CHAPTER 3

STRAIGHT LINE AND CIRCLE

Art. 23. Equation of a Straight Line

Let $P_1\ (x_1, y_1)$ be a fixed point and $P\ (x, y)$ a variable point on the line MN. If the line is not perpendicular to the x-axis (Fig. 23a), let its slope be m. Since the line passes through P_1 and P, by the definition of slope,

$$m = \frac{y - y_1}{x - x_1}.$$

This is an equation satisfied by the coördinates, x and y, of any point P on the line. Conversely, if the coördinates of any point

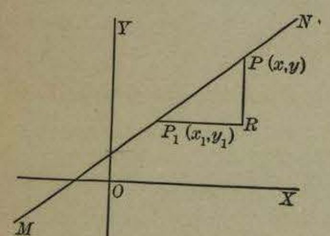

FIG. 23a.

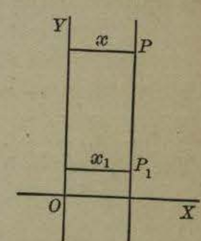

FIG. 23b.

$P\ (x, y)$ satisfy this equation, the slope of P_1P is m and consequently P lies on MN. It is therefore the equation of MN. It can be written

$$y - y_1 = m\,(x - x_1). \qquad (23a)$$

This is consequently the equation of a line through (x_1, y_1) *with slope* m.

Let the line MN cross the y-axis at $(0, b)$. Replacing x_1, y_1 in equation (23a) by 0, b, the equation becomes

$$y = mx + b. \qquad (23b)$$

The quantity b is called the intercept of the line on the y-axis. It is

numerically equal to the distance along the y-axis to the line, is positive when the line is above the origin and negative when it is below. Equation (23b) represents the line with slope m and intercept b on the y-axis.

If the line is perpendicular to the x-axis (Fig. 23b) its slope is infinite and equations (23a) and (23b) cannot be used. In this case the figure shows that

$$x = x_1. \qquad (23c)$$

Conversely, if the abscissa of a point is x_1, it lies on the line. Therefore (23c) is the equation of a line through (x_1, y_1) perpendicular to the x-axis.

Example 1. Find the equation of a line through $(-1, 2)$, the angle from the x-axis to the line being $30°$.

The slope of the line is

$$m = \tan\,(30°) = \tfrac{1}{3}\sqrt{3}.$$

The equation of the line is then

$$y - 2 = \tfrac{1}{3}\sqrt{3}\,(x + 1).$$

Ex. 2. Find the equation of the line through the points $(2, 0)$ and $(1, -3)$.

The slope of the line is

$$\frac{-3 - 0}{1 - 2} = +3.$$

Since the line passes through $(2, 0)$ and has a slope equal to 3, its equation is $y - 0 = 3\,(x - 2)$, whence

$$y = 3x - 6.$$

Ex. 3. Find the equation of the line through $(1, -1)$ perpendicular to the line through $(2, 3)$ and $(3, -2)$.

The slope of the line through $(2, 3)$ and $(3, -2)$ is -5. The slope of a perpendicular line is $-1/(-5) = \frac{1}{5}$. The equation of the line with this slope passing through $(1, -1)$ is

$$y + 1 = \tfrac{1}{5}\,(x - 1),$$

which is the equation required.

Ex. 4. Show that the equation $2x - 3y = 5$ represents a straight line. Find its slope.

Solving for y,

$$y = \tfrac{2}{3}x - \tfrac{5}{3}.$$

Comparing this with the equation $y = mx + b$, it is seen that the two are equivalent if $m = \frac{2}{3}$, $b = -\frac{5}{3}$. Therefore the given equation represents a straight line with slope $\frac{2}{3}$ and intercept $-\frac{5}{3}$ on the y-axis.

Exercises

1. The angle from the x-axis to a line is 60°. The line passes through $(-1, -3)$. Find its equation.
2. Find the equation of the line through $(2, -1)$ and $(3, 2)$.
3. Find the equation of the line through $(2, 3)$ and $(2, -4)$.
4. Find the equation of the line through $(1, 2)$ parallel to the x-axis.
5. Find the equation of the perpendicular bisector of the segment joining $(-3, 5)$ and $(-4, 1)$.
6. An equilateral triangle has its base in the x-axis and its vertex at $(3, 5)$. Find the equations of its sides.
7. A line is perpendicular to the segment between $(-4, -2)$ and $(2, -6)$ at the point one-third of the way from the first to the second point. Find its equation.
8. Find the equation of the line through $(3, 5)$ parallel to that through $(2, 5)$ and $(-5, -2)$.
9. One diagonal of a parallelogram joins the points $(4, -2)$ and $(-4, -4)$. One end of the other diagonal is $(1, 2)$. Find its equation and length.
10. A diagonal of a square joins the points $(1, 2)$, $(2, 5)$. Find the equations of the sides of the square.
11. The base of an isosceles triangle is the segment joining $(-2, 3)$ and $(3, -1)$. Its vertex is on the y-axis. Find the equations of its sides.
12. Show that the equation $2x - y = 3$ represents a straight line. Find its slope and construct the line.
13. Show that the equations

$$2x + 3y = 5, \quad 3x - 2y = 7$$

represent two perpendicular straight lines.

14. Perpendiculars are dropped from the point $(5, 0)$ upon the sides of the triangle whose vertices are the points $(4, 3)$, $(-4, 3)$, $(0, -5)$. Show that the feet of the perpendiculars lie on a line.

Art. 24. First Degree Equation

Any straight line is represented by an equation of the first degree in rectangular coördinates. In fact, if the line is not perpendicular to the x-axis, its equation has been shown to be

$$y - y_1 = m(x - x_1),$$

x_1, y_1 and m being constants. If it is perpendicular to the x-axis its equation is

$$x = x_1.$$

Since both of these equations are of the first degree in x and y, it follows that any straight line has an equation of the first degree in rectangular coördinates.

Conversely, *any equation of the first degree in rectangular coördinates represents a straight line.* For any equation of the first degree in x and y has the form

$$Ax + By + C = 0, \tag{24}$$

A, B, C being constant. If B is not zero, this equation is equivalent to

$$y = -\frac{A}{B}x - \frac{C}{B},$$

which represents a line with slope $-A/B$ and intercept $-C/B$ on the y-axis. If B is zero, the equation is equivalent to

$$x = -\frac{C}{A},$$

which represents a line perpendicular to the x-axis passing through the point $(-C/A, 0)$. Hence in any case an equation of the first degree represents a straight line.

Graph of First Degree Equation. — Since a first degree equation represents a straight line, its graph can be constructed by finding two points and drawing the straight line through them. The best points for this purpose are usually the intersections of the line and coördinate axes. The intersection A (Fig. 24*a*) with the x-axis is found by letting $y = 0$ and solving the equation of the line for the corresponding value of x. Similarly, the intersection B with the y-axis is found by letting $x = 0$ and solving for y. The abscissa of

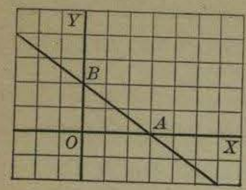

FIG. 24*a*.

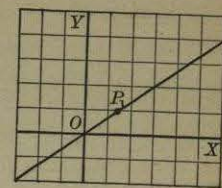

FIG. 24*b*.

A and the ordinate of B are called the *intercepts* of the line on the coördinate axes.

If the line passes through the origin (Fig. 24*b*) the points A and B coincide at the origin and it is necessary to find another point on the line. This is done by assigning any value to one of the coördinates and calculating the resulting value of the other coördinate.

Example 1. Plot the line $2x + 3y = 6$ and find its intercepts on the axes.

Substituting $y = 0$ gives $x = 3$ and substituting $x = 0$ gives $y = 2$. Hence the line passes through the points A (3, 0) and B (0, 2). Its intercept on the x-axis is 3 and on the y-axis 2.

Ex. 2. Construct the line whose equation is $2x - 3y = 0$.

When x is zero y is zero. The line then passes through the origin. When $y = 1$, $x = \frac{3}{2}$. Hence the line OP_1 (Fig. 24*b*) through the origin and the point $(\frac{3}{2}, 1)$ is the one required.

Slope of a Line. — If the line is perpendicular to the x-axis, its slope is infinite. If it is not perpendicular to the x-axis, its equation is

$$y = mx + b.$$

The slope is the coefficient of x in this equation. Consequently, if the equation of a line is solved for y, its slope is the coefficient of x.

Example 1. Find the slope of the line $3x - 5y = 7$.

Solving for y

$$y = \tfrac{3}{5}x - \tfrac{7}{5}.$$

The slope of the line is therefore $\frac{3}{5}$.

Ex. 2. Find the angle from the line $x + y = 3$ to the line $y = 2x + 5$.

The slope of the first line is -1, that of the second 2. The angle β between the lines is determined by the equation

$$\tan \beta = \frac{2 - (-1)}{1 + 2(-1)} = -3.$$

The negative sign signifies that the angle is negative or obtuse.

Ex. 3. Find the equation of the line through (3, 1) perpendicular to the line $2x + 4y = 5$.

The given equation can be written $y = -\frac{1}{2}x + \frac{5}{4}$. Its slope is consequently $-\frac{1}{2}$. The slope of a perpendicular line is 2. The equation of the line through (3, 1) with slope 2 is

$$y - 1 = 2(x - 3),$$

which is the equation required.

Exercises

Plot the straight lines represented by the following equations, find their slopes and intercepts:

1. $2y - 3 = 0$.
2. $5x + 7 = 0$.
3. $x + y = 2$.
4. $2x + 3y - 5 = 0$.
5. $3x - 6y + 7 = 0$.
6. $2x + 5y + 8 = 0$.
7. $4x + 3y = 0$.
8. $3x - 4y = 0$.

9. Show that the equation

$$(2x + 3y - 1)(x - 7y + 2) = 0$$

represents a pair of lines.

10. Show that $(x + 4y)^2 = 9$ represents two parallel lines.

11. Show that $x^2 = (y - 1)^2$ represents two lines perpendicular to each other.

12. Show that the lines $3x + 4y - 7 = 0$, $9x + 12y - 8 = 0$ are parallel.

13. Show that the lines $x + 2y + 5 = 0$, $4x - 2y - 7 = 0$ are perpendicular to each other.

14. Find the interior angles of the triangle formed by the lines

$$x = 0, \quad x - y + 2 = 0, \quad 2x + 3y - 21 = 0.$$

15. Find the equation of the line whose intercepts on the x and y axes are 2 and -3 respectively.

16. Find the equation of the line whose slope is 5 and intercept on the y-axis -4.

17. Find the equation of the line through $(3, -1)$ parallel to the line $x - y = 8$.

18. Find the equation of the line through $(2, -1)$ perpendicular to the line $9x - 8y + 6 = 0$.

19. Find the projection of the point $(2, -3)$ on the line $x - 4y = 5$.

20. Find the equation of the line perpendicular to $3x - 5y = 9$ and bisecting the segment joining $(-1, 2)$ and $(4, 5)$.

21. Find the lengths of the sides of the triangle formed by the lines $4x + 3y - 1 = 0$, $3x - y - 4 = 0$, $x + 4y - 10 = 0$.

22. A line passes through $(2, 2)$. Find its equation if the angle from it to $3x - 2y = 0$ is 45°.

23. Find the equation of the line through $(4, \frac{5}{3})$ and the intersection of the lines $3x - 4y - 2 = 0$, $12x - 15y - 8 = 0$.

24. Find the equation of the line through the intersection of the lines $2x - y + 5 = 0$, $x + y + 1 = 0$, and the intersection of the lines $x - y + 7 = 0$, $2x + y - 5 = 0$.

25. Find the locus of a point if the tangents from it to two fixed circles are of equal length.

26. What angle is made with the axis of y by a straight line whose equation is $\frac{1}{4}y + \frac{1}{3}x = 1$?

Art. 25. The Expression $Ax + By + C$

At each point P of the plane a first degree expression $Ax + By + C$ has a definite value obtained by putting for x and y the coördinates of P. Thus at the point $(1, 2)$ the expression has the value $A + 2B + C$. Points where the expression is zero constitute a line whose equation is $Ax + By + C = 0$. If the point P moves slowly the value of the expression changes continuously. A number changing continuously can only change sign by passing through zero. If the point P does not cross the line the expression cannot become zero and so cannot change sign. Therefore *at all points on one side of the line $Ax + By + C = 0$ the expression $Ax + By + C$ has the same sign.*

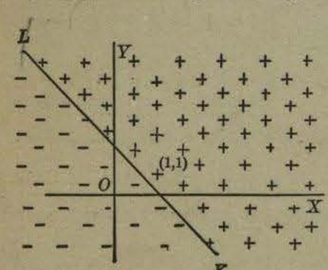

FIG. 25a.

Example 1. Determine the region in which $x + y - 1 > 0$.

The equation $x + y - 1 = 0$ represents the line LK (Fig. 25a).

At all points on one side of LK the expression then has the same sign. At $(1, 1)$

$$x + y - 1 = 1 + 1 - 1 = 1$$

which is positive. It is seen from the figure that $(1, 1)$ is above LK. Hence, at all points above LK, $x + y - 1$ is positive. At the origin

$$x + y - 1 = 0 + 0 - 1 = -1$$

which is negative. The origin is below the line. Hence at all points below LK the expression $x + y - 1$ is negative. The region in which $x + y - 1 > 0$ is therefore the part of the plane above the line.

Ex. 2. Determine the region in which $x + y > 0$, $x + 2y - 2 < 0$ and $x - y - 1 < 0$.

In Fig. 25b the lines $x + y = 0$, $x + 2y - 2 = 0$, $x - y - 1 = 0$ are marked (1), (2), (3) respectively. Proceeding as in the last example, it is found that $x + y > 0$ above (1), $x + 2y - 2 < 0$ below (2) and $x - y - 1 < 0$ on the left of (3). Hence the three inequalities hold in the shaded triangle which is the part common to the three regions.

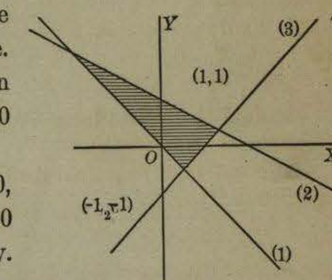

FIG. 25b.

Art. 26. Distance from a Point to a Line

We wish to find the distance from the point $P_1 (x_1, y_1)$ to the line LK whose equation is $Ax + By + C = 0$. In Fig. 26a let MP_1 be perpendicular to the x-axis and DP_1 to the line LK. Let ϕ be the angle from OX to LK. Then

(a) $DP_1 = QP_1 \cos\phi$
$= (MP_1 - MQ) \cos\phi.$

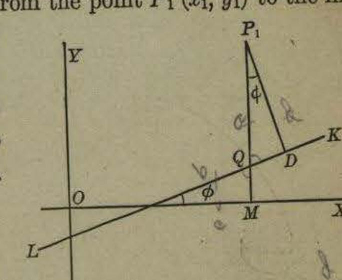

FIG. 26a.

From the figure it is seen that

(b) $$MP_1 = y_1.$$

Since Q is on the line LK, its coördinates, x_1 and MQ, must satisfy the equation of LK. Therefore

$$Ax_1 + B \cdot MQ + C = 0,$$

and consequently

$$(c) \qquad MQ = -\frac{Ax_1 + C}{B}.$$

$DP' = (MP_1 - MQ) \cos \phi$

The slope of LK is $\tan \phi = -A/B$, whence

$$(d) \qquad \cos \phi = \frac{B}{\pm\sqrt{A^2 + B^2}}.$$

Substituting the values from (b), (c), (d) in (a),

$$DP_1 = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}. \qquad (26)$$

Equation (26) *gives the distance from the point* (x_1, y_1) *to the line whose equation is* $Ax + By + C = 0$. The distance being positive, such a sign must be used in the denominator that the result is positive.

Example 1. Find the distance from the point (1, 2) to the line $2x - 3y = 6$.

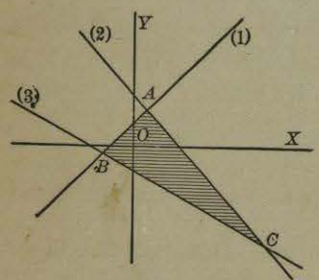

FIG. 26b.

The distance from any point (x_1, y_1) to the line is by (26)

$$\frac{2x_1 - 3y_1 - 6}{\pm\sqrt{13}}.$$

The distance from (1, 2) is then

$$\frac{2(1) - 3(2) - 6}{\pm\sqrt{13}} = \frac{10}{\sqrt{13}}.$$

Ex. 2. The lines (1) $y - x - 1 = 0$, (2) $x + y - 2 = 0$, (3) $x + 2y + 2 = 0$ determine a triangle ABC. Find the bisector of the angle A between the lines (1) and (2) (Fig. 26b).

The bisector is a locus of points equidistant from the lines (1) and (2). If (x, y) is any point of the bisector, x and y must then satisfy the equation

$$\frac{y - x - 1}{\pm\sqrt{2}} = \frac{x + y - 2}{\pm\sqrt{2}}.$$

The signs must be so chosen that these expressions are positive at points inside the triangle. At the origin these expressions become $-1/(\pm\sqrt{2})$, $-2/(\pm\sqrt{2})$. Hence the negative sign must be used in both denominators. The bisector required is therefore

$$\frac{y - x - 1}{-\sqrt{2}} = \frac{x + y - 2}{-\sqrt{2}}.$$

When simplified this becomes $x = \frac{1}{2}$.

Exercises

Determine the region occupied by points satisfying the inequalities in each of the following cases:

1. $2x + 3y - 6 > 0$.
2. $x < 3y$.
3. $x - y - 1 > 0$,
   $y - 2x > 0$.
4. $2x - y - 2 > 0$,
   $3x + 4y - 12 < 0$,
   $2y - 1 > 0$.
5. $y - 2x > 1$,
   $y - 2x < 3$,
   $2y + x > 1$,
   $2y + x < 3$.
6. $(x + 2y - 3)(2x - y + 3) > 0$.
7. $(x + 4y)^2 > 9$.

8. Inside the triangle determined by the lines $x + y = 0$, $2x - 3y - 1 = 0$, $y - 2 = 0$, what algebraic signs have the expressions $x + y$, $2x - 3y - 1$, $y - 2$?

9. Express by inequalities the inside of the triangle determined by the points (1, 1), (3, 4), (2, −2).

10. Find the distance from the point (3, 5) to the line $y = 4x - 8$.

11. Find the distance from (6, −2) to the line through (−1, 3) and (5, −1).

12. Find the distance between the two parallel lines, $4x + 3y - 10 = 0$ and $4x + 3y - 8 = 0$.

13. Find the equation of the bisector of the acute angle between the lines $2x - y = 1$, $x + 3y = 2$.

14. A triangle is formed by the lines $3x - 4y = 5$, $4x + 3y = 5$, $5x + 12y = 13$. Find the center of the inscribed circle.

15. Find the locus of a point whose distance from (2, 3) is equal to its distance from the line $x + 2y = 3$.

16. The sum of the distances from the point $P(x, y)$ to the lines $y = x$, $x + y = 4$, $y + 2 = 0$, is 4. Find an equation satisfied by the coördinates of P. Do all points whose coördinates satisfy this equation have the required property?

Art. 27. Equation of a Circle

A circle is the locus of a point at constant distance from a fixed point. The fixed point is the center of the circle and the constant distance is its radius.

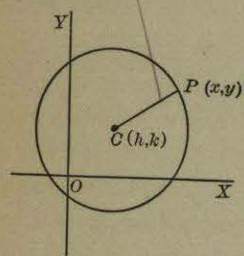

FIG. 27.

Let $C(h, k)$, Fig. 27, be the center of the circle and r its radius. If $P(x, y)$ is any point on the circle

$$r = CP = \sqrt{(x-h)^2 + (y-k)^2},$$

$$\text{or}\quad (x-h)^2 + (y-k)^2 = r^2. \qquad (27a)$$

This is an equation satisfied by the coördinates of any point on the circle. Conversely, if the coördinates x, y satisfy this equation, the point P is at the distance r from the center and consequently lies on the circle. Therefore it is the equation of the circle.

Example 1. Find the equation of the circle with center $(-2, 1)$ and radius 3. In this case, $h = -2$, $k = 1$, $r = 3$.

By $(27a)$ the equation of the circle is then

$$(x+2)^2 + (y-1)^2 = 9.$$

Ex. 2. Find the equation of the circle with center $(1, 1)$ which passes through the point $(3, -4)$.

The radius is the distance from the center to the point $(3, -4)$. Consequently

$$r = \sqrt{(3-1)^2 + (-4-1)^2} = \sqrt{29}.$$

The equation of the circle is therefore

$$(x-1)^2 + (y-1)^2 = 29.$$

Form of the Equation. — Expanding $(27a)$,

$$x^2 + y^2 - 2hx - 2ky + h^2 + k^2 - r^2 = 0.$$

This is an equation of the form

$$A(x^2 + y^2) + Bx + Cy + D = 0, \qquad (27b)$$

in which A, B, C, D are constant.

Conversely, if A is not zero, an equation of the form $(27b)$ represents a circle, if it represents any curve at all. To show this divide by A and complete the squares of the terms containing x and those containing y separately. The result is

$$\left(x + \frac{B}{2A}\right)^2 + \left(y + \frac{C}{2A}\right)^2 = \frac{B^2 + C^2 - 4AD}{4A^2}.$$

If the right side of this equation is positive, it represents a circle with center $(-B/2A, -C/2A)$ and radius $\sqrt{B^2 + C^2 - 4AD}/2A$. If the right side is zero, the radius is zero and the circle shrinks to a point. If the right side is negative, since the sum of squares of real numbers is positive, there is no real locus.

Example 1. Find the center and radius of the circle

$$2x^2 + 2y^2 - 3x + 4y = 1.$$

Dividing by 2 and completing the squares,

$$(x - \tfrac{3}{4})^2 + (y+1)^2 = \tfrac{33}{16}.$$

The center of the circle is $(\frac{3}{4}, -1)$ and its radius is $\frac{1}{4}\sqrt{33}$.

Ex. 2. Determine the locus of

$$x^2 + y^2 + 4x - 6y + 13 = 0.$$

Completing the squares,

$$(x+2)^2 + (y-3)^2 = 0.$$

The sum of two squares can only be zero when both are zero. The only real point on this locus is then $(-2, 3)$. The circle shrinks to a point.

Ex. 3. Discuss the locus of

$$x^2 + y^2 + 2y + 3 = 0.$$

Completing the squares,

$$x^2 + (y+1)^2 = -2.$$

There are no real values for which this is true. The locus is imaginary.

Art. 28. Circle Determined by Three Conditions

When a circle is given, h, k, r, in equation (27*a*), have definite values. Conversely, if values are assigned to h, k and r, a circle is determined. This is expressed by saying that the circle is determined by three independent constants. Any limitation on the circle, such as requiring it to pass through a point or be tangent to a line, can be expressed by an equation or equations connecting h, k and r. Now three numbers are determined by three independent equations. This is expressed by saying a circle can be found satisfying three independent conditions. Thus a circle can be passed through three points, can touch three lines, etc. Many problems consist in finding a circle doing three things. To find the equation of such a circle, express the three conditions by equations connecting h, k and r, solve and substitute the values in equation (27*a*).

Example 1. A circle is tangent to the x-axis, passes through (1, 1) and has its center on the line $y = x - 1$. Find its equation.

The circle is shown in Fig. 28*a*. Since the circle is tangent to the x-axis, $k = \pm r$. Since it passes through (1, 1), the circle lies above the x-axis and k is positive. Hence

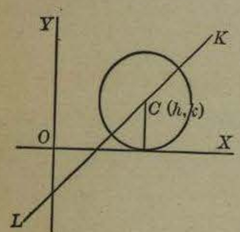

FIG. 28*a*.

(*a*) $$k = r.$$

Since the center is on the line $y = x - 1$, its coördinates must satisfy that equation. Consequently

(*b*) $$k = h - 1.$$

Since the circle passes through (1, 1),

(*c*) $$(1 - h)^2 + (1 - k)^2 = r^2.$$

The solution of (*a*), (*b*) and (*c*) is $h = 2$, $k = 1$, $r = 1$. The equation required is then

$$(x - 2)^2 + (y - 1)^2 = 1.$$

Ex. 2. Find the equation of the circle through (2, 3), (4, 1) and tangent to the line $4x - 3y = 15$ (Fig. 28*b*).

Since the circle is tangent to the line its center is at a distance r

from the line. The distance from the center (h, k) to $4x - 3y = 15$ is by equation (26)

(*a*) $$\frac{4h - 3k - 15}{\pm 5}.$$

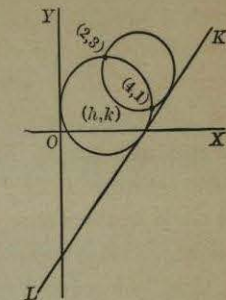

FIG. 28*b*.

Since the circle passes through (2, 3) its center is on the same side of the line as (2, 3). To make (*a*) positive at the center and therefore at (2, 3), the negative sign must be used in the denominator of (*a*). The condition of tangency is then

(*b*) $$\frac{4h - 3k - 15}{-5} = r.$$

Since the circle passes through (2, 3) and (4, 1),

(*c*) $$(2 - h)^2 + (3 - k)^2 = r^2,$$

(*d*) $$(4 - h)^2 + (1 - k)^2 = r^2.$$

Solving equations (*b*), (*c*) and (*d*) simultaneously, we get

$$h = 2, \quad k = 1, \quad r = 2 \text{ and } h = \tfrac{178}{49}, \quad k = \tfrac{129}{49}, \quad r = \tfrac{82}{49}.$$

These are consequently two circles satisfying the given conditions. Their equations are

$$(x - 2)^2 + (y - 1)^2 = 4,$$
$$(x - \tfrac{178}{49})^2 + (y - \tfrac{129}{49})^2 = (\tfrac{82}{49})^2.$$

Ex. 3. Find the circle through the three points (0, 1), (−1, −1) and (2, 0).

The coördinates of the three points must satisfy the equation of the circle. Hence

$$(0 - h)^2 + (1 - k)^2 = r^2,$$
$$(-1 - h)^2 + (-1 - k)^2 = r^2,$$
$$(2 - h)^2 + (0 - k)^2 = r^2.$$

Subtracting the first and third of these equations from the second we get

$$2h + 4k + 1 = 0,$$
$$6h + 2k - 2 = 0.$$

The solution of these equations is $h = \frac{1}{2}$, $k = -\frac{1}{2}$. These values substituted in either of the original equations give $r^2 = \frac{10}{4}$. The circle required is therefore

$$(x - \tfrac{1}{2})^2 + (y + \tfrac{1}{2})^2 = \tfrac{10}{4}.$$

Exercises

Find the center and radius of each of the following circles. Draw the curves.

1. $x^2 + y^2 = 25$.
2. $x^2 + y^2 = 4x$.
3. $2x^2 + 2y^2 - 6y + 1 = 0$.
4. $3x^2 + 3y^2 + 4x = 1$.
5. $x^2 + y^2 = x + y$.
6. $x^2 - 2ax + y^2 - 2ay = 0$.
7. $x^2 + y^2 - 2x + 4y + 2 = 0$.
8. $x^3 + y^2 - 2x + 1 = 0$.

9. What locus is represented by the equation $x^2 + y^2 + 2x + 2y + 2 = 0$?

10. What is the locus of the equation $x^2 + y^2 - 6x + 6y + 9 = 0$?

11. Find the equation of the circle through $(-2, 4)$ and having the same center as the circle $x^2 + y^2 - 5x + 4y - 1 = 0$.

12. Find the equation of the circle whose diameter is the segment joining $(-1, -2)$ and $(3, 4)$.

13. Find the equations of the circles through $(1, 2)$ and tangent to both coördinate axes.

14. Find the equations of the circles with centers at the origin and tangent to the circle $x^2 + y^2 - 4x + 4y + 7 = 0$.

15. Find the intersections of the circles

$$x^2 + y^2 = 2x + 2y,$$
$$x^2 + y^2 + 2x = 4.$$

16. Find the equation of the circle through the three points $(0, 3)$, $(3, 0)$ and $(0, 0)$.

17. Find the circles of radius 5 passing through the points $(2, -1)$ and $(3, -2)$.

18. The center of a circle passing through $(1, -2)$ and $(-2, 2)$ is on the line $8x - 4y + 9 = 0$. What is its equation?

19. A circle passes through the points $(0, 0)$, $(2, -2)$ and is tangent to the line $y + 4 = 0$. Find its equation.

20. A circle passes through the points $(-1, 0)$, $(0, 1)$ and is tangent to the line $x - y = 1$. Find its equation.

21. A circle is tangent to the lines $x = 3$, $x = 7$, and its center is on the line $y = 2x + 4$. What is its equation?

22. Find the equation of the circle circumscribed about the triangle formed by the three lines $x + y - 2 = 0$, $9x + 5y - 2 = 0$, $y + 2x - 1 = 0$.

23. Find the equation of the circle inscribed in the triangle formed by the lines $x + y = 1$, $y - x = 1$, $x - 2y = 1$.

24. Find the locus of points from which the tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 2x + 4y = 4$ are of equal length.

25. By subtracting the equation $x^2 + y^2 - 2x - 2y = 0$ from the equation $x^2 + y^2 + 2x - 6y + 2 = 0$ the equation of a line is obtained. Show that this line is the common chord of the two circles.

26. A point moves so that the sum of its distances from two vertices of an equilateral triangle is equal to its distance from the third vertex. Find an equation satisfied by its coördinates. Do all points whose coördinates satisfy this equation have the required property?