which clearly shows that this loss decreases as the number of valves increases, when a is kept constant. Therefore the suction and discharge chambers may be made to give a hydraulic advantage, either by using many valves of a given size or by making the total valve area na sufficiently large, since h' is thus diminished. The number of valves will usually be 8, 12, or 16.

As a numerical example, take a plunger force pump, like Fig. 194e, having a piston area $A = 0.84$ square feet, and a stroke of 1.25 feet, the number of single strokes per minute being 30. The volume of water lifted per second is hence $30 \times 0.82 \times 1.25/60 = 0.525$ cubic feet. Let the diameter of the suction pipe be 10 inches and the area of its cross-section $a_1 = 0.545$ square feet. The mean velocity in the suction pipe is then $0.525/0.545 = 0.96$ feet per second. Let there be 12 valves in the suction chamber, so that $n = 6$, and let the area of each valve opening be $a = 8$ square inches $= 0.0556$ square feet. The velocity through each of the open valves is then $V = 0.525/3 \times 0.0556 = 3.15$ feet per second. As Art. 92 does not give the values of m' for poppet valves, it may be here noted that the experiments of Bach* indicate that they range from 1.1 to 2.8, depending upon the height of valve lift and the width of the seat. Taking 2 as a mean value of m', the lost head in the pump is

$$h' = 0.01555\left[1 + 8 \times 3\left(\frac{0.545}{6 \times 0.0556}\right)^2\right]0.96^2 = 0.96 \text{ feet.}$$

The useful head h, when the lengths of the suction and discharge pipes are disregarded, is probably about 3 feet, so that the hydraulic efficiency is $e = h/(h+h') = 0.75$. If the lengths of the vertical suction and discharge pipes be each 20 feet and their diameters be 10 inches, the useful head h is about 43 feet and from (195) the value of h' is found to be about one foot, so that the hydraulic efficiency is about 0.97. The velocity-head $v_2^2/2g$ which is lost at the top of the discharge pipe is here only 0.01 feet, so that it is unnecessary to consider it in determining the efficiency.

This discussion shows that the losses of head in force pumps may be made very slight by running them at low speeds in order that the velocity v_1 may be small. It shows that the losses decrease as the areas of the valve openings and their number are increased. It shows

* Zeitschrift deutscher Ingenieur Verein, 1886, p. 421.

that, for vertical suction and discharge pipes, the efficiency increases with the useful lift h, if the velocity in the pipes is the same for different lifts. These conclusions are verified by experiments, some of which will be noted in the next article. Since the flow through the valves and pump cylinder is not quite steady, numerical computations like the above cannot, however, be expected to give more than rough approximate results; nevertheless such results are useful in indicating the influence of the resistances upon the efficiency.

Prob. 195. For the above numerical example, compute the horse-power required to run the pump when the useful lift is 43 feet, assuming that 3 percent of that power is expended in overcoming friction in the stuffing boxes.

Art. 196. Pumping Engines

The steam engine was invented and perfected through the desire to devise methods of pumping water better than those in which the power of men and horses was used. Worcester in 1633, and Papin in 1695, used the direct pressure of steam upon water in a cylinder, and Savery in 1700 used both such pressure and the partial vacuum caused by the condensation of the steam. Newcomen in 1705 used a piston, on one side of which steam was applied and condensed, the motion of the piston being communicated by a walking beam to the piston rod of a pump. Watt, about 1775, introduced the crank, the parallel motion, the cut-off, the governor, and other improvements; he also brought the steam to both sides of the piston, thus making the engine double-acting. The first important application of the steam engine was in operating pumps to drain mines, but it soon came into use in all branches of industry where power was needed. Its influence on modern progress has been great.

The modern pumping engine consists of one or more steam cylinders connected to the same number of pump cylinders by piston rods, so that the steam pressure is directly transmitted through them to the water. It is important that the pressure in the water cylinder should be maintained nearly constant during the length of the stroke, and hence the steam should not be used expansively in the usual way; to insure constant steam pressure some form of compensator is used. The water cylinders are usually of the plunger type, and these are connected to the same suction and discharge pipes, an air chamber being placed on the latter to relieve the pump chambers of shock and to insure steady flow. The boilers, steam cylinders, and water cylinders constitute one machine or apparatus called a pumping engine. The efficiency of this apparatus is low, for it is equal to the product of the efficiencies of its separate parts. The efficiency of the furnace and boiler is about 75 percent in the best designs, the efficiency of the steam cylinders about 30 percent, and that of the water cylinders about 80 percent, so that the efficiency of the pumping engine as a whole is only 18 percent. This means that only 18 percent of the energy of the fuel is utilized in lifting the water, and this figure is, indeed, a high one, for many pumping plants are operated with an efficiency of less than 10 percent.

The term "duty" is often employed as a measure of the performance of a pumping engine, instead of expressing it by an efficiency percentage. This term was devised by Watt, who defined duty as the number of foot-pounds of useful work produced by the consumption of 100 pounds of coal. On account of the variable quality of coal a more precise definition of duty was introduced in 1890 by a committee of the American Society of Mechanical Engineers, namely, that duty should be the number of foot-pounds of work produced by the expenditure of 1 000 000 British thermal heat units. One British thermal heat unit is that amount of energy which will raise one pound of pure water one Fahrenheit degree in temperature when the water is at or near the temperature of maximum density (Art. 3); this amount of energy is 778 foot-pounds, and this constant is called the mechanical equivalent of heat. The duty of a perfect pumping engine, in which no losses of any kind occur, would be 778 000 000 foot-pounds. The highest duty obtained in a test is about 180 000 000 foot-pounds, and the efficiency of such an engine is $180/778 = 0.23$.* Common pumping engines have duties ranging from 120 000 000 to 60 000 000, the corresponding efficiencies being from 15 to 7.5 percent. The modern definition of duty

* Transactions American Society of Civil Engineers, vol. 73, 1911.

agrees with that of Watt, if the coal used be of such quality that one pound of it possesses a potential energy of 10 000 British heat units, which is somewhat less than that obtainable from average coal. The higher the duty of a pumping engine the greater is the amount of work that can be performed by burning a given quantity of coal. A high-duty engine is hence economical and a low-duty engine is wasteful in coal consumption, but the first cost of the former is much greater than that of the latter.

A duty test of a pumping engine consists in determining the number of heat units furnished by a given quantity of coal, the quantity of water lifted by the pump, the leakage past the piston packing, the pressure-heads in the suction and discharge pipes, the indicated horse-power of the steam cylinders, and many other minor quantities needed for estimating the efficiency of the boiler and steam part of the apparatus. The usual method of determining the discharge is by the displacement of the piston or plunger; if A be the area of its cross-section, l the length of the stroke, N the number of single strokes during the test, and T the number of seconds during which the test lasted, then NAl is the total quantity of water lifted, and

$$q = cNAl/T$$

is the quantity lifted per second, c being a coefficient which takes account of the leakage or slip past the plunger. The value of c is to be found by removing one of the cylinder heads and admitting water on the other side of the plunger, and its value is usually from 0.99 to 0.95 in new pumps, The total pressure-head H is found from

$$H = (h_2 \pm h_1 + d)$$

where h_1 and h_2 are the pressure-heads corresponding to the mean readings of the gages on the suction and discharge pipes and d the vertical distance between the centers of the gages; here the plus sign is to be used when the corresponding pressure is below and the minus sign when it is above that of the atmosphere. The total work done by the pump during the trial is then $cNAl \cdot H$ and then the duty of the pumping engine

$$\text{Duty} = 1\,000\,000\, cNAlH/\text{heat units},$$

in which the denominator is determined by the thermodynamic tests made on the boiler and steam engine. The capacity of the pump, or the quantity of water lifted in 24 hours, is $24 \times 3600 \times q$.

The efficiency of pump cylinders, which are tested in the above manner, is usually found by dividing the work wqH done by them in one second by that done by the steam as determined by indicator cards taken from the steam cylinders. This method differs from that used in the previous articles, and gives results too small from the standpoint of hydraulic losses. A discussion by Webber * of several tests shows that this efficiency increases with the lift as follows:

Lift in feet,	5	15	30	100	170	270
Efficiency,	0.30	0.45	0.65	0.85	0.91	0.88

The highest value of 91 percent was obtained from a test of a Leavitt pumping engine having a duty of 111 549 000 foot-pounds, and a capacity of 4 400 000 gallons per 24 hours; the duration of this test was 15.1 hours.

Prob. 196. In a test lasting 12 hours, 27 502 000 heat units were produced under the boiler. The area of the plunger was 172 square inches, the length of the stroke was 18.9 inches, the number of single strokes was 76 000, and the leakage past the plunger packing was 5900 cubic feet. The pressure gage on the force pipe read 100 and the vacuum gage on the suction pipe read 9.3 pounds per square inch, the distance between the centers of these gages being 8 feet. The mean indicated horse-power of the steam cylinders was 128. Compute the discharge of the pump in cubic feet per second and its capacity in gallons per day. Compute the total pressure-head H. Compute the duty of the pumping engine. Compute the efficiency of the pump cylinders.

Art. 197. The Centrifugal Pump

The centrifugal pump is the reverse of a turbine wheel, and any reaction turbine, when run backwards by power applied to its axle, will raise water through its penstock. The centrifugal pump, like the turbine, is of modern origin and development. A rude form, devised by Ledemour in 1730, consisted of an inclined tube attached by arms to a vertical shaft; the lower

* Transactions American Society Mechanical Engineers, 1886, vol. 7, p. 602.

end of the tube being immersed, the water flowed from its upper end when the shaft was rotated. It was not, however, until about 1840 that the first true centrifugal pumps came into use, and they have since been perfected so as to be of great value in engineering operations, especially for low lifts.

Fig. 197 shows the principle of the arrangement and action of the centrifugal pump. The power is applied through the axis A to rotate the wheel BB in the direction indicated by the arrow. This wheel is formed of a number of curved vanes like those in a turbine wheel (Art. 174). The revolving vanes produce a partial vacuum, and this causes the water to rise in the suction pipe DD which enters through the center of the wheel case and delivers the water at the axis of the wheel. The water is then forced outward through the vanes and passes into the volute chamber CC, which is of varying cross-section in order to accommodate the increasing quantity of water that is delivered into it, and all of which passes up the discharge pipe E. The rotation of the wheel hence produces a negative pressure at the upper end of the suction pipe and a positive pressure in the volute chamber, and the water rises in the pipes in the same manner as in those of a suction and force pump. The height of the suction lift cannot usually exceed about 28 feet.

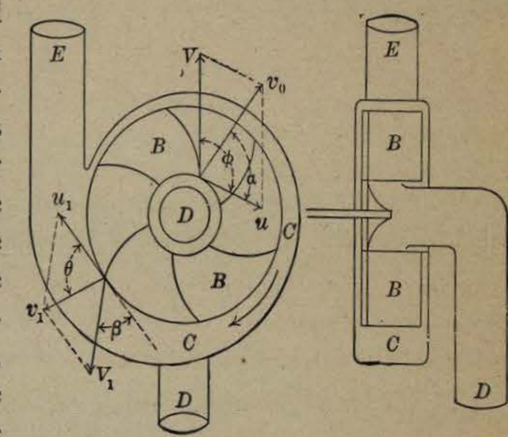

Fig. 197.

The parallelograms of velocity shown in Fig. 197 are the same as in the reaction turbine (Art. 174), and a similar notation will be used. The velocities of rotation of the inner and outer circumferences will be called u and u_1, the absolute velocities of the water as it enters and leaves the wheel are v_0 and v_1, and the corresponding relative velocities are V and V_1. The angles of entrance, approach, and exit are called ϕ, α, and β, while θ denotes the angle between v_1 and u_1. Let H_0 be the pressure-head at the top of the entrance pipe and H_1 that at the foot of the discharge pipe, while h_0 and h_1 are the heights of the suction and force lifts estimated downward and upward from the center of the wheel, and let h_a be the height of the water barometer. Then from formula (162)

$$V^2 - u^2 - V_1^2 + u_1^2 = 2g(H_1 - H_0)$$

and also from $(31)_2$, not considering frictional resistances,

$$H_1 = h_a + h_1 - \frac{v_1^2}{2g} \qquad H_0 = h_a - h_0 - \frac{v_0^2}{2g}$$

Combining these equations, and replacing $h_1 + h_0$ by h, where h is the total lift, the fundamental equation for the discussion of frictionless centrifugal pumps results. To introduce the frictional losses, however, $h + h'$ should be used instead of h, where h' is the total head lost in all the hydraulic resistances. Then

$$V^2 - V_1^2 - u^2 + u_1^2 + v_1^2 - v_0^2 = 2g(h + h') \qquad (197)_1$$

is the fundamental formula for the discussion of the centrifugal pump. Since there are no guides, the water enters the vanes radially, so that the approach angle α is a right angle, and hence $V^2 = u^2 + v_0^2$. Also the parallelogram of velocities at exit gives $V_1^2 = u_1^2 + v_1^2 - 2u_1v_1\cos\theta$. Inserting these values of V_2 and V_1^2 in $(197)_1$, it reduces to

$$u_1v_1 \cos\theta = g(h + h')$$

which is a necessary relation connecting u_1 and v_1.

A centrifugal pump must be run at a certain velocity in order to overcome the pressure-head $h + h'$ by means of the velocity-head $v_1^2/2g$ of the issuing water. Hence $h + h' = v_1^2/2g$, and equating this to the value of $h + h'$ established by the above formula, there results $u_1 \cos\theta = \frac{1}{2}v_1$. It hence follows from the parallelogram of velocities that V_1 and u_1 must be equal. Then $\theta = 90° - \frac{1}{2}\beta$, and

$$u_1 = \frac{v_1}{2\sin\frac{1}{2}\beta} \quad \text{or} \quad u_1 = \frac{\sqrt{2g(h+h')}}{2\sin\frac{1}{2}\beta} \qquad (197)_2$$

gives the required velocity of the outer circumference of the wheel. This velocity decreases as the exit angle β increases; when β is very small, u_1 is very large; when the vanes are radial at the outer circumference, β is 90° and $u_1 = \sqrt{g(h + h')}$. Hence the speed of the pump must increase with the square root of the pressure-head $h + h'$. Since $v_1 = q/a_1$, where a_1 is the area of the exit orifices normal to v_1, the velocity is also $u_1 = q/2a_1 \sin\frac{1}{2}\beta$, and therefore the discharge q increases directly with the speed.

Since the speed must increase with the lift, and since the losses of head increase with the speed, it follows that the efficiency of the centrifugal pump in general decreases with the lift. This theoretic conclusion has been verified by practical tests. Webber, in his discussion cited in the last article, gives the following as the mean results derived from a number of experiments, the efficiency computed being the ratio of the work done by the pump to that obtained from indicator cards taken on the cylinders of the steam motor:

Lift in feet,	5	10	20	40	60
Efficiency,	0.56	0.64	0.68	0.58	0.40

For a low lift the centrifugal pump has a hydraulic efficiency higher than these figures indicate, but, as in the case of the force pump, it is difficult to determine reliable values by numerical computations.

The centrifugal pump possesses an advantage over the force pump in having no valves and in being able to handle muddy water, for even gravel may pass through the vanes without injuring them. The above figure represents the principle rather than the actual details of construction. Usually the suction pipe is divided into two parts which enter the axis upon opposite sides of the wheel, and the volute chamber is often made wider than the wheel case, thus forming what is called a whirlpool chamber, which prevents some of the losses of head due to impact. The vanes are sometimes curved in the opposite direction to that shown in the figure, as by so doing the angle β is made larger and the speed of the pump is lessened, as is seen from formula $(197)_2$. The theory of the centrifugal pump is, however, much less definite than that of the reaction turbine, and experiment is the best guide to determine the advantageous shape of the vanes.

Multiple stage centrifugal pumps for work against high heads are extensively used.*†

Prob. 197. A centrifugal pump lifts 120 cubic feet of water per minute through a discharge pipe having a diameter of 1 foot. The outer diameter of the wheel is 2 feet, the exit angle is 90°, the number of revolutions per second is 60, and the water is lifted 18 feet. Compute the horse-power of the pump, and its hydraulic efficiency.

Art. 198. The Hydraulic Ram

The hydraulic ram is an apparatus which employs the dynamic pressure produced by stopping a column of moving water to raise a part of this water to a higher level than that of its source. The principle of its action was recognized by Whitehurst in 1772,‡ but the credit of perfecting the machine is due to Montgolfier, who in 1796 built the first self-acting ram. It has since been widely used for pumping small quantities of water from streams to houses, but is not so well adapted to lifting a large quantity; many attempts have been made in this direction, some of which give promise of much usefulness.

The principle of the action of the hydraulic ram is shown in Fig. 198, where A is the reservoir that furnishes the supply, BCD

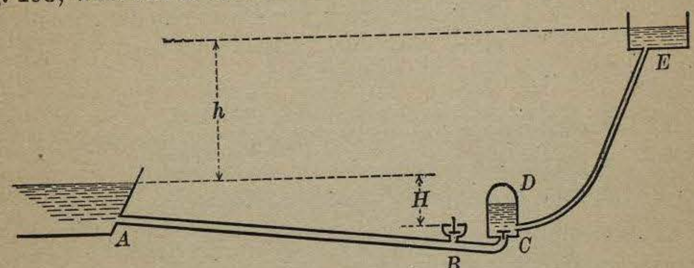

Fig. 198.

the ram, AB the drive pipe which carries the water to the ram, DE the discharge pipe through which a part of the water is raised to the tank E. The ram itself consists merely of the waste valve B through which a part of the water from the drive pipe

* Journal American Society of Mechanical Engineers, Jan. and March, 1910.

† Journal Western Society of Engineers, April, 1910.

‡ Transactions Royal Society, 1775, vol. 65, p. 277.

escapes, and the air vessel D which has a valve C that allows water to enter it through BC, but prevents its return. The waste valve B is either weighted or arranged with a spring so that it will open when acted upon by the static pressure due to the head H. As soon as it opens the water flows through it, but as the velocity increases the dynamic pressure due to the motion of the column AB (Art. 157) becomes sufficiently great to close the valve B. Then this dynamic pressure opens the valve C and compresses the air in the air chamber or forces water up the discharge pipe. A moment later when equilibrium has obtained in the air vessel, the valve C closes and the air pressure maintains the flow for a short period in the discharge pipe, while the water in the drive pipe comes to rest. Then the waste valve B opens again, and the same operations are repeated.

The algebraic discussion of the hydraulic ram is very difficult because it involves the time in which the waste valve closes and the law of its rate of closing. The investigation in Art. 157, however, clearly shows that the operations above described will take place if the drive pipe is long enough to produce a dynamic pressure sufficient to close the waste valve. Let l be the length of that pipe, v the velocity in it, p_0 the static unit pressure due to H, w the weight of a cubit unit of water, g the acceleration of gravity, and t the time in which the valve closes. Then, since there is no static pressure at the valve during the flow, the formula $(157)_1$ gives

$$p = 2wlv/gt - p_0$$

which is a good approximation to the excess of dynamic pressure over the static pressure p_0. It is seen that this excess p may be rendered very great by making l large and t small, and that its greatest value is

$$p = wuv/g - p_0$$

in which u is the velocity of sound in water. It is rare, however, that a drive pipe is sufficiently long to furnish the excess dynamic pressure given by the last formula.

The efficiency of the hydraulic ram is the ratio of the useful work done to the energy expended in the waste water. Let q be the quantity of water lifted per second through the height h

from the level of the reservoir A to that of the tank E. Let Q be the discharge per second through the waste valve and H the height through which it falls, then the efficiency of the ram and its pipes is

$$e = \frac{wqh}{wQH} = \frac{qh}{QH}$$

It is found by experiment that the efficiency decreases as the ratio h/H increases. Eytelwein found that e was 0.92 when h/H was unity, 0.67 when h/H was 5, and 0.23 when h/H was 20, but these values were probably derived by using a different formula for the efficiency.

Experiments in 1890 at Lehigh University on a Gould ram No. 2, in which the waste valve made 55 strokes per minute, gave a mean efficiency of 35 percent. The length of the supply pipe was 38 feet and its fall 12 feet, the length of the discharge pipe 60 feet, and the lift h was 12 feet, so that the ratio h/H was unity. These experiments showed also that the efficiency increased as the number of strokes per minute was decreased by lessening the weight on the waste valve. The maximum quantity of water raised per minute, however, occurred with a heavier waste valve than that which gave the maximum efficiency. The efficiency was also found to increase as the length of the stroke of the waste valve decreased.

The least possible fall in the drive pipe of the hydraulic ram is about $1\frac{1}{2}$ feet and the least length of drive pipe about 15 feet. It is customary to make the area of the discharge pipe from one-third to one-fourth that of the drive pipe, and with these proportions a fall of 10 feet will force water to a height of nearly 150 feet. A common rule of manufacturers is that about one-seventh of the water flowing down the drive pipe may be raised to a height five times that of the fall in the drive pipe; this is a rough rule only, for the length of the discharge pipe is one of the controlling factors as well as its vertical rise.

The Rife hydraulic engine is a water ram on a large scale, two or more being connected to the same discharge pipe, so that the flow in it is nearly continuous.* Three of these engines are said to raise 864 000 gallons of water per day to an elevation of 150 feet, the fall in the drive pipe being 30 feet. The diameter of the drive pipe is 8 inches and that of the discharge pipe is 4 inches; the waste valve weighs

* Engineering News, 1896, vol. 36, p. 429.

50 pounds, and it is provided with an adjusting lever in order that its effective weight may be regulated so as to cause the maximum discharge to be delivered.

Prob. 198. A hydraulic ram raises $32\frac{1}{2}$ pounds of water in 5 minutes through a discharge pipe 60 feet long. The drive pipe is 38 feet long and the amount of water wasted in 5 minutes is $41\frac{1}{2}$ pounds. The fall of the drive pipe is 12 feet and the vertical rise of the discharge pipe above the ram is 24 feet. Compute the efficiency of the ram.

Art. 199. Other Kinds of Pumps

The lift and force pumps described in Arts. 193 and 194 are called displacement pumps, because the volume of water lifted in one stroke is that displaced by the piston or plunger. If there be no leakage past the piston packing, and if no air is mingled with the water, the discharge in a given time may be very accurately determined by counting the number of strokes and multiplying this number by the displacement in one stroke. On account of the reciprocating motion of the piston these forms are often called reciprocating pumps. There is always a loss of energy due to putting the piston into motion at the beginning of each stroke, and to avoid this many forms of rotary pumps have been devised; yet notwithstanding this loss the plunger force pump is probably the most efficient and economical of all kinds.

A rotary or impeller pump is one in which the moving parts have a circular motion only, and the centrifugal pump described in Art. 197 is of this kind. Numerous other rotary pumps have been invented, but none is widely used except the centrifugal one. Fig. 199*a* shows one where the moving parts consist of two wheels which are rotated in opposite directions as indicated by the arrows; this motion produces a partial vacuum whereby the water rises in the suction pipe D, and is then carried between the teeth and the case and forced up the discharge pipe E. Fig. 199*b* shows a form where the moving parts are two lobes in contact with each other and each in contact with the inclosing case. In the left-hand diagram the water rising in the pipe D is flowing toward the right, but a moment later the lobe B has assumed