi$ .  
 12.  $x = \frac{a}{2}(2 \cos \theta + \cos 2\theta)$ ,  
 $y = \frac{a}{2}(2 \sin \theta + \sin 2\theta)$ .  
 13.  $x + y = 0$ ,  $2x - 3y = 0$ .  
 14.  $x^2 + y^2 = 11$ .  
 20.  $3x^2 + y^2 = 2$ .  
 21.  $r^2 \cos 2\theta = 2$ .  
 22.  $x^2 - y^2 = 8$ .  
 23.  $\frac{x^2}{4(\sqrt{2}-1)} - \frac{y^2}{4(\sqrt{2}+1)} = 1$ .

## Page 154.

4. Distance from the  $x$ -axis  $\sqrt{y^2 + z^2}$ , distance from the origin  $\sqrt{x^2 + y^2 + z^2}$ .  
 5. In the  $xy$ -plane  $(1, 2, 0)$ , on the  $x$ -axis  $(1, 0, 0)$ .  
 6. The projections on the  $y$ -axis are  $1, -2, 1$ .  
 7.  $76^\circ 22'$ ,  $76^\circ 22'$ ,  $19^\circ 28'$ .  
 8.  $5, -4$ , and  $3$ .  
 9.  $(\frac{1}{2}, -\frac{1}{2}, \frac{3}{2})$ .

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3.  $(1, 2, 3)$ .  
 4.  $(\frac{1}{3}, 3, -2)$ ,  $(\frac{1}{2}, -\frac{1}{4}, \frac{1}{4})$ .  
 5. The projection on the  $xy$ -plane is  $[2, 2, 0]$ .  
 7.  $(5, 2, 5)$ .

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3.  $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$ .  
 4.  $45^\circ$  and  $135^\circ$ .  
 5.  $\frac{1}{2}\sqrt{2}, \frac{1}{2}\sqrt{2}, 0$ .  
 6.  $54^\circ 44'$ .  
 7.  $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ .  
 8.  $70^\circ 32'$ ,  $48^\circ 12'$ ,  $48^\circ 12'$ .  
 9.  $71^\circ 34'$ ,  $71^\circ 34'$ ,  $36^\circ 52'$ .  
 11.  $56^\circ 1'$ .

## Pages 166, 167.

1.  $\cos \alpha = \frac{1}{\pm \sqrt{21}}$ ,  $\cos \beta = \frac{2}{\pm \sqrt{21}}$ ,  $\cos \gamma = \frac{4}{\pm \sqrt{21}}$ .

The positive square roots correspond to one direction along the normal, the negative to the other. If a particular direction is desired, the proper sign is easily determined. In the following answers only one set of cosines is given.

2.  $\cos \alpha = \frac{1}{\sqrt{11}}$ ,  $\cos \beta = -\frac{1}{\sqrt{11}}$ ,  
 $\cos \gamma = \frac{3}{\sqrt{11}}$ .
5.  $\cos \alpha = \frac{2}{3}$ ,  $\cos \beta = 0$ ,  $\cos \gamma = \frac{1}{3}$ .  
6.  $\cos \alpha = \cos \beta = 0$ ,  $\cos \gamma = 1$ .
7.  $x + y\sqrt{2} + z = 0$ .  
8.  $3x - 5y + 4z + 2 = 0$ .
3.  $\cos \alpha = \cos \beta = \cos \gamma = \frac{1}{3}\sqrt{3}$ .  
4.  $\cos \alpha = \frac{2}{\sqrt{13}}$ ,  $\cos \beta = -\frac{3}{\sqrt{13}}$ ,  
 $\cos \gamma = 0$ .
9.  $\frac{x}{1} + \frac{y}{2} + \frac{z}{3} = 1$ .  
12.  $70^\circ 32'$ .  
13.  $29^\circ 40'$ .

## Page 170.

17.  $x^2 + y^2 + z^2 = a^2$ ,  $r^2 + z^2 = a^2$ ,  
 $\rho = a$ .  
18.  $x^2 + y^2 + z^2 = 2az$ ,  
 $r^2 + z^2 = 2az$ ,  $\rho = 2a \cos \phi$ .  
19.  $x^2 + y^2 = a^2$ ,  $r = a$ ,  $\rho = a \csc \phi$ .  
20.  $x^2 + y^2 = z^2$ ,  $r = z$ ,  $\phi = \frac{\pi}{4}$ .  
21.  $y^2 + z^2 = 2az$ .  
22.  $x^2 = az$ .  
23.  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .
24.  $\frac{x^2}{a^2} + \frac{y^2 + z^2}{b^2} = 1$ .  
25.  $\frac{x^2}{a^2} - \frac{y^2 + z^2}{b^2} = 1$ , or  
 $\frac{x^2 + z^2}{a^2} - \frac{y^2}{b^2} = 1$ .  
26.  $y^2 + z^2 = ax$ .  
27.  $(\sqrt{x^2 + z^2} - b)^2 + y^2 = a^2$ .  
A circle with center on the  
 $x$ -axis is rotated about the  
 $y$ -axis.

## Pages 177, 178.

1. There are two sets of direction cosines differing in algebraic sign.  
One set is:

- $\cos \alpha = -\frac{2}{3}$ ,  $\cos \beta = \frac{2}{3}$ ,  $\cos \gamma = \frac{1}{3}$ .
2.  $\cos \alpha = \frac{1}{\sqrt{6}}$ ,  $\cos \beta = -\frac{1}{\sqrt{6}}$ ,  
 $\cos \gamma = \frac{2}{\sqrt{6}}$ .
4.  $\cos \alpha = \cos \beta = 0$ ,  $\cos \gamma = 1$ .  
5.  $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z+1}{3}$ .
3.  $\cos \alpha = -\frac{7}{\sqrt{78}}$ ,  $\cos \beta = \frac{2}{\sqrt{78}}$ ,  
 $\cos \gamma = \frac{5}{\sqrt{78}}$ .
6.  $\frac{x}{3} = \frac{y-1}{1} = \frac{z-2}{5}$ .  
7.  $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{-1}$ .
8.  $60^\circ$ .  
9.  $58^\circ 31'$ .

## Page 180.

1. The projection on the  $xy$ -plane is

$$x^2 + y^2 + xy - x - y = 0, z = 0.$$

2. The projection on the  $yz$ -plane is

$$z = \pm a, x = 0.$$

3. The projection on the  $xz$ -plane is  
 $z^2 = 2x^2 - 2x + 1, y = 0$ .
11.  $(-1, 1, 2)$ ,  $(-\frac{1}{3}, \frac{1}{3}, \frac{4}{3})$ .

## Pages 181, 182.

1. The projection on the  $xy$ -plane is  $x + y = 3, z = 0$ .  
2. The projection on the  $yz$ -plane is  $y = \sin(\frac{1}{2}z), x = 0$ .  
3. The projection on the  $xz$ -plane is  $x = z \sin z, y = 0$ .  
5.  $x = a\theta \cos \theta, y = a\theta \sin \theta, z = k\theta$ .  
6.  $z = k\theta$ .  
7.  $y = x\sqrt{2}, z = x - .000064x^2, z = .7071y - .000032y^2$ .