wheels have penstocks and shafting similar to those of units 1–10, but the wheels are of the Jonval type, the flow being inward and downward. The wheel case has the form of a flattened sphere, the water entering from one side and passing through the guides to a single turbine 64 inches in diameter and 23.5 inches deep. After leaving the wheel, the water passes to two draft tubes, each about 58 inches in diameter, and is discharged near the invert of the tail race at an angle of 45° to the horizontal axis of the wheel pit. The wheel case is supported on these two draft tubes as on two legs, while the penstock is supported on iron lugs in the same way as those of units 4–10. By these draft tubes the head on the wheel is increased to 144 feet, this being the difference from the water level in the head race to that in the tail race. The balancing pistons are below the wheels, and are supported from an independent pipe instead of from the penstock. Each shaft is also supplied with an oil step-bearing, which is designed to support, if necessary, the entire revolving weight at the normal speed of 250 revolutions per minute.

Prob. 182*a*. Compute the hydraulic efficiency of the turbines described above. Compute the velocity v_0 with which the water enters the lower wheel and the velocity v_1 with which it leaves the same when the speed is 250 revolutions per minute.

Prob. 182*b*. Compute the efficiency of a reaction wheel under a head of 3.5 meters when the radius of the exit orifices is 0.64 meters, the coefficient of velocity 0.95, and the number of revolutions per minute is 130.

Prob. 182*c*. Design an outward-flow reaction turbine which shall use 8 cubic meters of water per second under a head of 12.4 meters, taking the entrance angle ϕ as 90°.

Prob. 182*d*. A dynamo delivering 4100 kilowatts has an efficiency of 97.5 percent, while the efficiency of the turbine is 81.3 percent and that of the approaches to the turbine is 99.7 percent. The turbine is of the Jonval type, and the difference between the levels of head and tail race is 14.4 meters. How many cubic meters of water are used per second?

Prob. 182*e*. Consult engineering periodicals and describe other large power plants for the development of electrical energy which have been installed at Niagara Falls, especially that of the Canadian Niagara Power Company and that of the Ontario Power Company.

CHAPTER 15

NAVAL HYDROMECHANICS

Art. 183. General Principles

In this chapter is to be discussed in a brief and elementary manner the subject of the resistance of water to the motion of vessels, and the general hydrodynamic principles relating to their propulsion. The water may be at rest and the vessel in motion, or both may be in motion as in the case of a boat going up or down a river. In either event the velocity of the vessel relative to the water need only be considered, and this will be called v. The simplest method of propulsion is by the oar or paddle; then come the paddle wheel, and the jet and screw propellers. The action of the wind upon sails will not be here discussed, as it is outside of the scope of this book.

The unit of linear measure used on the ocean is generally the nautical mile, while one nautical mile per hour is called a knot. One nautical mile is about 6080 feet, so that knots may be transformed into feet per second by multiplying by 1.69, and feet per second may be transformed into knots by multiplying by 0.592. On rivers the speed is estimated in statute miles per hour, and the corresponding multipliers will be 1.47 and 0.682. One kilometer per hour equals 0.621 miles per hour or 0.91 feet per second. On the ocean the weight of a cubic foot of water is to be taken as about 64 pounds (it is often used as 64.32 pounds, so that the numerical value is the same as $2g$), and in rivers at 62.5 pounds.

The speed of a ship at sea was formerly roughly measured by observations with the log, which is a triangular piece of wood attached to a cord which is divided by tags into lengths of about $50\frac{2}{3}$ feet. The log being thrown into the water, it remains stationary, the ship moves away from it, and the number of tags run out in half a minute is counted; this number is the same as the number of knots per hour at which the ship is moving, since $50\frac{2}{3}$ feet is the same part of a knot that a half minute is of an hour. The patent log, which is a small self-recording current meter, drawn in the water behind the ship, is, however, now generally used, this being rated at intervals (Art. 40). In experimental work more accurate methods of measuring the velocity are necessary, and for this purpose the boat may run between buoys whose distance apart has been found by triangulation from measured bases on shore.

The Pitot tube has recently been applied to the determination of the velocity of a ship through the water. By the use in connection with this tube of a recording mechanism similar to that described in Art. 38 for the Venturi meter it would seem possible to automatically record on dials both the speed through the water as well as the total number of miles passed over. By the use of a chart an autographic record of variations in the speed could also be kept. Practical difficulties in the way of keeping the mouths of the Pitot tubes free from obstructions have already been to a certain extent overcome.*

When a boat or ship is to be propelled through water, the resistances to be overcome increase with its velocity, and consequently, as in railroad trains, a practical limit of speed is soon attained. These resistances consist of three kinds: the dynamic pressure caused by the relative velocity of the boat and the water, the frictional resistance of the surface of the boat, and the wave resistance. The first of these can be entirely overcome, as indicated in Art. 155, by giving to the boat a "fair" form; that is, such a form that the dynamic pressure of the impulse near the bow is balanced by that of the reaction of the water as it closes in around the stern. It will be supposed in the following pages that the boat has this form, and hence this first resistance need not be further considered. The second and third sources of resistance will be discussed later.

* Engineering News, May 4, 1911.

The total force of resistance which exists when a vessel is propelled with the velocity v can be ascertained by drawing it in tow at the same velocity, and placing on the tow line a dynamometer to register the tension. An experiment by Froude on the Greyhound, a steamer of 1157 tons, gave for the total resistance the following figures: *

Speed in knots,	4	6	8	10	12
Resistance in tons,	0.6	1.4	2.5	4.7	9.0

which show that at low speeds the resistance varies about as the square of the velocity, and at higher speeds in a faster ratio. For speeds of 15 to 25 knots, the usual velocity of ocean steamers, the law of resistance is not so well known, but as an approximation it is usually taken as varying with the square of the velocity.

Prob. 183. What horse-power was expended in the above test of the Greyhound when the speed was 12 knots per hour?

Art. 184. Frictional Resistances

When a stream or jet moves over a surface, its velocity is retarded by the frictional resistances, or if the velocity be maintained uniform, a constant force is overcome. In pipes, conduits, and channels of uniform section the velocity is uniform, and consequently each square foot of the surface or bed exerts a constant resisting force, the intensity of which will now be approximately computed. This resistance will be the same as the force required to move the same surface in still water, and hence the results will be directly applicable to the propulsion of ships.

Let F be the force of frictional resistance per square foot of surface of the bed of a channel, p its wetted perimeter, l its length, h its fall in that length, a the area of its cross-section, and v the mean velocity of flow. The force of friction over the entire surface then is Fpl, and the work per second lost in friction is $Fplv$. The work done by the water per second is Wh or $wavh$. Equating these two expressions for the work, there results

$$F = w(a/p)(h/l) = wrs$$

* Thearle's Theoretical Naval Architecture (London, 1876), p. 347.

in which r is the hydraulic radius and s the slope of the water surface. Now inserting for rs its value from formula (113), there results

$$F = wv^2/c^2$$

in which w is the weight of a cubic foot of water and c is the coefficient in the Chezy formula, the values of which are given in Chap. 9 and the accompanying tables. Inasmuch as the velocities along the bed of a channel are somewhat less than the mean velocity v, the values of F thus determined will probably be slightly greater than the actual resistance.

For smooth iron pipes the following are computed values of the frictional resistance in pounds per square foot of surface:

Velocity, feet per second =	2	4	6	10	15
for 1 foot diameter	$F = 0.023$	0.080	0.17	0.43	0.92
for 4 feet diameter	$F = 0.015$	0.053	0.11	0.28	0.59

These figures indicate that the resistance is subject to much variation in pipes of different diameters; it is not easy to conclude from them, or from formula (113), what the force of resistance is for plane surfaces over which water is moving.

Experiments made by moving flat plates in still water so that the direction of motion coincides with the plane of the surface have furnished conclusions regarding the laws of fluid friction similar to those deduced from the flow of water in pipes. It is found that the total resistance is approximately proportional to the area of the surface, and approximately proportional to the square of the velocity. Accordingly the force of resistance per square foot may be written

$$F = fv^2, \tag{184}$$

in which v is the velocity in feet per second and f is a number depending upon the nature of the surface. The following are average values of f for large surfaces, as given by Unwin:*

Varnished surface,	$f = 0.00250$
Painted and planed plank,	$f = 0.00339$
Surface of iron ships,	$f = 0.00351$
Fine sand surface,	$f = 0.00405$
New well-painted iron plate,	$f = 0.00473$

* Encyclopedia Britannica, 9th Ed., vol. 12, p. 483; 11th Ed., vol. 14, p. 57.

Undoubtedly the value of f is subject to variations with the velocity, but the experiments on record are so few that the law and extent of its variation cannot be formulated. It should, however, be remarked that the formulas and constants here given do not apply to low velocities, for the reasons given in Art. 124. At the same time they are only approximately applicable to high velocities. A low velocity of a body moving in an unlimited stream may be regarded as 1 foot per second or less, a high velocity as 25 or 30 feet per second.

It may be noted that the above-mentioned experiments indicate that the value of F is greater for small surfaces than for large ones. For instance, a varnished board 50 feet long gave $f = 0.00250$, while one 20 feet long gave $f = 0.00278$, and one 8 feet long gave $f = 0.00325$, the motion being in all cases in the direction of the length. The resistance is the same whatever be the depth of immersion, for the friction is uninfluenced by the intensity of the static pressure. This is proved by the circumstance that the flow of water in a pipe is found to depend only upon the head on the outlet end, and not upon the pressure-heads along its length.

The frictional resistance of a boat or ship may be roughly estimated by taking $0.004v^2$ and multiplying it by the immersed area. For instance, if this area be 8000 square feet, the frictional resistance at a velocity of 10 feet per second is 3200 pounds, but at a velocity of 20 feet per second it is 12 800 pounds; the horse-powers needed to overcome these resistances are 58 and 464, respectively. To these must be added the power necessary to overcome the friction of the air and that wasted in the production of waves.

The above discussion refers to the case of boats moving in the ocean and lakes or in a stream of large width and depth. In a canal the resistance is much greater, and it depends upon the ratio of the cross-section of the canal to that of the immersed portion of the boat. It depends also on the depth of the water. The "drag" of a ship in shoal water is very pronounced. For some experiments on the suction of vessels consult.* When the width of the canal is about five times that of the boat and the area of its cross-section about seven times that of the boat, the resistance is but slightly greater than in an

* Transactions American Society of Naval Architects and Marine Engineers, vol. 17, 1909.

unlimited stream. For smaller ratios the resistance rapidly increases, and when two boats pass each other in a small canal, the utmost power of the horses may be severely taxed. The reason for this increased resistance appears to be largely due to the fact that the velocity of the water relative to the boat increases with the diminution of the cross-section of the canal. Thus, if a and A be the areas of the cross-section of the canal and of the immersed part of the boat, the effective area of the water cross-section is $a - A$, and the water flowing backward through this area must have a higher relative velocity as A increases. The value of F given by formula (184) is accordingly increased to $fv^2/(1 - (A/a))^2$.

Prob. 184*a*. What horse-power is required to overcome the frictional resistance of a boat moving at the rate of 9 knots per hour when the area of its immersed surface is 320 square feet?

Prob. 184*b*. A canal has a cross-section of 360 square feet, while that of a canal boat is 60 square feet. Show that when two boats pass each other, the resistance of each is increased about 60 percent.

Art. 185. Work Required for Propulsion

When a boat or ship moves through still water with a velocity v, it must overcome the pressure due to impulse of the water and the resistance due to the friction of its surface on the water and air. If the surface be properly curved, there is no resultant pressure due to impulse, as shown in Art. 155. The resistance caused by friction of the immersed surface on the water can be estimated, as explained above. If A be the area of this surface in square feet, the work per second required to overcome this resistance is

$$k = AFv = fAv^3 \qquad (185)$$

The work, and hence the horse-power, required to move a boat accordingly varies approximately as the cube of its velocity. By the help of the values of f given in the last article an approximate estimate of the work can be made for particular cases. The resistance of the air, which in practice must be considered, will be here neglected.

To illustrate this law let it be required to find how many tons of coal will be used by a steamer in making a trip of 3000 miles in 6 days, when it is known that 800 tons are used in making

the trip in 10 days. As the power used is proportional to the amount of coal, and as the distances traveled per day in the two cases are 500 miles and 300 miles, the law gives $T/480 = (5/3)^3$, whence $T = 2220$ tons. By the increased speed the expense for fuel is increased 277 percent, while the time is reduced 40 percent. If the value of wages, maintenance, interest, etc., saved on account of the reduction in time, will balance the extra expense for fuel, the increased speed is profitable. That such a compensation occurs in many instances is apparent from the constant efforts to reduce the time of trips of passenger steamers.

When a boat moves with the velocity v in a current which has a velocity u in the same direction, the velocity of the boat relative to the water is $v - u$, and the resistance is proportional to $(v - u)^2$ and the work to $(v - u)^3$. If the boat moves in the opposite direction to the current, the relative velocity is $v + u$, and of course v must be greater than u or no progress would be made. In all cases of the application of the formulas of this article and the last, v is to be taken as the velocity of the boat relative to the water.

Another source of resistance to the motion of boats and ships is the production of waves. This is due in part to a different level of the water surface along the sides of the ship due to the variation in static pressure caused by the velocity, and in part to other causes. It is plain that waves, eddies, and foam cause energy to be dissipated in heat, and that thus a portion of the work furnished by the engines of the boat is lost. This source of loss is supposed to consume from 10 to 40 percent of the total work, and it is known to increase with the velocity. On account of the uncertainty regarding this resistance, as well as those due to the friction of the water and air, practical computations on the power required to move boats at given velocities can only be expected to furnish approximate results.

The investigations of Rankine on this difficult subject led to the conclusion announced in 1858 in the anagram (20*a*, 4*b*, 6*c*, 9*d*, 34*e*, 8*f*, 4*g*, 16*h*, 10*i*, 5*l*, 3*m*, 15*n*, 14*o*, 4*p*, 3*q*, 14*r*, 13*s*, 25*t*, 4*u*, 2*v*, 2*w*, 1*x*, 4*y*).* The meaning of this anagram was published in 1861: "The resistance of a sharp-ended ship exceeds the resistance of a current of water of

* Philosophical Magazine, September, 1858.

the same velocity in a channel of the same length and mean girth by a quantity proportional to the square of the greatest breadth divided by the square of the length of the bow and stern."

Prob. 185. Compute the horse-power required to maintain a velocity of 18 knots per hour, taking $A = 7473$ square feet and $f = 0.004$.

ART. 186. THE JET PROPELLER

The method of jet propulsion consists in allowing water to enter the boat and acquire its velocity, and then to eject it backwards at the stern by means of a pump. The reaction thus produced propels the boat forward. To investigate the efficiency of this method, let W be the weight of water ejected per second, V its velocity relative to the boat, and v the velocity of the boat itself. The absolute velocity of the issuing water is then $V - v$, and it is plain without further discussion that the maximum efficiency will be obtained when this is 0, or when $V = v$, as then there will be no energy remaining in the water which is propelled backward. It is, however, to be shown that this condition can never be realized and that the efficiency of jet propulsion is low.

The effective work which is exerted on the boat by the reaction of the issuing water is

$$k = W\frac{(V-v)v}{g}$$

and the work lost in the absolute velocity of the water is

$$k' = W\frac{(V-v)^2}{2g}$$

The sum of these is the total theoretic work, or

$$K = W\frac{V^2-v^2}{2g}$$

Therefore the efficiency of jet propulsion is expressed by

$$e = \frac{k}{K} = \frac{2v}{V+v}$$

This becomes equal to unity when $v = V$ as before indicated, but then it is seen that the work k becomes 0 unless W is infinite. The value of W is waV, if a be the area of the orifices through which

the water is ejected; and hence in order to make e unity and at the same time perform work it is necessary that either V or a should be infinity. The jet propeller is therefore like a reaction wheel (Art. 172), and it is seen upon comparison that the formula for efficiency is the same in the two cases.

By equating the above value of the useful work to that established in the last article there is found

$$fgAv^2 = waV(V-v)$$

and if this be solved for V, and the resulting value be substituted in the formula for e, it reduces to

$$e = \frac{4}{3+\sqrt{1+(4fgA/wa)}}$$

which again shows that e approaches unity as the ratio of a to A increases. The area of the orifices of discharge must hence be very large in order to realize both high power and high efficiency. For this reason the propulsion of vessels by this method has not proved economical, although in the case of the boat Waterwitch, built in England about 1860, a fair speed was attained. In nature the same result is seen, for no marine animal except the cuttle-fish uses this principle of propulsion. Even the cuttle-fish cannot depend upon his jet to escape from his enemies, but for this relies upon his supply of ink with which he darkens the water about him.

Prob. 186. Compute the velocity and efficiency of a jet propeller driven by a 1-inch nozzle under a pressure of 150 pounds per square inch when $A = 1000$ square feet and $f=0.004$. Compute also the efficiency when the diameter of the nozzle is 3 inches.

ART. 187. PADDLE WHEELS

The method of propulsion by rowing and paddling is well known to all. The power is furnished by muscular energy within the boat, the water is the fulcrum upon which the blade of the oar acts, and the force of reaction thus produced is transmitted to the boat and urges it forward. If water were an unyielding substance, the theoretic efficiency of the oar should be unity, or, as in any lever, the work done by the force at the rowlock should equal the work performed by the motive force exerted by the man on the handle of the oar. But as the water is yielding, some of it is driven backward by the blade of the oar, and thus energy is lost.

The paddle or side wheel so extensively used in river navigation is similar in principle to the oar. The power is furnished by motor within the boat, the blades or vanes of the wheel tend to drive the water backward, and the reaction thus produced urges the boat forward. On first though it might be supposed that the efficiency of the method would be governed by laws similar to those of the undershot wheel, and such would be the case if the vessel were stationary and the wheel were used as an apparatus for moving the water. In fact, however, the theoretic efficiency of the paddle wheel on a boat is much higher than that of the undershot motor.

The work exerted by the steam-engine upon the paddle wheels may be represented by PV, in which P is the pressure produced by the vanes upon the water, and V is their velocity of revolution; and the work actually imparted to the boat may be represented by Pv, in which v is its velocity with respect to the water. Accordingly the efficiency of the paddle wheel, neglecting losses due to foam and waves, is

$$e=\frac{v}{V}=\frac{v}{v+v_1}$$

in which v_1 is the difference $V-v$, or the so-called "slip." If the slip be 0, the velocities V and v are equal, and the theoretic efficiency of the wheel is unity. The value of V is determined from the radius r of the wheel and its number of revolutions per second; thus $V=2\pi rn$.

On account of the lack of experimental data it is difficult to give information regarding the practical efficiency of paddle wheels considered from a hydromechanic point of view. Owing to the water which is lifted by the blades, and to the foam and waves produced, much energy is lost. They are, however, very advantageous on account of the readiness with which the boat can be stopped and reversed. When the wheels are driven by separate engines, as is sometimes done on river boats, perfect control is secured, as they can be revolved in opposite directions when desired. Paddle wheels with feathering blades are more efficient than those with fixed radial ones, but practically they are found to be cumbersome, and liable to get out of order. In ocean navigation the screw has now almost entirely replaced the paddle wheel on account of its higher efficiency.

Prob. 187. The radius of the blades of a paddle wheel is 10.5 feet and the number of revolutions per minute is 24. If the efficiency is 75 percent, what is the velocity of the boat in miles per hour? Show that for this case the slip is 33 percent of the velocity of the boat.

Art. 188. The Screw Propeller

The screw propeller consists of several helicoidal blades attached at the stern of a vessel to the end of a horizontal shaft which is made to revolve by steam power. The dynamic pressure of the reaction developed between the water and the helicoidal surface drives the vessel forward, the theoretic work of the screw being the product of this pressure by the distance traversed. The pitch of the screw is the distance, parallel to the shaft, between any point on a helix and the corresponding point on the same helix after one turn around the axis, and the pitch may be constant at all distances from the axis, or it may be variable. If the water were unyielding, the vessel would advance a distance equal to the pitch at each revolution of the shaft; actually, the advance is less than the pitch, the difference being called the "slip." The effect thus is that the pressure P existing between the helical surfaces and the water moves the vessel with the velocity v, while the theoretic velocity which should occur is V, being the pitch of the screw multiplied by the number of revolutions per second. The work expended is hence PV or $P(v+v_1)$, if v_1 be the slip per second, and the work utilized is Pv. Accordingly the efficiency of screw propulsion is, approximately,

$$e=\frac{v}{v+v_1}$$

which is the same expression as before found for the paddle wheel. Here, as in the last article, all the pressure exerted by the blades upon the water is supposed to act backward in a direction parallel to the shaft of the screw, and the above conclusion is approximate because this is actually not the case, and also because the action of friction has not been considered. The practical advantage of the screw over the paddle wheel has been found to be very great, and this is probably due to the circumstance that less energy is wasted in lifting the water and in forming waves.

The pressure P which is exerted by the helicoidal blades upon the water is the same as the thrust or stress in the shaft, and the value of this may be approximately ascertained by regarding it as due to the reaction of a stream of water of cross-section a and velocity v, or $P = wa(v+v_1)v/g$

Another expression for this may be found from the indicated work k of the steam cylinders of the engines; thus

$$P = k/v$$

Numerical values computed from these two expressions do not, however, agree well, the latter giving in general a much less value than the former.

In Art. 185 the work to be performed in propelling a vessel of fair form having the submerged surface A was found to be

$$k = fAv^3$$

If the value of v is taken from this equation and inserted in the expression for efficiency, there obtains

$$e = \frac{1}{1 + v_1(Af/k)^{\frac{1}{3}}}$$

which shows that e increases as v_1, f, and A decrease, and as k increases. Or for given values of f and A the efficiency decreases with the speed.

It has been observed in a few instances that the "slip" v_1 is negative, or that V, as computed from the number of revolutions and pitch of the screw, is less than v. This is probably due to the circumstance that the water around the stern is following the vessel with a velocity v', so that the real slip is $V - v + v'$ instead of $V - v$. The existence of negative slip is usually regarded as evidence of poor design.

Twin screws are frequently used, and since these revolve in opposite directions, the vessel can be more readily controlled. Fig. 188 shows the position of the twin screws with respect to the rudder. On some of the recent high-powered turbine-driven steamships two and three screws all mounted on a single shaft have been employed. Two sets of engines, and two shafts, one on each side of the rudder, are often employed as in Fig. 188, but a different arrangement of the shafts with respect to the hull of the ship permits the screws to be placed at considerable distances apart on the shafts, thus obtaining a greater efficiency than in the case of the single screw.

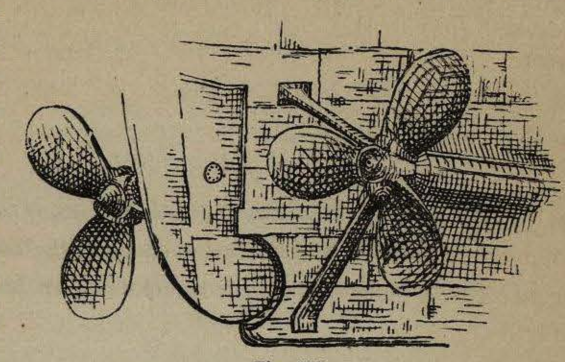

Fig. 188.

Prob. 188. A steamer having a submerged surface of 30 000 square feet is propelled at 18 knots per hour by an expenditure of 6000 horse-powers. If the pitch of the screw is 20 feet, its number of revolutions 120 per minute, and $f = 0.004$, compute the number of lost horse-powers.

Art. 189. Stability of a Ship

In Art. 14 the general principles regarding the stability of a floating body were stated, and these are of great importance in the design of ships. The center of gravity is, of course, always above the center of buoyancy, and the metacenter must be above the center of gravity in order to insure stability. The distance between the metacenter and the center of gravity is denoted by m, and if the body be inclined slightly to the vertical at the angle θ, the moment of the couple formed by the weight W of the body which acts downward through the center of gravity and the upward pressure W of the displaced water which acts through the center of buoyancy is $Wm \tan\theta$. Hence $m \tan\theta$ is a measure of the stability of the body, and the greater its value, the greater is the tendency of the body to return to the upright position.

The metacentric height m cannot, however, be made very great, for the rapidity of rolling increases with it. When a floating body or ship is displaced from its vertical position, it rolls to and fro with isochronous oscillations like those of a pendulum, and the time of one oscillation from port to starboard is given by the formula

$$t = \pi\sqrt{r^2/mg}$$

in which r is the radius of gyration of the weight of the ship about a horizontal longitudinal axis passing through its center of gravity. Hence if m is large t is small and the ship rolls quickly;

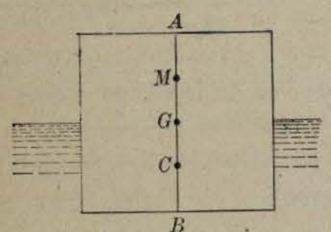

Fig. 189*a*.

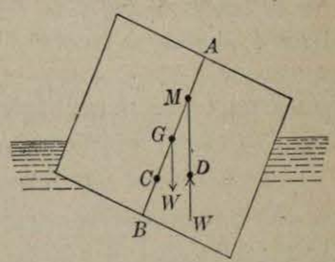

Fig. 189*b*.

but if m is small t is large and the ship rolls slowly. The metacentric height m for ocean vessels usually ranges from 2 to 15 feet, about 6 or 8 feet being the usual value.

The determination of the values of m and r for a ship is a laborious process, owing to its curved shape and the irregular distribution of its weight and cargo. The process will here be applied to the simple case of a rectangular prism of uniform density. Let h be the height and b the breadth of the prism, and l its length perpendicular to the plane of the drawing in Fig. 189*a*. When the prism is in the vertical position, its depth of flotation is sh, if s is its specific gravity (Art. 13), and this is also the length of the immersed portion of the axis AB when the prism is inclined to the vertical at the angle θ, as in Fig. 189*b*. In the latter position the center of buoyancy D, being the center of gravity of the displaced water, is easily located, and

$$x = \frac{b^2 \tan\theta}{12sh} \qquad y = \frac{sh}{2} + \frac{b^2 \tan^2\theta}{24sh}$$

are its coordinates with respect to B, x being measured normal and y

parallel to AB. The distance m from the center of gravity g to the metacenter M is then found to be

$$m = \frac{b^2}{12sh}(1 + \tfrac{1}{2}\tan^2\theta) - \tfrac{1}{2}h(1 - s)$$

If m is positive, the metacenter is above the center of gravity and the equilibrium is stable, for the moment $Wm \tan\theta$ restores the prism to the vertical position; if m is zero, the equilibrium is indifferent; if m is negative, the equilibrium is unstable, and the prism falls over.

The square of the radius of gyration of the prism with respect to a horizontal longitudinal axis through G is its polar moment of inertia $\frac{1}{12}l(bh^3+hb^3)$ divided by its volume lbd, whence $r^2 = \frac{1}{12}(h^2+b^2)$. For example, if h is 5 feet, b is 8 feet, and s is 0.5, the value of r^2 is 7.42 feet². The value of m to be used in the above formula for the time of one roll is that obtained by making θ equal to zero, since that formula is strictly true only for small deviations from the vertical. For the above data this value of m is $+0.88$ feet, the plus sign denoting stability, and hence the time of one oscillation from port to starboard is $t = 1.61$ seconds. It is seen that t can be increased either by increasing r^2 or by decreasing m; since a decrease in m is unfavorable to stability, it is usually preferable to increase r^2. For instance, in loading a ship the cargo may be placed along the sides rather than near the middle of the hold, and this will increase r^2, as the width of a ship is always greater than its depth. The general rule to promote stability and prevent quick rolling is hence to place the cargo as far as possible from the center of gravity.

The above formula for m shows that the moment $Wm \tan\theta$ which restores the floating prism to the vertical increases with the angle θ up to a maximum value, then decreases, and when D arrives vertically beneath G, it becomes zero and the prism upsets. For the case where $h = 5$ feet, $b = 8$ feet, and $s = 0.5$, the value of $m \tan\theta$ is 0.00 feet for $\theta = 0°$, 0.16 feet for $\theta = 10°$, 0.37 feet for $\theta = 20°$, and 0.72 feet for $\theta = 30°$; at $\theta = 32°$ the corner of the prism becomes immersed so that the formula no longer holds, but up to this point the moment constantly increases. From the above expression for m the solution of Prob. 14 is readily made.

Prob. 189*b*. An open rectangular wooden box caisson of length l, breadth b, and depth d has sides of mean thickness b_1 and a bottom of thickness d_1. Deduce formulas for the metacentric height m and the squared radius of gyration r^2. Compute m, r^2, and t for a numerical case.

Art. 190. Action of the Rudder

The action of the rudder in steering a vessel involves a principle that deserves discussion. In Fig. 190 is shown a plan of a boat with the rudder turned to the starboard side, at an angle θ with the line of the keel. The velocity of the vessel being v, the action of the water upon the rudder is the same as if the vessel were at rest and the water in motion with the velocity v. Let W be the weight of water which produces dynamic pressure against the rudder, due to the impulse $W \cdot v/g$ (Art. 152). The component of this pressure normal to the rudder is

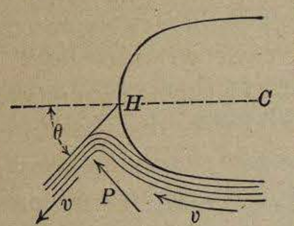

Fig. 190.

$$P = Wv \sin\theta/g$$

and its effect in turning the vessel about the center of gravity C is measured by its moment with reference to that point. Let b be the breadth of the rudder and d the distance CH between the center of gravity and the hinge of the rudder, then the lever arm of the force P is

$$l = \tfrac{1}{2}b + d \cos\theta$$

and accordingly the turning moment is

$$M = \tfrac{1}{2}W(b \sin\theta + d \sin 2\theta)v/g$$

To determine that value of θ which produces the greatest effect in turning the boat the derivative of M with respect to θ must vanish, which gives

$$\cos\theta = -\frac{b}{8d} + \sqrt{\frac{1}{2} + \frac{b^2}{64d^2}}$$

and from this the value of θ is found to be approximately 45°, since d is always much larger than b.

Values of the angle θ for several values of the ratio b/d may now be computed as follows:

$b/d =$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{100}$	0
$\cos\theta =$	0.6825	0.6916	0.6947	0.7069	0.7071
$\theta =$	$46^\circ 58'$	$46^\circ 15'$	$46^\circ 00'$	$45^\circ 01'$	45°

which shows that about 45° is the advantageous angle. In practice it is usual to arrange the mechanism of the rudder so that it can only be turned to an angle of about 42° with the keel, for it is found that the power required to turn it the additional 3° or 4° is not sufficiently compensated by the slightly greater moment that would be produced. The reasoning also shows that intensity of the turning moment increases with v, so that the rudder acts most promptly when the boat is moving rapidly. For the same reason a rudder on a steamer propelled by a screw is not required to be so broad as one on a boat driven by paddle wheels, for the effect of the screw is to increase the velocity of the impinging water, and hence also to increase the dynamic pressure against the rudder.

Prob. 190. Explain how it is that a boat can sail against the wind. What is the influence of the keel in this motion?

Art. 191. Tides and Waves

The complete discussion of the subject of waves might, like many other branches of hydraulics, be expanded so as to embrace an entire treatise, while there can be here given only the briefest outline of a few of the most important principles. There are two classes or kinds of waves, the first including the tidal waves and those produced by earthquakes or other sudden disturbances, and the second those due to the wind. The daily tidal wave generated by the attraction of the moon and sun originates in the South Pacific Ocean, whence it travels in all directions with a velocity dependent upon the depth of water and the configuration of the continents, and which in some regions is as high as 1000 miles per hour. Striking against the coasts, the tidal waves cause currents in inlets and harbors, and if the circumstances were such that their motion could become uniform and permanent, these might be governed by the same laws which apply to the flow of water in channels. Such, however, is rarely the case; and accordingly the subject of tidal currents is one of much complexity and not capable of general formulation.

The velocity of a tidal wave on the ocean is $\sqrt{gD}$, where D is the depth of the water. When such a wave rolls over the land, the greatest velocity it can have is $\sqrt{gd}$, where d is its depth, this being the case of the bore (Art. 139). The velocity of a wave which is produced by a sudden disturbance in a channel of uniform width has also been found to be $\sqrt{gD}$, where D is the depth of the water.

Rolling waves produced by the wind travel with a velocity which is small compared with those above noted, although in water where the disturbance can extend to the bottom, it is generally supposed that their velocity is $\sqrt{gD}$. Upon the ocean the maximum length of such waves is estimated at 550 feet and their velocity at about 53 feet per second. For this class of waves it is found by observation that each particle of water upon the surface moves in an elliptic or circular orbit, whose time of revolution is the same as the time of one wave length.

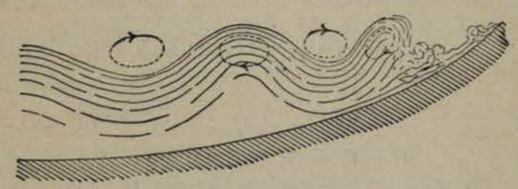

Fig. 191.

Thus the particles on the crest of a wave are moving forward in the direction of the motion of the wave, while those in the trough are moving backward. When such waves advance into shallow water, their length and speed decrease, but the time of revolution of the particles in their orbits remains unaltered, and as a consequence the slopes become steeper and the height greater, until finally the front slope becomes vertical and the wave breaks with roar and foam. Below the surface the particles revolve also in elliptic orbits, which grow smaller in size toward the bottom. The curve formed by the vertical section of the surface of a wave at right angles to its length is of a cycloidal nature.

The force exerted by ocean waves when breaking against sea walls is very great, as already mentioned in Art. 155, and often proves destructive. If walls can be built so that the waves are reflected without breaking, as is sometimes possible in deep water, their action is rendered less injurious. Upon the ocean waves move in the same direction as the wind, but along shore it is observed that they generally move normally toward it, whatever may be the direction in which the wind is blowing. The force of wave action is felt at depths of over 100 feet below the surface, for sand has been brought up from depths

of 80 feet and dropped upon the decks of vessels. Shoals also cause a marked increase in the height of waves, even when such shoals are 500 feet or more below the water surface.

Prob. 191*a*. In a channel 6.5 feet wide, and of a depth decreasing 1.5 feet per 1000 feet, Bazin generated a wave by suddenly admitting water at the upper end. At points where the depths were 2.16, 1.85, 1.46, and 0.80 feet, the velocities were observed to be 8.70, 8.67, 7.80, and 6.69 feet per second. Do these velocities agree with the theoretic law?

Prob. 191*b*. Show that the values of f given in Art. 175 for use in the formula $F=fv^2$ are to be multiplied by 5.255 when v is in meters per second and F in kilograms per square meter.

Prob. 191*c*. Compute the metric horse-power required for a velocity of 25 kilometers per hour for a boat which has a submerged area of 237 square meters.

Prob. 191*d*. A ship rolls from starboard to port in 7.5 seconds. If the metacentric height m is 2.4 meters, what is the value of the transverse radius of gyration of the ship? How much must the radius of gyration be increased in order to increase the time of rolling 15 percent?