Art. 176. Theory of Reaction Turbines

The theory of reaction turbines may be said to include two problems: first, given all the dimensions of a turbine and the head under which it works, to determine the maximum efficiency, and the corresponding speed, discharge, and power; and second, having given the head and the quantity of water, to design a turbine of high efficiency. This article deals only with the first problem, and it should be said at the outset that it cannot be fully solved theoretically, even for the best-conditioned wheels, on account of losses in foam, friction, and leakage. The investigation will be limited to the case of full gate, since when the gate is partially depressed, a loss of energy results from the sudden expansion of the entering water.

The notation will be the same as that used in Chaps. 11 and 12, and as shown in Figs. 174*b* and 174*c*; the reasoning will apply to both outward- and inward-flow turbines. Let r be the radius of the circumference where the water enters the wheel and r_1 that of the circumference where it leaves, let u and u_1 be the corresponding velocities of revolution; then $ur_1 = u_1r$. Let v_0 be the absolute velocity with which the water leaves the guides and enters the wheel, and V its velocity of entrance relative to the wheel; let α be the approach angle and ϕ the entrance angle which these velocities make with the direction of u. At the exit circumference let V_1 be the relative velocity with which the water leaves the guides, and v_1 its absolute velocity; let β be the exit angle which V_1 makes with this circumference. Let a_0, a, and a_1 be the areas of the guide orifices, the entrance, and the exit orifices of the wheel, respectively, measured perpendicular to the directions of v_0, V, and V_1. Let d_0, d, and d_1 be the depths of these orifices; when the gate is fully raised, d_0 becomes equal to d.

The areas a_0, a, a_1, neglecting the thickness of the guides and vanes, and taking the gate as fully open, have the values

$$a_0 = 2\pi rd \sin\alpha \qquad a = 2\pi rd \sin\phi \qquad a_1 = 2\pi r_1 d_1 \sin\beta$$

and since these areas are fully filled with water,

$$q = v_0 \cdot 2\pi rd \sin\alpha = V \cdot 2\pi rd \sin\phi = V_1 \cdot 2r_1 d_1 \sin\beta \qquad (176)_1$$

These relations, together with the formulas of the last article and the geometrical conditions of the parallelograms of velocities, include the entire theory of the reaction turbine.

In order that the efficiency of the turbine may be as high as possible the water must enter tangentially to the vanes, and the absolute velocity of the issuing water must be as small as possible. The first condition will be fulfilled when u and v_0 are proportional to the sines of the angles $\phi - \alpha$ and ϕ. The second will be secured by making $u_1 = V_1$ in the parallelogram at exit, as then the diagonal v_1 becomes very small. Hence

$$\frac{u}{v_0} = \frac{\sin(\phi - \alpha)}{\sin\phi} \qquad u_1 = V_1 \qquad (176)_2$$

are the two conditions which should obtain in order that the hydraulic efficiency may be a maximum.

Now making $V_1 = u_1$ in the third quantity of $(176)_1$ and equating it to the first, there results

$$\frac{u_1}{v_0} = \frac{rd \sin\alpha}{r_1 d_1 \sin\beta} \qquad \text{and} \qquad \frac{u}{v_0} = \frac{r^2 d \sin\alpha}{r_1^2 d \sin\beta}$$

Also making $V_1 = u_1$ in $(175)_1$ and substituting for V^2 its value $u^2 + v_0^2 - 2uv_0 \cos\alpha$ from the triangle at A between u and v_0, there is found the important relation

$$uv_0 \cos\alpha = gh \qquad (176)_3$$

which gives another condition between u and v_0. The velocity v_0, with which the water enters, hence depends upon the speed of the wheel as well as upon the head h.

Thus three equations between two unknown quantities u and v_0 have been deduced for the case of maximum hydraulic efficiency, namely,

$$\frac{u}{v_0} = \frac{\sin(\phi - \alpha)}{\sin\phi} \qquad \frac{u}{v_0} = \frac{r^2 d \sin\alpha}{r_1^2 d_1 \sin\beta} \qquad uv_0 = \frac{gh}{\cos\alpha}$$

If the values of the velocities u and v_0 be found from the first and third equations, they are

$$u = \sqrt{\frac{gh \sin(\phi - \alpha)}{\cos\alpha \sin\phi}} \qquad v_0 = \sqrt{\frac{gh \sin\phi}{\cos\alpha \sin(\phi - \alpha)}} \qquad (176)_4$$

the first of which is the advantageous velocity of the circumference where the water enters, and the second is the absolute velocity with which the water leaves the guides and enters the wheel. In order, however, that these expressions may be correct, the first and second values of u/v_0 must also be equal, and hence

$$\frac{\sin(\phi - \alpha)}{\sin\phi} = \frac{r^2 d \sin\alpha}{r_1^2 d_1 \sin\beta} \qquad (176)_5$$

which is the necessary relation between the dimensions and angles of the wheel in order that this theory may apply.

For a turbine so constructed and running at the advantageous speed the theoretic hydraulic efficiency is

$$e = 1 - \frac{v_1^2}{2gh} = 1 - \frac{2u_1^2 \sin^2 \frac{1}{2}\beta}{gh}$$

and substituting for u_1 its value in terms of u from $(176)_4$, and having regard to $(176)_5$, this becomes

$$e = 1 - \frac{d}{d_1} \tan\alpha \tan\tfrac{1}{2}\beta \qquad (176)_6$$

The discharge under the same conditions is $q = a_0 v_0$, and lastly the work of the wheel per second is $k = wqhe$.

The result of this investigation is that the general problem of investigating a given turbine cannot be solved theoretically, unless it be so built as to approximately satisfy the condition in $(176)_5$. If this be the case, it may be discussed by the formulas deduced. Even then no very satisfactory conclusions can be drawn from the numerical values, since the formulas do not take into account the loss by friction and that of leakage. To determine the actual efficiency, best speed, and power of a given turbine, the only way is to actually test it by the method described in Art. 149. The above formulas are, however, of great value in the discussion of the design of turbines. More exact formulas, from a theoretical standpoint, may be derived by using the condition $V_1 = u_1 \cos\beta$ instead of $V_1 = u_1$ to determine the exit velocity v_1 (Art. 168), but these are very complex in form, and numerical values computed from them differ but little from those found from the formulas here established.

When the coefficient of discharge of a turbine is known (Art. 175), the advantageous speed and corresponding discharge may be closely computed. For this purpose the condition $u_1 = V_1 = q/a_1$ is to be used. Inserting in this the value of q from $(175)_2$ and solving for u_1, there is found

$$u_1^2 = \frac{c^2 \cdot 2gh}{1 + c^2\dfrac{r^2}{r_1^2} + \dfrac{a_1^2}{a_0^2} - \dfrac{a_1^2}{a^2} - c^2}$$

which gives the advantageous velocity of the circumference where the water leaves the wheel, and then by $(175)_2$ the discharge can be obtained. As an example, take the case of Holyoke test No. 275, where $r_1 = 27\frac{1}{2}$ inches, $r = 21\frac{1}{2}$ inches, $h = 23.8$ feet, $a_0 = 2.066$, $a = 5.526$, $a_1 = 1.949$ square feet, $\alpha = 25\frac{1}{2}°$, $\phi = 90°$, $\beta = 11\frac{3}{4}°$. Assuming $c = 0.95$, as the turbine is similar to that investigated in the last article, the above formula gives $u_1 = 31.24$ feet per second, which corresponds to 130 revolutions per minute, and this agrees well with the actual number 138. The efficiency found by the test at that speed was 0.79, which is a very much less value than the above theoretic formula gives, since this formula was derived without taking into account the friction losses within and without the wheel.

Prob. 176. For the case of the last problem $r = 4.67$, $r_1 = 3.95$, $d = 1.01$, $d_1 = 1.23$, $h = 13.4$ feet, $\alpha = 9°.5$, $\phi = 119°$, $\beta = 11°$. Compute the areas a_0, a, a_1, and the advantageous speed. Compute also the velocity with which the water enters the wheel.

Art. 177. Design of Reaction Turbines

The design of an outward- or inward-flow turbine for a given head and discharge includes the determination of the dimensions r, r_1, d, d_1, and the angles α, β, and ϕ. These may be selected in very many different ways, and the formulas of the last article furnish a guide how to make a selection so as to secure a high degree of efficiency.

First, it is seen from $(176)_6$ that the approach angle α and the exit angle β should be small, but that, as in other wheels, β has a greater influence than α. However, β must usually be greater for an inward-flow than for an outward-flow wheel in order to make the orifices of exit of sufficient size. For the entrance angle ϕ a good value is 90°, and in this case the velocity u is always that due to one-half the head, as seen from $(176)_4$. The radii r and r_1

should not differ too much, as then the frictional resistance of the flowing water and the moving wheel would be large. It is also seen that the efficiency is increased by making the exit depth d_1 greater than the entrance depth d, but usually these cannot greatly differ, and are often taken equal.

Secondly, it is seen that the dimensions and angles should be such as to satisfy the formula $(176)_5$, since if this be not the case losses due to impact at entrance will occur which will render the other formulas of little value.

As a numerical illustration let it be required to design an outward-flow reaction turbine which shall use 120 cubic feet per second under a head of 18 feet and make 100 revolutions per minute. Let the entrance angle ϕ be taken at 90°, then from formula $(176)_4$ the advantageous velocity of the inner circumference is

$$u = \sqrt{32.16 \times 18} = 24.06 \text{ feet per second,}$$

and hence the inner radius of the wheel is

$$r = \frac{60 \times 24.06}{2\pi \times 100} = 2.298 \text{ feet.}$$

Now let the outer radius of the wheel be 3 feet, and also let the depths d and d_1 be equal; then from $(176)_5$

$$\frac{\sin\beta}{\tan\alpha} = \left(\frac{2.298}{3.000}\right)^2 = 0.5866$$

If the approach angle α be taken as 30°, the value of the exit angle β to satisfy this equation is 19° 48′, and from $(176)_6$ the hydraulic efficiency is 0.899. If, however, α be 24°, the value of β is β 15° 08′ and the hydraulic efficiency is 0.941; these values of α and β will hence be selected.

The depth d is to be chosen so that the given quantity of water may pass out of the guide orifices with the proper velocity. This velocity is, from $(176)_4$,

$$v_0 = 24.06/\cos 24° = 26.34 \text{ feet per second;}$$

and hence the area of the guide orifices should be

$$a_0 = 120/26.34 = 4.556 \text{ square feet,}$$

from which the depth of the orifices and wheel is

$$d = 4.556/2\pi r \sin 24° = 0.776 \text{ feet.}$$

As a check on the computations the velocities V and V_1, with the corresponding areas a and a_0, may be found, and d be again determined in two ways. Thus,

$V = v_0 \sin 24° = 10.71$ $\qquad V_1 = u_1 = ur_1/r = 31.42$ feet per second.

$a = 120/10.71 = 11.204$ $\qquad a_1 = 120/31.42 = 3.820$ square feet.

$d = 11.204/2\pi r = 0.776$ $\qquad d_1 = 3.820/2\pi r_1 \sin\beta = 0.776$ feet.

And this completes the preliminary design, which should now be revised so that the several areas may not include the thickness of the guides and vanes (Art. 178).

Although the hydraulic efficiency of this reaction turbine is 94 percent, the practical efficiency will probably not exceed 80 percent. About 2 percent of the total work will be lost in axle friction. The losses due to the friction of the water in passing through the guides and vanes, together with that of the wheel revolving in water, and perhaps also a loss in leakage, will probably amount to more than one-tenth of the total work. All of these losses influence the advantageous velocity, so that a test would be likely to show that the highest efficiency would obtain for a speed somewhat less than 100 revolutions per minute.

Prob. 177. Design an inward-flow reaction turbine which shall use 120 cubic feet of water per second under a head of 18 feet while making 100 revolutions per minute, taking ϕ = 68°, α = 10°, and β = 21°. Also taking ϕ = 75°, α = 15°, and β = 20°.

Art. 178. Guides and Vanes

The discussions in the last two articles have neglected the thickness of the guides and vanes. As these, however, occupy a considerable space, a more correct investigation will here be made to take them into account. Let t be the thickness of a guide and n their number, t_1 the thickness of a vane and n_1 their number. Then the areas a_0, a, and a_1 perpendicular to the directions of v_0, V, and V_1 are strictly

$$a_0 = (2\pi r \sin\alpha - nt)d \qquad a = (2\pi r \sin\phi - n_1t_1)d$$

$$a_1(2\pi r_1 \sin\beta - n_1t_1)d_1$$

and the expressions for the discharge in $(176)_1$ are

$$q = a_0v_0 = aV = a_1V_1$$

and, since V_1 equals u_1, these give

$$\frac{u_1}{v_0} = \frac{a_0}{a_1} \qquad \frac{u}{v_0} = \frac{a_0 r}{a_1 r_1}$$

also, the necessary condition in $(176)_5$ becomes

$$\frac{\sin(\phi - \alpha)}{\sin\phi} = \frac{a_0 r}{a_1 r_1}$$

and the greatest hydraulic efficiency of the turbine when running at the advantageous speed is given by

$$e = 1 - 2\frac{r_1^2}{r^2}\frac{\sin(\phi - \alpha)}{\sin\phi}\frac{\sin^2\frac{1}{2}\beta}{\cos\alpha}$$

in which, of course, $\sin(\phi - \alpha)/\sin\phi$ may be replaced by its equivalent $a_0 r/a_1 r_1$. The advantageous speed is, as before, given by formula $(176)_4$

To discuss a special case, let the example of the last article be again taken. An outward-flow turbine is to be designed to use 120 cubic feet of water under a head of 18 feet while making 100 revolutions per minute, the gate being fully opened. The preliminary design has furnished the values $r = 2.298$ feet, $r_1 = 3.000$ feet, $d = d_1 = 0.766$ feet, $\phi = 90°$, $\alpha = 24°$, $\beta = 15°\ 08'$. It is now required to revise these so that 24 guides and 36 vanes may be introduced. Each of these will be made one-half an inch thick, but on the inner circumference of the wheel the vanes will be thinned or rounded so as to prevent shock and foam that might be caused by the entering water impinging against their ends (see Fig. 182e). If the radii and angles remain unchanged, the effect of the vanes will be to increase the depth of the wheel, which is now 0.702 feet wide and 0.776 feet deep. As these are good proportions, it will perhaps be best to keep the depth and the radii unchanged, and to see how the angles and the efficiency will be affected.

Since the vanes are to be thinned at the inner circumference, the area a is unaltered and its value is simply $2\pi rd \sin\phi$. Hence ϕ remains 90° and V is unchanged. This requires that the area a should remain the same as before. The area a_1 is also the same, as its value is q/u_1. Accordingly the equations result

$$4.556 = (2\pi r \sin\alpha - 24t)\, d \qquad 3.820 = (2\pi r_1 \sin\beta - 36t_1)\, d_1$$

in which α and β are alone unknown. Inserting the numerical values and solving, $\alpha = 28°\ 26'$ and $\beta = 19°\ 55'$, both being increased by about $4\frac{1}{2}°$. The efficiency is now found to be 0.898, a decrease of 0.043, due to the introduction of the guides and vanes.

The efficiency may be slightly raised by making the outer depth d_1 greater than the inner depth d. For instance, let $d_1 = 0.816$ while d remains 0.776; then β is found to be $19°\ 06'$, and $e = 0.906$. But another way is to thin down the vanes at the exit circumference and thus maintain the full area a_1 with a small angle β. If this be done in the present case d_1 may be kept at 0.776 feet, β be reduced to about 16°, and the efficiency will then be about 0.92 or 0.93.

No particular curve for the guides and vanes is required, but it must be such as to be tangent to the circumferences at the designated angles. The area between two vanes on any cross-section normal to the direction of the velocity should also not be greater than the area at entrance; in order to secure this vanes are frequently made much thicker at the middle than at the ends (see Fig. 182e).

Prob. 178. Find the advantageous speed and the probable discharge and power of the turbine designed above when under a head of 50 feet.

Art. 179. Downward-flow Turbines

Downward- or parallel-flow turbines are those in which the water passes through the wheel without changing its distance from the axis of revolution. In Fig. 179a is a semi-vertical section of the guide and wheel passages, and also a development of a portion of a cylindrical section showing the inner arrangement. The formula for the discharge can be adapted to this by making $u_1 = u$. In this turbine there is no action of centrifugal force, so that the relative exit velocity V_1 is equal to the relative entrance velocity V.

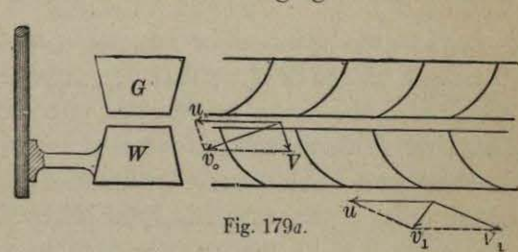

Fig. 179a.

The great advantage of this form of turbine is that it can be set some distance above the tail race and still obtain the power due to the total fall. This distance cannot exceed 34 feet, the height of the water barometer, and usually it does not exceed 25 feet. Fig. 179*b* shows in a diagrammatic way a cross-section of the penstock P, the guide passages G, the wheel W, and the air-tight draft tube T, from which the water escapes by a gate E to the tail race. The pressure-head H_1 on the exit orifice is here negative, so that the air pressure equivalent to this head is added to the water pressure in the penstock, and hence the discharge through the guides occurs as if the wheel were set at the level of the tail race. Strictly speaking, a vacuum, more or less complete, is formed just below the wheel into which the water drops with a low absolute velocity, having surrendered to the wheel nearly all its energy. Draft tubes are also often used with inward-flow turbines when these are set above the tail race.

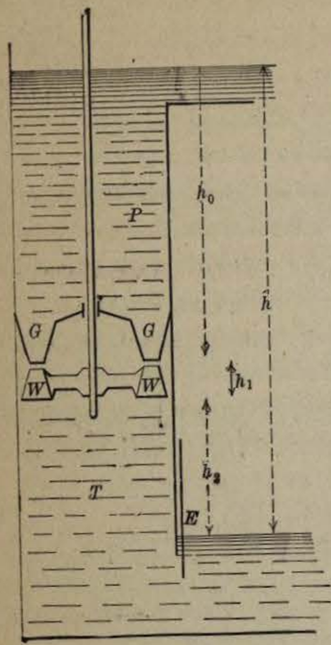

Fig. 179*b*

Let h be the total head between the water levels in the head and tail races, h_0 the depth of the entrance orifices of the wheel below the upper level, h_1 the vertical height of the wheel, and h_2 the height of the exit orifices above the tail race; so that $h = h_0+h_1+h_2$. Let H and H_1 be the heads which measure the absolute pressures at the entrance and exit orifice of the wheel, and h_a the height of the water barometer. Let v_0 be the absolute velocity with which the water leaves the guides and enters the vanes, and V and V_1 the relative velocities at entrance and exit. Then from the theorem of energy in steady flow (Art. 31),

$$v_0^2 = 2g(h_a + h_0 - H)$$

$$V_1^2 = V^2 + 2g(h_1 + H - H_1)$$

Adding these two equations there results

$$v_0^2 - V^2 + V_1^2 = 2g(h_0 + h_1 + h_a - H_1)$$

But $h_a - H_1$ is equal to h_2, and hence

$$v_0^2 - V^2 + V_1^2 = 2gh$$

This formula is the same as $(175)_1$ if u be made equal to u_1, and hence all the formulas of the last three articles apply to the downward-flow reaction turbine by making equal the velocities u and u_1, as also the radii r and r_1.

Let r be the mean radius and u the mean velocity of the entrance and exit orifices of the wheel, let d be the width of the entrance orifices and d_1 that of the exit orifices. Let α be the approach angle which the direction of the entering water makes with that of the velocity u, or the angle which the guides make with the upper plane of the wheel (Fig. 179*a*); let ϕ be the entrance angle which the vanes make with that plane, and β the acute exit angle which they make with the lower plane. Then the values of the advantageous velocity u and the entering velocity v_0 are

$$u = \sqrt{\frac{gh\sin(\phi-\alpha)}{\cos\alpha\sin\phi}} \qquad v_0 = \sqrt{\frac{gh\sin\phi}{\cos\alpha\sin(\phi-\alpha)}}$$

and the necessary relation between the angles of the vanes and the dimensions of the wheel is

$$\frac{\sin(\phi-\alpha)}{\sin\phi} = \frac{d\sin\alpha}{d_1\sin\beta} \cdot \frac{a_0}{a_1}$$

while the hydraulic efficiency of the turbine is

$$e = 1 - 2\frac{a_0}{a_1}\frac{\sin^2\frac{1}{2}\beta}{\cos\alpha} = 1 - \frac{d}{d_1}\tan\alpha\tan\tfrac{1}{2}\beta$$

To these equations is to be added the condition that the pressure-head H_1 cannot be less than that of a vacuum, and on account of air leakage it must be practically greater; thus

$$H_1 > 0 \quad \text{and} \quad h_2 < h_a$$

that is, the height of the wheel orifices above the tail race must be less than the height of the water barometer.

As an example of design, let $\phi = 90°$ and $\alpha = 30°$. Then $u = \sqrt{gh}$, or the velocity due to one-half the head; and $v_0 = \sqrt{\frac{4}{3}gh}$, or a velocity due to two-thirds of the head. From the above formulas, taking $d_1 = \frac{3}{2}d$, the value of β is 22° 38′ and the efficiency is found to be 0.92. This value will be lowered by the introduction of guides and vanes, as well as by friction, so that perhaps not more than 0.80 will be obtained in practice.

Prob. 179. A downward-flow turbine with draft tube has its exit orifices 7.5 feet above the level of the tail race, and it uses 87 cubic feet of water per second under a head of 25 feet. What horse-power will this turbine deliver when its efficiency, as measured by the friction brake, is 76 percent?

Art. 180. Impulse Turbines

Whenever a turbine is so arranged that the channels between the vanes are not fully filled with water, it ceases to act as a reaction turbine and becomes an impulse turbine. A turbine set above the level of the tail race becomes an impulse turbine when the gate is partially lowered, unless the gates are arranged so as to cover the exit orifices instead of being, as usual, in front of the entrance orifices.

The velocity with which the water leaves the guides in an impulse turbine is simply $2\sqrt{gh_0}$, where h_0 is the head on the guide orifices. The rules and formulas in Art. 168 apply in all respects, and for a well-designed wheel the entrance angle ϕ is double the approach angle α, the advantageous speed and corresponding hydraulic efficiency are

$$u = \sqrt{\frac{gh_0}{2\cos^2\alpha}} \qquad e = 1 - \left(\frac{r_1 \sin\frac{1}{2}\beta}{r\cos\alpha}\right)^2$$

while the discharge is $q = a_0\sqrt{2gh_0}$, and the work of the turbine per second is $k = wqh_0e$.

As an example, suppose that the reaction turbine designed in Art. 177 were to act as an impulse turbine, the angles α and β remaining at 24° and 15° 08′, and the radii r and r_1 being 2.298 and 3.000 feet. It would then be necessary that ϕ should be 48° instead of 90° in order to secure the best results. Under a head of 18 feet the velocity of flow from the guides would be 34.02 feet per second instead of 26.34. The velocity of the inner circumference would be 18.63 feet per second instead of 24.06, so that the number of revolutions per minute would be about 77 instead of 100. The efficiency would be 0.96, or almost exactly the same as before. If, however, the angle ϕ were to remain 90°, the efficiency of the turbine would be materially lowered, since then the water could not enter tangentially upon the vanes, and a loss in energy of the entering water due to the impact would necessarily result.

Impulse turbines revolve more slowly than reaction turbines under the same head, but the relative entrance velocity V is greater, and hence more energy is liable to be spent in shock and foam. In impulse turbines the entrance angle ϕ should be double the approach angle α, but in reaction turbines it is often greater than 3α, and its value depends upon the exit angle β; hence the vanes in impulse turbines are of sharper curvature for the same values of α and β. In impulse turbines the efficiency is not lowered by a partial closing of the gates, whereas the sudden enlargement of section causes a material loss in reaction turbines. The advantageous speed of an impulse turbine remains the same for all positions of the gate, but with a reaction turbine it is very much slower at part gate than at full gate. For many kinds of machinery it is important to maintain a constant speed for different amounts of power, and with a reaction turbine this can only be done by a great loss in efficiency. When the water supply is low, the impulse turbine hence has a marked advantage in efficiency. A further merit of the impulse turbine is that it may be arranged so that water enters only through a part of the guides, while this is impossible in reaction turbines. On the other hand, reaction turbines can be set below the level of the tail race or above it, using a draft tube in the latter case, and still secure the power due to the total fall, whereas an impulse turbine must always be set above the tail-race level and loses all the fall between that level and the guide orifices.

Prob. 180*a*. Compare the advantageous speeds of impulse and reaction turbines when the velocity of the water issuing from the guide orifices is the same.

Prob. 180*b*. Design an outward-flow impulse turbine which shall use 120 cubic feet of water per second under a head of 18 feet and make 100 revolutions per minute. Compare the dimensions and angles with those of the reaction turbine designed for the same data in Art. 177.