Prob. 201*c*. What is the efficiency of a bucket pump which lifts 2000 liters of water per minute through a height of 3.5 meters with an expenditure of 2.5 metric horse-powers?

Prob. 201*d*. When the height of the mercury barometer is 760 millimeters, water at a temperature of 0° centigrade is raised by suction in a perfect vacuum to a height of 10.33 meters (Art. 193). Under the same atmospheric pressure, how high can it be raised when the temperature is 32° centigrade?

Prob. 201*e*. What metric horse-power is required to raise 4 000 000 liters per day through a height of 75 meters when the diameter of the pipe is 20 centimeters and its length 500 meters?

Prob. 201*f*. The calorie is the metric thermal unit, this being the energy required to raise one kilogram of water one degree centigrade when the temperature of the water is near that of maximum density. How many calories are equivalent to 1 000 000 British thermal units?

APPENDIX

Art. 202. Hydraulic-Electric Analogies

It is well known that there are certain analogies between the flow of water in pipes and that of the electric current in wires, and some of these will here be briefly explained from a hydraulic point of view. The electric analog of a water pump is the dynamo, both being driven by mechanical power and both transforming it into other forms of energy. The analog of a water wheel is the electric motor, each of which delivers mechanical power by virtue of the energy transmitted to it through the water pipe or electric wire. While the water is flowing from the pump to the wheel much of its energy is lost in overcoming frictional resistances, whereby heat is produced; while the electricity is flowing from the dynamo to the electric motor some of its energy is lost in overcoming molecular resistances, whereby heat is produced. The steady flow of water corresponds to the continuous flow of electricity in one direction, or to the direct current, and the following discussion compares hydraulic phenomena with those of the direct electric current. The phenomena of the alternating current have also certain hydraulic analogies in the flow of water, but these will not be discussed here.

Let q represent electric current, R the electric resistance of a wire of length l, cross-section a, and diameter d, and p the electromotive force under which the current is pushed through the wire. Then Ohm's law gives, if s is the specific resistance of the material of the wire,

$$p = Rq = s\frac{l}{a}q = A\frac{l}{d^2}q \qquad (202)_1$$

in which A is a constant depending only on the material of the wire. This equation shows that the electric pressure p varies directly with the length of the wire, inversely as the square of its diameter, and directly as the current. By increasing the length of the wire or by decreasing its diameter, the electromotive force required to maintain a given electric current is increased. Similarly in a water pipe the friction-head required to maintain a given discharge increases directly as the length of the pipe, and is greater for a small pipe than for a large one (Art. 90).

In Art. 105 it was pointed out that the distribution of water flow among several diversions of a pipe follows laws analogous to those of the electric current. It was there shown that the discharge q divides between the diversions inversely as their resistances, provided $\sqrt{fl/d^5}$ be taken as the measure of resistance. In electric flow the direct current is the analog of the discharge in the water pipe, but Ohm's law shows that the resistance is the simpler quantity $f'l/d^2$. The hydraulic analog of electro-motive force is often taken to be the lost friction-head or its corresponding unit pressure, and this will be followed here. The loss in water pressure is represented by the hydraulic gradient (Art. 99), and the loss in electric pressure is often represented in a similar way, the gradient being a straight line in both cases.

In order to make an algebraic comparison of the two phenomena, take the expression for friction-head in (90) and replace h'' by p/w, where p is the loss of unit pressure in the length l, and w is the weight of a cubic unit of water; also replace v by q/a, and a by $\frac{1}{4}\pi d^2$. Then formula (90) becomes

$$p = \frac{8fw}{g}\frac{l}{d^5}q^2 = B\frac{l}{d^5}q^2 \qquad (202)_2$$

in which the constant B depends upon the roughness of the surface and the force of gravity. Accordingly the lost pressure varies directly as the length of the pipe, inversely as the fifth power of its diameter, and directly as the square of the discharge.

Thus, in the case of a single water pipe or electric wire,

$$\text{for electric flow} \quad p = A\frac{l}{d^2}q$$

$$\text{for hydraulic flow} \quad p = B\frac{l}{d^5}q^2$$

If each of these flows be divided among n diversions, as in Fig. 201, the expressions for the pressure become

$$\text{for electric flow} \quad p = \frac{Al}{nd^2}q$$

$$\text{for hydraulic flow} \quad p = \frac{Bl}{n^2d^5}q^2$$

so that the drop of the gradient is far more rapid in the latter case; thus, when n is 3, the electromotive force for three wires is one-third of that for a single wire, but the hydraulic pressure for three pipes is one-ninth of that for a single pipe.

The conclusion to be derived from this comparison is that the analogies between hydraulic and electric flow are rough ones and cannot embrace all the quantities involved. The only perfect analogy is that p varies directly as l; the analogy between hydraulic discharge and electric current is perfect only as regards its distribution between branches or diversions; the analogy between hydraulic and electric resistance is an imperfect one that is liable to lead to confusion. Although a decrease in size of the pipe or wire causes an increase in resistance, the law of increase is quite different in the two cases. If hydraulic resistance be defined as in Art. 105, then the lost pressure p is not proportional to resistance, but to its square root, while the lost electric pressure p varies directly as electric resistance.

For the viscous flow of water in pipes (Art. 110), where the resistances are those of sliding friction only,

$$p = \frac{4wc_1}{\pi}\frac{l}{d^4}q = B_1\frac{l}{d^4}q,$$

which shows that the lost pressure is proportional to q as in Ohm's law; so that the analogy is closer than in the common motion of water, where the greater part of the loss is due to impact. The resistance, however, varies inversely as the square of the area of the pipe, while in electric flow it varies inversely as the first power of the area. Thus this analogy breaks down, as all analogies connecting electric and mechanical phenomena are found to do sooner or later.*

There are also analogies between the economic problems of electricity and those of hydraulics. For a wire line for the electric transmission of power, let C be the annual expenditure in interest and sink-

* Heavyside, Electromagnetic Theory (London, 1894), vol. 1, p. 232.

ing fund charges on account of the cost of the wire and D be the annual loss on account of the energy wasted in heating the wire, both for a wire of diameter unity. Then the total annual loss is $Cd^2 + D/d^2$, and this is a minimum when D/d^2 equals Cd^2; that is, the size of the wire which gives the greatest economy is such that the annual value of the energy lost in heat equals the annual expenditure on the cost of the wire line. In a similar manner, let C and D represent the same quantities for a pipe line carrying water to a power plant, both for a pipe of diameter unity. Then, since the thicknesses of pipes vary as their diameters and their costs as the squares of the diameters, $Cd^2 + D/d^5$ is the total annual loss, and this is a minimum when D/d^5 equals $\frac{2}{5}Cd^2$; that is, the size of pipe which gives greatest economy is such that the annual value of the energy lost in friction equals two-fifths of the annual expenditure on the cost of the pipe line.*

Prob. 202. A copper wire having a specific resistance of 0.0000016 ohms is one centimeter in diameter. A steel rail having a specific resistance of 0.0000145 ohms has a section area of 54.8 square centimeters. A certain transmission line consists of 9 kilometers of the copper wire and 3 kilometers of the steel rail. Compute the loss in voltage required to maintain a direct current of 150 amperes. If the pressure at the beginning of the line is 2500 volts and the rail is at the middle of the line, draw the electric gradient.

Art. 203. Miscellaneous Problems

The following problems introduce subjects that have not been specifically treated in the preceding pages. Teachers who wish to offer prize problems to their classes may perhaps find some of these suitable for that purpose.

Prob. 203*a*. A wooden water tank 18 feet in diameter and 24 feet high is to be hooped with iron bands which may be safely spaced 6 inches apart at the middle of the height. How far apart should they be spaced at the bottom?

Prob. 203*b*. A house is 60 feet lower than a spring A and 30 feet higher than a spring B. A pipe from A to the house runs near B. Explain a method by which the water from B can be drawn into the pipe and be delivered at the house.

Prob. 203*c*. A river having a width of 300 feet on the surface, a cross-section of 1800 square feet, a hydraulic radius of 5.3 feet, and a slope of 1 on 10 000, discharges 10 400 cubic feet per second. If it be frozen over to the depth of one foot, what will be its discharge?

* Adams, Proceedings American Society of Civil Engineers, May, 1907.

Prob. 203*d*. From a pumping station water is forced by direct pressure through a compound pipe, consisting of 7500 feet of 14-inch pipe, 4100 feet of 12-inch pipe, and 780 feet of 8-inch pipe, to a 6-inch pipe on which there are three hydrants A, B, and C. A is 133 feet from the end of the 8-inch pipe and 115 feet above the gage at the pumping station; B is 433 feet from the end of the 8-inch pipe and 135 feet above the gage; C is 733 feet from the end of the 8-inch pipe and 125 feet above the gage. To each of these hydrants is attached 50 feet of $2\frac{1}{2}$-inch rubber-lined hose with a 1-inch smooth nozzle at the end. When the gage at the pumping station reads 175 pounds per square inch, to what heights will the three streams be thrown from the three nozzles?

Prob. 203*e*. When a body falls vertically in water, its velocity soon becomes constant. For a smooth sphere an approximate formula for this velocity is $v\sqrt{2gd(s-1)}$, in which d is the diameter of the sphere and s its specific gravity. Compute the velocity v for a sphere having a diameter of 0.001 feet and a specific gravity of 1.25.

Prob. 203*f*. The velocity with which water flows through a sand filter bed varies directly as the head (Art. 110). If V is the velocity in meters per day, d the effective size of the sand grains in millimeters, h the head, l the thickness of the sand bed, and t the centigrade temperature,

$$V = 1000\,(0.70 + 0.03t)\,(h/l)d^2$$

is the formula deduced by Hazen.* When $t = 32°.4$ centigrade, $d = 0.4$ millimeters, $l = 4$ feet, and $h = 0.4$ feet, find how many million gallons per day will pass through one acre of filter beds.

Prob. 203*g*. A bent U tube of uniform size is partly filled with water. Let the water in one leg be depressed a certain distance, causing that in the other to rise the same distance. When the depressing force is removed, the water oscillates up and down in the legs of the tube, the times of oscillation being isochronous. If l be the entire length of the water in the tube, show that the time of one oscillation is $\pi\sqrt{l/2g}$. If the legs are inclined to the horizontal at the angles θ and ϕ, show that the time of one oscillation is $\pi\sqrt{l/g\,(\sin\theta + \sin\phi)}$.

Prob. 203*h*. The bottom of a canal has the width $2b$, and it is desired to shape the banks so that the hydraulic radius of the cross-section may be constant. Show that the equation of the curve is

$$y = r\log_e\,(x + \sqrt{x^2 - r^2})/(b + \sqrt{b^2 - r^2})$$

in which y is the depth of the water, x the half width of the water surface, and r the constant hydraulic radius.

Prob. 203*i*. A river having a slope of 1 on 2500 runs due east. A line drawn due north at a point A on the river strikes at B, 5000 feet from A,

* Report Massachusetts State Board of Health, 1892, p. 553.

the edge of a large swamp which it is desired to drain. The level of the water in this swamp is 0.5 feet below the river surface at A, and it is desired to lower that level 1.5 feet more. For this purpose a ditch is to be dug running from A in a straight line on a uniform slope until it joins the river at a point C eastward from A. The discharge of this ditch, in order to properly drain the swamp, will be 25 cubic feet per second, its side slopes are to be 1 on 1, the mean velocity is not to exceed 2.5 feet per second, and the coefficient c in the Chezy formula is estimated at 70. Find the length and width of the most economical ditch.

Art. 204. Answers to Problems

Below will be found answers to some of the problems given in the preceding pages, the numbers of the problems being placed in parentheses. In general it is not a good plan for a student to solve a problem in order to obtain a given answer. One object of solving problems is, of course, to obtain correct results, but the correctness of those results should be established by methods of verification rather than by the authority of a given answer. It is more profitable that a number of students should obtain different answers to a problem and engage in a discussion as to the correctness of their solutions than that all discussion should be stopped because a certain answer is given in the text. However satisfactory it may be to know in advance the result of the solution of an exercise, let the student bear in mind that after commencement day answers to problems will not be given.

(1) One horse-power. (3) 147.2 pounds. (4) See Table 4. (7) See Index. (8) 29.56 inches. (9*b*) 9.54 kilograms per square centimeter. (9*d*) 5575 kilograms. (12) 40.6, 1.56, 2.65. (15) 28 300 pounds. (17) 4.01 feet. (20*b*) 3.07. (20*c*) 2945 kilograms. (21) 56.9 feet per second. (25) $v = 32.1$ feet per second. (27) 19.3 pounds. (32) 24.9 seconds. (33*c*) 0.73. (35) 1.96 and 166 cubic feet. (36) 0.017 inches. (37) 1.15 feet. (39) $v = 4.00$ feet per second. (41) See Engineering News, May 4, 1911. (45) $c = 1.06$. (48) $c = 0.605$. (49) 17.2 feet. (50) 10.5 cubic feet per second. (51) 0.034 cubic feet per second. (55) 103. (59*a*) $c_1 = 0.98$. (60) 0.361 feet per second. (62) 0.0109 feet. (67) 7.10 and 6.97 cubic feet per second. (71) 0.74 percent. (72) 0.581. (72*a*) 1.30 centimeters. (75) 0.126 feet. (76) 0.13 and 7.60 feet. (77) 0.28 feet. (78) $c = 0.90$ and $h_1 = 0.70$. (80) $c = 0.802$. (81) 6.67 feet. (83) 0.963. (84) 1.06. (89) 0.29 feet. (95) 3.06 and 4.94 inches. (98) About 6 cubic feet per second. (112) 2.8 feet. (114) 4.4 feet. (115) 7.32 feet per second. (116) 1.28×0.64 feet. (118) 57 400 000 gallons. (120) $d = 3.09$ feet. (127*b*) 0.48 meters. (129) 546 cubic feet per second. (132) 1.76 feet per second. (134) 760 cubic feet per second. (140) $d_1 = 12.5$ feet. (141*d*) $H = 0.41$ meters. (145) 0.9. (146) 13.5 horse-powers. (147) 1.32 horse-powers. (148) 257 feet. (149) 35.4 percent. (151*c*) 18 400 kilowatts. (152) 3.96 gallons. (155) About 120 pounds. (159) 34.5 feet per second. (162*a*) $e = 0.83$. (164) From 48 to 50 horse-powers. (165) 13.6. (171*a*) 30.1 kilowatts. (172) 16 feet. (175) 4.117 and 4.120. (178) 167. (182*e*) 27.0 cubic meters. (183) 743 horse-powers. (185) 1530 horse-powers. (191*d*) $r = 11.6$ meters. (198) $e = 0.78$. (200) 17.8 horse-powers. (201*d*) $9\frac{1}{4}$ meters.

Evolvi varia problemata. In scientiis enim ediscendis prosunt exempla magisquam præcepta. Qua de causa in his fusius expatiatus sum. — Newton.

Art. 205. Mathematical Tables

Tables A, B, C, D give constants often needed in computations.

Table E gives squares of numbers from 1.00 to 9.99, the arrangement being the same as that of the logarithmic table. By properly moving the decimal point, four-place squares of other numbers are also readily taken out. For example, the square of 0.874 is 0.7639, and that of 87.4 is 7639, correct to four significant figures.

Table F gives areas of circles for diameters ranging from 1.00 to 9.99, arranged in the same manner, and by properly moving the decimal point, four-place areas for all circles can be found. For instance, if the diameter is 4.175 inches, the area is 13.69 square inches; if the diameter is 0.535 feet, the area is 0.2248 square feet.

Table G gives trigonometric functions of angles and Table H the logarithms of these functions. The term "arc" means the length of a circular arc of radius unity, while "coarc" is the complement of the arc, or a quadrant minus the arc. If θ is the number of degrees in any angle, the value of arcθ is $\pi\theta/180$.

Table J gives four-place common logarithms of numbers, and these are of great value in hydraulic computations (Art. 8). Table K, taken from the author's "Elements of Precise Surveying and Geodesy," gives nine-place constants and their logarithms.

For other tables used in hydraulic computations see American Civil Engineers' Pocket Book (New York, 1912). Barlow's Tables (London, 1907) give eight-place values of squares, cubes, square roots, cube roots, and reciprocals of numbers from 1 to 10 000.

TABLE A. FUNDAMENTAL HYDRAULIC CONSTANTS

English Measures

Name	Symbol	Number	Logarithm
Pounds of water in one cubic foot	w	62.5	1.7959
Pounds of water in one U. S. gallon	$w/7.481$	8.355	0.9220
Pounds per square inch due to one atmosphere		14.7	1.1673
Pounds per square inch due to one foot of head	$w/144$	0.434	$\bar{1}.6375$
Feet of head for pressure of one pound per square inch	$144/w$	2.304	0.3625
Cubic feet in one U. S. gallon	$231/1728$	0.1337	$\bar{1}.1261$
U. S. gallons in one cubic foot	$1728/231$	7.481	0.8739
Acceleration of gravity in feet per second per second	g	32.16	1.5073
	$\sqrt{2g}$	8.020	0.9042
	$\frac{2}{3}\sqrt{2g}$	5.347	0.7281
	$1/2g$	0.01555	$\bar{2}.1916$
	$\frac{1}{4}\pi\sqrt{2g}$	6.299	0.7993

TABLE B. FUNDAMENTAL HYDRAULIC CONSTANTS

Metric Measures

Name	Symbol	Number	Logarithm
Kilograms of water in one cubic meter	w	1000	3.0000
Kilograms of water in one liter	$w/1000$	1	0.0000
Kilograms per square centimeter due to one atmosphere		1.033	0.0142
Kilograms per square centimeter due to one meter head	$w/10000$	0.1	$\bar{1}.0000$
Meters of head for pressure of one kilogram per square centimeter	$10000/w$	10	1.0000
Cubic meters in one liter	$1/1000$	0.001	$\bar{3}.0000$
Liters in one cubic meter	$1000/1$	1000	3.0000
Acceleration of gravity in meters per second per second	g	9.800	0.9912
	$\sqrt{2g}$	4.427	0.6461
	$\frac{2}{3}\sqrt{2g}$	2.951	0.4700
	$1/2g$	0.05104	$\bar{2}.7077$
	$\frac{1}{4}\pi\sqrt{2g}$	3.477	0.5412

TABLE C. METRIC EQUIVALENTS OF ENGLISH UNITS

English Unit	Metric Equivalent	Logarithm
1 Inch	2.5400 centimeters	0.40483
1 Foot	0.3048 meters	$\bar{1}.48402$
1 Square Inch	6.4520 square centimeters	0.80969
1 Square Foot	0.09290 square meters	$\bar{2}.96803$
1 Cubic Foot	0.02832 cubic meters	$\bar{2}.45209$
1 U. S. Gallon	3.7854 liters	0.57812
1 Imperial Gallon	4.5438 liters	0.65742
1 Pound	0.4536 kilograms	$\bar{1}.65667$
1 Pound per Square Inch	0.07030 kilograms per square centimeter	$\bar{2}.84697$
1 Pound per Cubic Foot	16.017 kilograms per cubic meter	1.20457
1 Foot-pound	0.1383 kilogram-meters	$\bar{1}.14069$
1 Horse-power	1.0139 cheval-vapeur	0.00599
Fahrenheit Temperature F°	Centigrade temperature $C° = \frac{5}{9}(F° - 32°)$	

TABLE D. ENGLISH EQUIVALENTS OF METRIC UNITS

Metric Unit	English Equivalent	Logarithm
1 Centimeter	0.3937 inches	$\bar{1}.59517$
1 Meter	3.2808 feet	0.51598
1 Square Centimeter	0.1550 square inches	$\bar{1}.19031$
1 Square Meter	10.764 square feet	1.03197
1 Cubic Meter	35.314 cubic feet	1.54791
1 Liter	0.2642 U. S. gallons	$\bar{1}.42188$
1 Liter	0.2201 imperial gallons	$\bar{1}.34258$
1 Kilogram	2.2046 pounds	0.34333
1 Kilogram per Square Centimeter	14.224 pounds per square inch	1.15303
1 Kilogram per Cubic Meter	0.06244 pounds per cubic foot	$\bar{2}.79543$
1 Kilogram-meter	7.2329 foot-pounds	0.85931
1 Cheval-vapeur	0.9863 horse-powers	$\bar{1}.99041$
Centigrade Temperature C°	Fahrenheit temperature $F° = 32° + 1.8\ C°$	

TABLE E. SQUARES OF NUMBERS

n	0	1	2	3	4	5	6	7	8	9	Diff.
1.0	1.000	1.020	1.040	1.061	1.082	1.103	1.124	1.145	1.166	1.188	22
1.1	1.210	1.232	1.254	1.277	1.300	1.323	1.346	1.369	1.392	1.416	24
1.2	1.440	1.464	1.488	1.513	1.538	1.563	1.588	1.613	1.638	1.664	26
1.3	1.690	1.716	1.742	1.769	1.796	1.823	1.850	1.877	1.904	1.932	28
1.4	1.960	1.988	2.016	2.045	2.074	2.103	2.132	2.161	2.190	2.220	30
1.5	2.250	2.280	2.310	2.341	2.372	2.403	2.434	2.465	2.496	2.528	32
1.6	2.560	2.592	2.624	2.657	2.690	2.723	2.756	2.789	2.822	2.856	34
1.7	2.890	2.924	2.958	2.993	3.028	3.063	3.098	3.133	3.168	3.204	36
1.8	3.240	3.276	3.312	3.349	3.386	3.423	3.460	3.497	3.534	3.572	38
1.9	3.610	3.648	3.686	3.725	3.764	3.803	3.842	3.881	3.920	3.960	40
2.0	4.000	4.040	4.080	4.121	4.162	4.203	4.244	4.285	4.326	4.368	42
2.1	4.410	4.452	4.494	4.537	4.580	4.623	4.666	4.709	4.752	4.796	44
2.2	4.840	4.884	4.928	4.973	5.018	5.063	5.108	5.153	5.198	5.244	46
2.3	5.290	5.336	5.382	5.429	5.476	5.523	5.570	5.617	5.664	5.712	48
2.4	5.760	5.808	5.856	5.905	5.954	6.003	6.052	6.101	6.150	6.200	50
2.5	6.250	6.300	6.350	6.401	6.452	6.503	6.554	6.605	6.656	6 708	52
2.6	6.760	6.812	6.864	6.917	6.970	7.023	7.076	7.129	7.182	7.236	54
2.7	7.290	7.344	7.398	7.453	7.508	7.563	7.618	7.673	7.728	7.784	56
2.8	7.840	7.896	7.952	8.009	8.066	8.123	8.180	8.237	8.294	8.352	58
2.9	8.410	8.468	8.526	8.585	8.644	8.703	8.762	8.821	8.880	8.940	60
3.0	9.000	9.060	9.120	9.181	9.242	9.303	9.364	9.425	9.486	9.548	62
3.1	9.610	9.672	9.734	9.797	9.860	9.923	9.986	10.05	10.11	10.18	6
3.2	10.24	10.30	10.37	10.43	10.50	10.56	10.63	10.69	10.76	10.82	7
3.3	10.89	10.96	11.02	11.09	11.16	11.22	11.29	11.36	11.42	11.49	7
3.4	11.56	11.63	11.70	11.76	11.83	11.90	11.97	12.04	12.11	12.18	7
3.5	12.25	12.32	12.39	12.46	12.53	12.60	12.67	12.74	12.82	12.89	7
3.6	12.96	13.03	13.10	13.18	13.25	13.32	13.40	13.47	13.54	13.62	7
3.7	13.69	13.76	13.84	13.91	13.99	14.06	14.14	14.21	14.29	14.36	8
3.8	14.44	14.52	14.59	14.67	14.75	14.82	14.90	14.98	15.05	15.13	8
3.9	15.21	15.29	15.37	15.44	15.52	15.60	15.68	15.76	15.84	15.92	8
4.0	16.00	16.08	16.16	16.24	16.32	16.40	16.48	16.56	16.65	16.73	8
4.1	16.81	16.89	16.97	17.06	17.14	17.22	17.31	17.39	17.47	17.56	8
4.2	17.64	17.72	17.81	17.89	17.98	18.06	18.15	18.23	18.32	18.40	9
4.3	18.49	18.58	18.66	18.75	18.84	18.92	19.01	19.10	19.18	19.27	9
4.4	19.36	19.45	19.54	19.62	19.71	19.80	19.89	19.98	20.07	20.16	9
4.5	20.25	20.34	20.43	20.52	20.61	20.70	20.79	20.88	20.98	21.07	9
4.6	21.16	21.25	21.34	21.44	21.53	21.62	21.72	21.81	21.90	22.00	9
4.7	22.09	22.18	22.28	22.37	22.47	22.56	22.66	22.75	22.85	22.94	10
4.8	23.04	23.14	23.23	23.33	23.43	23.52	23.62	23.72	23.81	23.91	10
4.9	24.01	24.11	24.21	24.30	24.40	24.50	24.60	24.70	24.80	24.90	10
5.0	25.00	25.10	25.20	25.30	25.40	25.50	25.60	25.70	25.81	25.91	10
5.1	26.01	26.11	26.21	26.32	26.42	26.52	26.63	26.73	26.83	26.94	10
5.2	27.04	27.14	27.25	27.35	27.46	27.56	27.67	27.77	27.88	27.98	11
5.3	28.09	28.20	28.30	28.41	28.42	28.62	28.73	28.84	28.94	29.05	11
5.4	29.16	29.27	29.38	29.48	29.59	29.70	29.81	29.92	30.03	30.14	11
n	0	1	2	3	4	5	6	7	8	9	Diff.

TABLE E. SQUARES OF NUMBERS (*Continued*)

n	0	1	2	3	4	5	6	7	8	9	Diff.
5.5	30.25	30.36	30.47	30.58	30.69	30.80	30.91	31.02	31.14	31.25	11
5.6	31.36	31.47	31.58	31.70	31.81	31.92	32.04	32.15	32.26	32.38	11
5.7	32.49	32.60	32.72	32.83	32.95	33.06	33.18	33.29	33.41	33.52	12
5.8	33.64	33.76	33.87	33.99	34.11	34.22	34.34	34.46	34.57	34.69	12
5.9	34.81	34.93	35.05	35.16	35.28	35.40	35.52	35.64	35.76	35.88	12
6.0	36.00	36.12	36.24	36.36	36.48	36.60	36.72	36.84	36.97	37.09	12
6.1	37.21	37.33	37.45	37.58	37.70	37.82	37.95	38.07	38.19	38.32	12
6.2	38.44	38.56	38.69	38.81	38.94	39.06	39.19	39.31	39.44	39.56	13
6.3	39.69	39.82	39.94	40.07	40.20	40.32	40.45	40.58	40.70	40.83	13
6.4	40.96	41.09	41.22	41.34	41.47	41.60	41.73	41.86	41.99	42.12	13
6.5	42.25	42.38	42.51	42.64	42.77	42.90	43.03	43.16	43.30	43.43	13
6.6	43.56	43.69	43.82	43.96	44.09	44.22	44.36	44.49	44.62	44.76	13
6.7	44.89	45.02	45.16	45.29	45.43	45.56	45.70	45.83	45.97	46.10	14
6.8	46.24	46.38	46.51	46.65	46.79	46.92	47.06	47.20	47.33	47.47	14
6.9	47.61	47.75	47.89	48.02	48.16	48.30	48.44	48.58	48.72	48.86	14
7.0	49.00	49.14	49.28	49.42	49.56	49.70	49.84	49.98	50.13	50.27	14
7.1	50.41	50.55	50.69	50.84	50.98	51.12	51.27	51.41	51.55	51.70	14
7.2	51.84	51.98	52.13	52.27	52.42	52.56	52.71	52.85	53.00	53.14	15
7.3	53.29	53.44	53.58	53.73	53.88	54.02	54.17	54.32	54.46	54.61	15
7.4	54.76	54.91	55.06	55.20	55.35	55.50	55.65	55.80	55.95	56.10	15
7.5	56.25	56.40	56.55	56.70	56.85	57.00	57.15	57.30	57.46	57.61	15
7.6	57.76	57.91	58.06	58.22	58.37	58.52	58.68	58.83	58.98	59.14	15
7.7	59.29	59.44	59.60	59.75	59.91	60.06	60.22	60.37	60.53	60.68	16
7.8	60.84	61.00	61.15	61.31	61.47	61.62	61.78	61.94	62.09	62.25	16
7.9	62.41	62.57	62.73	62.88	63.04	63.20	63.36	63.52	63.68	63.84	16
8.0	64.00	64.16	64.32	64.48	64.64	64.80	64.96	65.12	65.29	65.45	16
8.1	65.61	65.77	65.93	66.10	66.26	66.42	66.59	66.75	66.91	67.08	16
8.2	67.24	67.40	67.57	67.73	67.90	68.06	68.23	68.39	68.56	68.72	17
8.3	68.89	69.06	69.22	69.39	69.56	69.72	69.89	70.06	70.22	70.39	17
8.4	70.56	70.73	70.90	71.06	71.23	71.40	71.57	71.74	71.91	72.08	17
8.5	72.25	72.42	72.59	72.76	72.93	73.10	73.27	73.44	73.62	73.79	17
8.6	73.96	74.13	74.30	74.48	74.65	74.82	75.00	75.17	75.34	75.52	17
8.7	75.69	75.86	76.04	76.21	76.39	76.56	76.74	76.91	77.09	77.26	18
8.8	77.44	77.62	77.79	77.97	78.15	78.32	78.50	78.68	78.85	79.03	18
8.9	79.21	79.39	79.57	79.74	79.92	80.10	80.28	80.46	80.64	80.82	18
9.0	81.00	81.18	81.36	81.54	81.72	81.90	82.08	82.26	82.45	82.63	18
9.1	82.81	82.99	83.17	83.36	83.54	83.72	83.91	84.09	84.27	84.46	18
9.2	84.64	84.82	85.01	85.19	85.38	85.56	85.75	85.93	86.12	86.30	19
9.3	86.49	86.68	86.86	87.05	87.24	87.42	87.61	87.80	87.98	88.17	19
9.4	88.36	88.55	88.74	88.92	89.11	89.30	89.49	89.68	89.87	90.06	19
9.5	90.25	90.44	90.63	90.82	91.01	91.20	91.39	91.58	91.78	91.97	19
9.6	92.16	92.35	92.54	92.74	92.93	93.12	93.32	93.51	93.70	93.90	19
9.7	94.09	94.28	94.48	94.67	94.87	95.06	95.26	95.45	95.65	95.84	20
9.8	96.04	96.24	96.43	96.63	96.83	97.02	97.22	97.42	97.61	97.81	20
9.9	98.01	98.21	98.41	98.60	98.80	99.00	99.20	99.40	99.60	99.80	20
n	0	1	2	3	4	5	6	7	8	9	Diff.

TABLE F. AREAS OF CIRCLES

d	0	1	2	3	4	5	6	7	8	9	Diff.
1.0	.7854	.8012	.8171	.8332	.8495	.8659	.8825	.8992	.9161	.9331	
1.1	.9503	.9677	.9852	1.003	1.021	1.039	1.057	1.075	1.094	1.112	
1.2	1.131	1.150	1.169	1.188	1.208	1.227	1.247	1.267	1.287	1.307	19
1.3	1.327	1.348	1.368	1.389	1.410	1.431	1.453	1.474	1.496	1.517	21
1.4	1.539	1.561	1.584	1.606	1.629	1.651	1.674	1.697	1.720	1.744	22
1.5	1.767	1.791	1.815	1.839	1.863	1.887	1.911	1.936	1.961	1.986	24
1.6	2.011	2.036	2.061	2.087	2.112	2.138	2.164	2.190	2.217	2.243	26
1.7	2.270	2.297	2.324	2.351	2.378	2.405	2.433	2.461	2.488	2.516	27
1.8	2.545	2.573	2.602	2.630	2.659	2.688	2.717	2.746	2.776	2.806	29
1.9	2.835	2.865	2.895	2.926	2.956	2.986	3.017	3.048	3.079	3.110	30
2.0	3.142	3.173	3.205	3.237	3.269	3.301	3.333	3.365	3.398	3.431	32
2.1	3.464	3.497	3.530	3.563	3.597	3.631	3.664	3.698	3.733	3.767	34
2.2	3.801	3.836	3.871	3.906	3.941	3.976	4.012	4.047	4.083	4.119	35
2.3	4.155	4.191	4.227	4.264	4.301	4.337	4.374	4.412	4.449	4.486	36
2.4	4.524	4.562	4.600	4.638	4.676	4.714	4.753	4.792	4.831	4.870	38
2.5	4.909	4.948	4.988	5.027	5.067	5.107	5.147	5.187	5.228	5.269	40
2.6	5.309	5.350	5.391	5.433	5.474	5.515	5.557	5.599	5.641	5.683	41
2.7	5.726	5.768	5.811	5.853	5.896	5.940	5.983	6.026	6.070	6.114	43
2.8	6.158	6.202	6.246	6.290	6.335	6.379	6.424	6.469	6.514	6.560	44
2.9	6.605	6.651	6.697	6.743	6.789	6.835	6.881	6.928	6.975	7.022	46
3.0	7.069	7.116	7.163	7.211	7.258	7.306	7.354	7.402	7.451	7.499	48
3.1	7.548	7.596	7.645	7.694	7.744	7.793	7.843	7.892	7.942	7.992	49
3.2	8.042	8.093	8.143	8.194	8.245	8.296	8.347	8.398	8.450	8.501	51
3.3	8.553	8.605	8.657	8.709	8.762	8.814	8.867	8.920	8.973	9.026	52
3.4	9.079	9.133	9.186	9.240	9.294	9.348	9.402	9.457	9.511	9.566	54
3.5	9.621	9.676	9.731	9.787	9.842	9.898	9.954	10.01	10.07	10.12	56
3.6	10.18	10.24	10.29	10.35	10.41	10.46	10.52	10.58	10.64	10.69	6
3.7	10.75	10.81	10.87	10.93	10.99	11.04	11.10	11.16	11.22	11.28	6
3.8	11.34	11.40	11.46	11.52	11.58	11.64	11.70	11.76	11.82	11.88	6
3.9	11.95	12.01	12.07	12.13	12.19	12.25	12.32	12.38	12.44	12.50	6
4.0	12.57	12.63	12.69	12.76	12.82	12.88	12.95	13.01	13.07	13.14	7
4.1	13.20	13.27	13.33	13.40	13.46	13.53	13.59	13.66	13.72	13.79	7
4.2	13.85	13.92	13.99	14.05	14.12	14.19	14.25	14.32	14.39	14.45	7
4.3	14.52	14.59	14.66	14.73	14.79	14.86	14.93	15.00	15.07	15.14	7
4.4	15.21	15.27	15.34	15.41	15.48	15.55	15.62	15.69	15.76	15.83	7
4.5	15.90	15.98	16.05	16.12	16.19	16.26	16.33	16.40	16.47	16.55	7
4.6	16.62	16.69	16.76	16.84	16.91	16.98	17.06	17.13	17.20	17.28	7
4.7	17.35	17.42	17.50	17.57	17.65	17.72	17.80	17.87	17.95	18.02	8
4.8	18.10	18.17	18.25	18.32	18.40	18.47	18.55	18.63	18.70	18.78	8
4.9	18.86	18.93	19.01	19.09	19.17	19.24	19.32	19.40	19.48	19.56	8
5.0	19.63	19.71	19.79	19.87	19.95	20.03	20.11	20.19	20.27	20.35	8
5.1	20.43	20.51	20.59	20.67	20.75	20.83	20.91	20.99	21.07	21.16	8
5.2	21.24	21.32	21.40	21.48	21.57	21.65	21.73	21.81	21.90	21.98	8
5.3	22.06	22.15	22.23	22.31	22.40	22.48	22.56	22.65	22.73	22.82	8
5.4	22.90	22.99	23.07	23.16	23.24	23.33	23.41	23.50	23.59	23.67	9
d	0	1	2	3	4	5	6	7	8	9	Diff.

TABLE F. AREAS OF CIRCLES (*Continued*)

d	0	1	2	3	4	5	6	7	8	9	Diff.
5.5	23.76	23.84	23.93	24.02	24.11	24.19	24.28	24.37	24.45	24.54	9
5.6	24.63	24.72	24.81	24.89	24.98	25.07	25.16	25.25	25.34	25.43	9
5.7	25.52	25.61	25.70	25.79	25.88	25.97	26.06	26.15	26.24	26.33	9
5.8	26.42	26.51	26.60	26.69	26.79	26.88	26.97	27.06	27.15	27.25	9
5.9	27.34	27.43	27.53	27.62	27.71	27.81	27.90	27.99	28.09	28.18	9
6.0	28.27	28.37	28.46	28.56	28.65	28.75	28.84	28.94	29.03	29.13	9
6.1	29.22	29.32	29.42	29.51	29.61	29.71	29.80	29.90	30.00	30.09	10
6.2	30.19	30.29	30.39	30.48	30.58	30.68	30.78	30.88	30.97	31.07	10
6.3	31.17	31.27	31.37	31.47	31.57	31.67	31.77	31.87	31.97	32.07	10
6.4	32.17	32.27	32.37	32.47	32.57	32.67	32.78	32.88	32.98	33.08	10
6.5	33.18	33.29	33.39	33.49	33.59	33.70	33.80	33.90	34.00	34.11	10
6.6	34.21	34.32	34.42	34.52	34.63	34.73	34.84	34.94	35.05	35.15	10
6.7	35.26	35.36	35.47	35.57	35.68	35.78	35.89	36.00	36.10	36.21	10
6.8	36.32	36.42	36.53	36.64	36.75	36.85	36.96	37.07	37.18	37.28	11
6.9	37.39	37.50	37.61	37.72	37.83	37.94	38.05	38.16	38.26	38.37	11
7.0	38.48	38.59	38.70	38.82	38.93	39.04	39.15	39.26	39.37	39.48	11
7.1	39.59	39.70	39.82	39.93	40.04	40.15	40.26	40.38	40.49	40.60	11
7.2	40.72	40.83	40.94	41.06	41.17	41.28	41.40	41.51	41.62	41.74	11
7.3	41.85	41.97	42.08	42.20	42.31	42.43	42.54	42.66	42.78	42.89	11
7.4	43.01	43.12	43.24	43.36	43.47	43.59	43.71	43.83	43.94	44.06	12
7.5	44.18	44.30	44.41	44.53	44.65	44.77	44.89	45.01	45.13	45.25	12
7.6	45.36	45.48	45.60	45.72	45.84	45.96	46.08	46.20	46.32	46.45	12
7.7	46.57	46.69	46.81	46.93	47.05	47.17	47.29	47.42	47.54	47.66	12
7.8	47.78	47.91	48.03	48.15	48.27	48.40	48.52	48.65	48.77	48.89	12
7.9	49.02	49.14	49.27	49.39	49.51	49.64	49.76	49.89	50.01	50.14	12
8.0	50.27	50.39	50.52	50.64	50.77	50.90	51.02	51.15	51.28	51.40	13
8.1	51.53	51.66	51.78	51.91	52.04	52.17	52.30	52.42	52.55	52.68	13
8.2	52.81	52.94	53.07	53.20	53.33	53.46	53.59	53.72	53.85	53.98	13
8.3	54.11	54.24	54.37	54.50	54.63	54.76	54.89	55.02	55.15	55.29	13
8.4	55.42	55.55	55.68	55.81	55.95	56.08	56.21	56.35	56.48	56.61	13
8.5	56.75	56.88	57.01	57.15	57.28	57.41	57.55	57.68	57.82	57.95	13
8.6	58.09	58.22	58.36	58.49	58.63	58.77	58.90	59.04	59.17	59.31	14
8.7	59.45	59.58	59.72	59.86	59.99	60.13	60.27	60.41	60.55	60.68	14
8.8	60.82	60.96	61.10	61.24	61.38	61.51	61.65	61.79	61.93	62.07	14
8.9	62.21	62.35	62.49	62.63	62.77	62.91	63.05	63.19	63.33	63.48	14
9.0	63.62	63.76	63.90	64.04	64.18	64.33	64.47	64.61	64.75	64.90	14
9.1	65.04	65.18	65.33	65.47	65.61	65.76	65.90	66.04	66.19	66.33	14
9.2	66.48	66.62	66.77	66.91	67.06	67.20	67.35	67.49	67.64	67.78	15
9.3	67.93	68.08	68.22	68.37	68.51	68.66	68.81	68.96	69.10	69.25	15
9.4	69.40	69.55	69.69	69.84	69.99	70.14	70.29	70.44	70.58	70.73	15
9.5	70.88	71.03	71.18	71.33	71.48	71.63	71.78	71.93	72.08	72.23	15
9.6	72.38	72.53	72.68	72.84	72.99	73.14	73.29	73.44	73.59	73.75	15
9.7	73.90	74.05	74.20	74.36	74.51	74.66	74.82	74.97	75.12	75.28	15
9.8	75.43	75.58	75.74	75.89	76.05	76.20	76.36	76.51	76.67	76.82	16
9.9	76.98	77.13	77.29	77.44	77.60	77.76	77.91	78.07	78.23	78.38	16
d	0	1	2	3	4	5	6	7	8	9	Diff.

TABLE G. TRIGONOMETRIC FUNCTIONS

Angle Deg.	Arc	Sin	Tan	Sec	Cosec	Cot	Cos	Coarc	
0	0.	0.	0.	1.	∞	∞	1.	1.5708	90
1	0.0175	0.0175	0.0175	1.0002	57.299	57.290	0.9998	.5533	89
2	.0349	.0349	.0349	1.0006	28.654	28.636	.9994	.5359	88
3	.0524	.0523	.0524	1.0014	19.107	19.081	.9986	.5184	87
4	.0698	.0698	.0699	1.0024	14.336	14.301	.9976	.5010	86
5	.0873	.0872	.0875	1.0038	11.474	11.430	.9962	.4835	85
6	0.1047	0.1045	0.1051	1.0055	9.5668	9.5144	0.9945	1.4661	84
7	.1222	.1219	.1228	1.0075	8.2055	8.1443	.9925	.4486	83
8	.1396	.1392	.1405	1.0098	7.1853	7.1154	.9903	.4312	82
9	.1571	.1564	.1584	1.0125	6.3925	6.3138	.9877	.4137	81
10	.1745	.1736	.1763	1.0154	5.7588	5.6713	.9848	.3963	80
11	0.1920	.01908	0.1944	1.0187	5.2408	5.1446	0.9816	1.3788	79
12	.2094	.2079	.2126	1.0223	4.8097	4.7046	.9781	.3614	78
13	.2269	.2250	.2309	1.0263	4.4454	4.3315	.9744	.3439	77
14	.2443	.2419	.2493	1.0306	4.1336	4.0108	.9703	.3265	76
15	.2618	.2588	.2679	1.0353	3.8637	3.7321	.9659	.3090	75
16	0.2793	0.2756	0.2867	1.0403	3.6280	3.4874	0.9613	1.2915	74
17	.2967	.2924	.3057	1.0457	3.4203	3.2709	.9563	.2741	73
18	.3142	.3090	.3249	1.0515	3.2361	3.0777	.9511	.2566	72
19	.3316	.3256	.3443	1.0576	3.0716	2.9042	.9455	.2392	71
20	.3491	.3420	.3640	1.0642	2.9238	2.7475	.9397	.2217	70
21	0.3665	0.3584	0.3839	1.0711	2.7904	2.6051	0.9336	1.2043	69
22	.3840	.3746	.4040	1.0785	2.6695	2.4751	.9272	.1868	68
23	.4014	.3907	.4245	1.0864	2.5593	2.3559	.9205	.1694	67
24	.4189	.4067	.4452	1.0946	2.4586	2.2460	.9135	.1519	66
25	.4363	.4226	.4663	1.1034	2.3662	2.1445	.9063	.1345	65
26	0.4538	0.4384	0.4877	1.1126	2.2812	2.0503	0.8988	1.1170	64
27	.4712	.4540	.5095	1.1223	2.2027	1.9626	.8910	.0996	63
28	.4887	.4695	.5317	1.1326	2.1301	1.8807	.8829	.0821	62
29	.5061	.4848	.5543	1.1434	2.0627	1.8040	.8746	.0647	61
30	.5236	.5000	.5774	1.1547	2.0000	1.7321	.8660	.0472	60
31	0.5411	0.5150	0.6009	1.1666	1.9416	1.6643	0.8572	1.0297	59
32	.5585	.5299	.6249	1.1792	1.8871	1.6003	.8480	1.0123	58
33	.5760	.5446	.6494	1.1924	1.8361	1.5399	.8387	0.9948	57
34	.5934	.5592	.6745	1.2062	1.7883	1.4826	.8290	.9774	56
35	.6109	.5736	.7002	1.2208	1.7434	1.4281	.8192	.9599	55
36	0.6283	0.5878	0.7265	1.2361	1.7013	1.3764	0.8089	0.9425	54
37	.6458	.6018	.7536	1.2521	1.6616	1.3270	.7986	.9250	53
38	.6632	.6157	.7813	1.2690	1.6243	1.2799	.7880	.9076	52
39	.6807	.6293	.8098	1.2868	1.5890	1.2349	.7771	.8901	51
40	.6981	.6428	.8391	1.3054	1.5557	1.1918	.7660	.8727	50
41	0.7156	0.6561	0.8693	1.3250	1.5243	1.1504	0.7547	0.8552	49
42	.7330	.6691	.9004	1.3456	1.4945	1.1106	.7431	.8378	48
43	.7505	.6820	.9325	1.3673	1.4663	1.0724	.7314	.8203	47
44	.7679	.6947	.9657	1.3902	1.4396	1.0355	.7193	.8029	46
45	.7854	.7071	1.	1.4142	1.4142	1.	.7071	.7854	45
	Coarc	Cos	Cot	Cosec	Sec	Tan	Sin	Arc	Angle Deg.

TABLE H. LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

Angle Deg.	Log Arc	Log Sin	Log Tan	Log Sec	Log Cosec	Log Cot	Log Cos	Log Coarc	
0	−∞	−∞	−∞	0.	∞	∞	0.	0.1961	90
1	$\bar{2}$.2419	$\bar{2}$.2419	$\bar{2}$.2419	0.0001	1.7581	1.7581	$\bar{1}$.9999	.1913	89
2	.5429	.5428	.5431	.0003	.4572	.4569	.9997	.1864	88
3	.7190	.7188	.7194	.0006	.2812	.2806	.9994	.1814	87
4	.8439	.8436	.8446	.0011	.1564	.1554	.9989	.1764	86
5	.9408	.9403	.9420	.0017	.0597	.0580	.9983	.1713	85
6	$\bar{1}$.0200	^{1}.0192	$\bar{1}$.0216	0.0024	0.9808	0.9784	$\bar{1}$.9976	0.1662	84
7	.0870	.0859	.0891	.0032	.9141	.9109	.9968	.1610	83
8	.1450	.1436	.1478	.0042	.8564	.8522	.9958	.1557	82
9	.1961	.1943	.1997	.0054	.8057	.8003	.9946	.1504	81
10	.2419	.2397	.2463	.0066	.7603	.7537	.9934	.1450	80
11	$\bar{1}$.2833	$\bar{1}$.2806	$\bar{1}$.2887	0.0081	0.7194	0.7113	$\bar{1}$.9919	0.1395	79
12	.3211	.3179	.3275	.0096	.6821	.6725	.9904	.1340	78
13	.3558	.3521	.3634	.0113	.6479	.6366	.9887	.1284	77
14	.3880	.3837	.3968	.0131	.6163	.6032	.9869	.1227	76
15	.4180	.4130	.4281	.0151	.5870	.5719	.9849	.1169	75
16	$\bar{1}$.4460	$\bar{1}$.4403	$\bar{1}$.4575	0.0172	0.5597	0.5425	$\bar{1}$.9828	0.1111	74
17	.4723	.4659	.4853	.0194	.5341	.5147	.9806	.1052	73
18	.4971	.4900	.5118	.0218	.5100	.4882	.9782	.0992	72
19	.5206	.5126	.5370	.0243	.4874	.4630	.9757	.0931	71
20	.5429	.5341	.5611	.0270	.4659	.4389	.9730	.0870	70
21	$\bar{1}$.5641	$\bar{1}$.5543	$\bar{1}$.5842	0.0298	0.4457	0.4158	$\bar{1}$.9702	0.0807	69
22	.5843	.5736	.6064	.0328	.4264	.3936	.9672	.0744	68
23	.6036	.5919	.6279	.0360	.4081	.3721	.9640	.0680	67
24	.6221	.6093	.6486	.0393	.3907	.3514	.9607	.0614	66
25	.6398	.6259	.6687	.0427	.3741	.3313	.9573	.0548	65
26	$\bar{1}$.6569	$\bar{1}$.6418	$\bar{1}$.6882	0.0463	0.3582	0.3118	$\bar{1}$.9537	0.0481	64
27	.6732	.6570	.7072	.0501	.3430	.2928	.9499	.0412	63
28	.6890	.6716	.7257	.0541	.3284	.2743	.9459	.0343	62
29	.7043	.6856	.7438	.0582	.3144	.2562	.9418	.0272	61
30	.7190	.6990	.7614	.0625	.3010	.2386	.9375	.0200	60
31	$\bar{1}$.7332	$\bar{1}$.7118	$\bar{1}$.7788	0.0669	0.2882	0.2212	$\bar{1}$.9331	0.0127	59
32	.7470	.7242	.7958	.0716	.2758	.2042	.9284	0.0053	58
33	.7604	.7361	.8125	.0764	.2639	.1875	.9236	$\bar{1}$.9978	57
34	.7734	.7476	.8290	.0814	.2524	.1710	.9186	.9901	56
35	.7859	.7586	.8452	.0866	.2414	.1548	.9134	.9822	55
36	$\bar{1}$.7982	$\bar{1}$.7692	$\bar{1}$.8613	0.0920	0.2308	0.1387	$\bar{1}$.9080	$\bar{1}$.9743	54
37	.8101	.7795	.8771	.0977	.2205	.1229	.9023	.9662	53
38	.8217	.7893	.8928	.1035	.2107	.1072	.8965	.9579	52
39	.8329	.7989	.9084	.1095	.2011	.0916	.8905	.9494	51
40	.8439	.8081	.9238	.1157	.1919	.0762	.8843	.9408	50
41	$\bar{1}$.8547	$\bar{1}$.8169	$\bar{1}$.9392	0.1222	0.1831	0.0608	$\bar{1}$.8778	$\bar{1}$.9321	49
42	.8651	.8255	.9544	.1289	.1745	.0456	.8711	.9231	48
43	.8753	.8338	.9697	.1359	.1662	.0303	.8641	.9140	47
44	.8853	.8418	.9848	.1431	.1582	.0152	.8569	.9046	46
45	.8951	.8495	0.	.1505	.1505	0.	.8495	.8951	45
	Log Coarc	Log Cos	Log Cot	Log Cosec	Log Sec	Log Tan	Log Sin	Log Arc	Angle Deg.

Table J. Logarithms of Numbers

n	0	1	2	3	4	5	6	7	8	9	Diff.
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	42
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	38
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	35
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	32
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	30
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	28
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	27
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	25
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	24
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	22
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	21
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	20
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	19
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	18
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	18
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	17
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	17
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	16
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	15
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	15
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	14
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	14
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	13
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	13
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	13
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	12
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	12
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	12
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	11
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	11
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	11
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	11
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	10
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	10
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	10
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	10
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	9
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	9
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	9
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	9
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	8
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	8
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	8
n	0	1	2	3	4	5	6	7	8	9	Diff.

Table J. Logarithms of Numbers (*Continued*)

n	0	1	2	3	4	5	6	7	8	9	Diff.
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	8
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	7
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	7
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	
72	8573	8579	8485	8591	8597	8603	8609	8615	8621	8627	
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	6
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	5
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	
n	0	1	2	3	4	5	6	7	8	9	Diff.

TABLE K. CONSTANTS AND THEIR LOGARITHMS

Name (Radius of circle or sphere = 1)	Symbol	Number	Logarithm
Area of circle	π	3.141 592 654	0.497 149 873
Circumference of circle	2π	6.283 185 307	0.798 179 868
Surface of sphere	4π	12.566 370 614	1.099 209 864
	$\frac{1}{6}\pi$	0.523 598 776	$\bar{1}$.718 998 622
Quadrant of circle	$\frac{1}{4}\pi$	0.785 398 163	$\bar{1}$.895 089 881
Area of semicircle	$\frac{1}{2}\pi$	1.570 796 327	0.196 119 877
Volume of sphere	$\frac{4}{3}\pi$	4.187 790 205	0.622 088 609
	π^2	9.869 604 401	0.994 299 745
	$\pi^{\frac{1}{2}}$	1.772 453 851	0.248 574 936
Degrees in a radian	$180/\pi$	57.295 779 513	1.758 122 632
Minutes in a radian	$10800/\pi$	3 437.746 771	3.536 273 883
Seconds in a radian	$648000/\pi$	206 264.806	5.314 425 133
	$1/\pi$	0.318 309 886	$\bar{1}$.502 850 127
	$1/\pi^{\frac{1}{2}}$	0.564 189 584	$\bar{1}$.751 425 064
	$1/\pi^2$	0.101 321 184	$\bar{1}$.005 700 255
Circumference/360	arc 1°	0.017 453 293	$\bar{2}$.241 877 368
	sin 1°	0.017 452 406	$\bar{2}$.241 855 318
Circumference/21600	arc 1′	0.000 290 888	4.463 726 117
	sin 1′	0.000 290 888	$\bar{4}$.463 726 111
Circumference/1296000	arc 1″	0.000 004 848	$\bar{6}$.685 574 867
	sin 1″	0.000 004 848	$\bar{6}$.685 574 867
Base Naperian system of logs	e	2.718 281 828	0.434 294 482
Modulus common system of logs	M	0.434 294 482	$\bar{1}$.637 784 311
Naperian log of 10	$1/M$	2.302 585 093	0.362 215 689
	hr	0.476 936 3	$\bar{1}$.678 460 4
Probable error constant	$hr\sqrt{2}$	0.674 489 7	$\bar{1}$.828 975 4

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