Power may be obtained from the ocean waves, which are constantly rising and falling, by a suitable arrangement of wheels and levers, and some inventions in this direction have given fair promise of success. One in operation on the coast of England about 1890 consisted of a large buoy which rose and fell with the waves on a fixed vertical shaft fastened in the rock bottom. As the buoy moved up and down it operated a system of levers and wheels which drove an air-compressor, and this in turn ran a dynamo that generated electric power. The rise of the ocean tide also affords opportunity for impounding water which may be used to generate power when the tide falls. Plants for this purpose are to be located along tidal rivers where opportunities for impounding occur, the wheels being idle during the rise of the tide, and in operation during its fall. Owing to this intermittent generation of power, it will be necessary to provide for its storage, so that industries using it may be in continuous operation.

Prob. 171*a*. A wheel using 10.5 cubic meters of water per minute under an effective head of 23.4 meters has an efficiency of 75 percent. What metric horse-power does it deliver? What is its power in kilowatts?

Prob. 171*b*. A breast wheel has $c_1 = 0.95$, $h_0 = 1.3$ meters, and $\alpha = 12°$. If its diameter is 3.5 meters, compute the most advantageous number of revolutions per minute.

Prob. 171*c*. An inward-flow impulse wheel has $\phi = 104°$, $\alpha = 52°$, and $\beta = 12°$, its inner diameter being 0.82 meters and its outer diameter 1.22 meters. If this wheel uses 0.86 cubic meters of water per second under an effective head of 7.9 meters, compute its efficiency and its probable effective horse-power.

Prob. 171*d*. A pipe 3200 meters long and 40 centimeters in diameter delivers water through two nozzles against a hurdy-gurdy wheel. When the diameter of one nozzle is 5 centimeters, find the diameter of the other nozzle in order that the energy of the two jets may be a maximum. If the head on the nozzles is 107 meters and the efficiency of the wheels is 81 percent, compute the horse-power which the wheels will deliver.

CHAPTER 14

TURBINES

Art. 172. The Reaction Wheel

The reaction wheel, invented by Barker about 1740, consists of a number of hollow arms connected with a hollow vertical shaft, as shown in Fig. 172. The water issues from the ends of the arms in a direction opposite to that of their motion, and by the dynamic pressure due to its reaction the energy of the water is transformed into useful work. Let the head of water CC in the shaft be h; then the pressure-head BB which causes the flow from the arms is greater than h, on account of the centrifugal force due to the rotation of the wheel. Let u_1 be the absolute velocity of the exit orifices, and V_1 be the velocity of discharge relative to the wheel; then, as shown in Art. 29, and also in Art. 162,

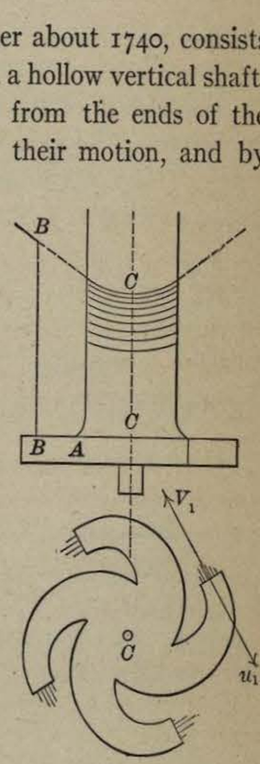

Fig. 172.

$$V_1 = \sqrt{2gh + u_1^2}$$

The absolute velocity v_1 of the issuing water now is

$$v_1 = V_1 - u_1 = \sqrt{2gh + u_1^2} - u_1$$

It is seen at once that the efficiency can never reach unity unless $v_1 = 0$, which requires that $V_1 = u_1$. This, however, can only occur when $u_1 = \infty$, since the above formula shows that V_1 must be greater than u_1 for any finite values of h and u_1. To deduce an expression for the efficiency the work of the wheel

$W(h - v_1^2/2g)$ is to be divided by the theoretic energy of the water Wh, and this gives

$$e = 1 - \frac{v_1^2}{2gh} = 1 - \frac{(V_1 - u_1)^2}{V_1^2 - u_1^2} = \frac{2u_1}{V_1 + u_1} \qquad (172)_1$$

which shows, as before, that e equals unity when $V_1 = u_1 = \infty$. If $V_1 = 2u_1$, the value of e is 0.667; if $V_1 = 3u_1$, the value of e is reduced to 0.50.

This investigation indicates that the efficiency of a reaction wheel increases with its speed. If a_1 be the area of the exit orifices and w the weight of a cubic unit of water, the weight of the water discharged in one second is wa_1V_1, which becomes infinite when $V_1 = u_1 = \infty$. Nothing approaching this can be realized, and on account of losses due to friction, a very high speed is impracticable. The reaction wheel, indeed, is like the jet propeller in regard to efficiency (Art. 186).

To consider the effect of friction in the arms, let c_1 be the coefficient of velocity (Chap. 7), so that

$$V_1 = c_1\sqrt{2gh + u_1^2}$$

Then the effective work of the wheel is

$$k = W\frac{(c_1\sqrt{2gh + u_1^2} - u_1)u_1}{g}$$

and the corresponding efficiency of the wheel is

$$e = \frac{c_1u_1\sqrt{2gh + u_1^2} - u_1^2}{gh}$$

The value of u_1, which renders this a maximum, is

$$u_1^2 = \frac{gh}{\sqrt{1 - c_1^2}} - gh$$

and this reduces the value of the efficiency to

$$e = 1 - \sqrt{1 - c_1^2} \qquad (172)_2$$

If $c_1 = 1$, there is no loss in friction, and $u_1 = \infty$ and $e = 1$, as before deduced. If $c_1 = 0.94$, the advantageous velocity u_1 is very nearly $\sqrt{2gh}$, and e is 0.66; hence the influence of friction in diminishing the efficiency is very great. In order to make c_1 large, the end of the arm where the water enters must be well rounded to prevent contraction, and the interior surface must be smooth. If the inner end has sharp, square edges, as in a standard tube (Art. 78), c_1 is 0.82, and e is 0.43.

The reaction wheel is not now used as a hydraulic motor on account of its low efficiency. Even when run at high speeds the efficiency is low on account of the greater friction and resistance of the air. By experiments on a wheel one meter in diameter under a head of 1.3 feet Weisbach found a maximum efficiency of 67 percent when the velocity of revolution u_1 was $\sqrt{2gh}$. When u_1 was $2\sqrt{2gh}$, the efficiency was nothing, or all the energy was consumed in frictional resistances.

The reaction wheel is here introduced at the beginning of the discussion of turbines mainly to call attention to the fact that the discharge varies with the speed. Although sometimes called a turbine, it can scarcely be properly considered as belonging to that class of hydraulic motors.

Prob. 172. The sum of the exit orifices of a reaction wheel is 4.25 square inches, their radius is 1.75 feet, and their velocity 32.1 feet, per second. Compute the head necessary to furnish 1.6 horse-powers, when $c_1 = 0.95$.

Art. 173. Classification of Turbines

A turbine wheel may be defined as one in which the water enters around the entire circumference instead of upon one portion, so that all the moving vanes are simultaneously acted upon by the dynamic pressure of the water as it changes its direction and velocity. The turbine was invented by Fourneyron in 1827, and owing to its compactness, cheapness, and high efficiency, it has largely replaced the older forms of water wheels. Turbines are usually horizontal wheels, and like the impulse wheels of the last chapter, they may be outward-flow, inward-flow, or downward-flow, with respect to the manner in which the water passes through them. In the outward-flow type the water enters the wheel around the entire inner circumference and passes out around the entire outer circumference (Fig. 174*b*). In the inward-flow type the motion is the reverse (Fig. 174*c*). In the downward-flow type the water enters around the entire upper annular openings, passes downward between the moving vanes, and leaves through the lower annulus (Fig. 179*a*). In all cases the water in leaving the wheel should have a low absolute velocity, so that most of its energy may be surrendered to the turbine in the form of useful work.

The supply of water to a turbine is regulated by a gate or gates, which can partially or entirely close the orifices where the water enters or leaves. The guides and wheel, with the gates and the surrounding casings, are made of iron. Numerous forms with different kinds of gates and different proportions of guides and vanes are in the market. They are made of all sizes from 6 to 60 inches in diameter, and larger sizes are built for special cases. The great turbines at Niagara are of the outward-flow type, the inner diameter of a wheel being 63 inches and each twin turbine furnishing about 5000 horse-powers (Art. 182). The smaller sizes of turbines used in the United States are mostly of the inward-flow type or of a combined inward- and downward-flow type.

The three typical classes of turbines above described are often called by the names of those who first invented or perfected them; thus the outward-flow is called the Fourneyron, the inward-flow the Francis, and the downward-flow the Jonval turbine. There are also many turbines in the market in which the flow is a combination of inward and downward motion, the water entering horizontally and inward, and leaving vertically, the vanes being warped surfaces. The usual efficiency of turbines at full gate is from 70 to 85 percent, although 90 percent has in some cases been derived. When the gate is partly closed, the efficiency in general decreases, and when the gate opening is small, it becomes very low. This is due to the loss of head consequent upon the sudden change of cross-section; and therein lies the disadvantage of the turbine, for when the water supply is low, it is important that it should utilize all the power available. A compilation of turbine tests with descriptions of the various forms of wheels has been made by Horton and issued by the United States Geological Survey.*

Another classification is into impulse and reaction turbines.

* Water Supply and Irrigation Paper, No. 180, 1906.

In an impulse turbine the water enters the wheel with a velocity due to the head at the point of entrance, just as it does from the nozzle which drives an impulse wheel (Art. 168). In a reaction turbine, however, the velocity of the entering water may be greater or less than that due to the head on the orifices of entrance, and, as in the reaction wheel, it is also influenced by the speed. This is due to the fact that in a reaction turbine the static pressure of the water is partially transmitted into the moving wheel, provided that the spaces between the vanes are fully filled. Any turbine may be made to act either as an impulse or a reaction turbine. If it be arranged so that the water passes through the vanes without filling them, it is an impulse turbine; if it be placed under water, or if by other means the flowing water is compelled to completely fill all the passages, it acts as a reaction turbine. As will be seen later, the theory of the reaction turbine is quite different from that of the impulse turbine.

Prob. 173. If the efficiency of a turbine is 75 percent when delivering 5000 horse-powers under a head of 136 feet, how many cubic feet of water per minute pass through it?

Art. 174. Reaction Turbines

A reaction turbine is driven by the dynamic pressure of flowing water which at the same time may be under a certain degree of static pressure. If in the reaction wheel of Fig. 172 the arms be separated from the penstock at A, and be so arranged that BA revolves around the axis while AC is stationary, the resulting apparatus may be called a reaction turbine. The static pressure of the head CC can still be transmitted through the arms, so

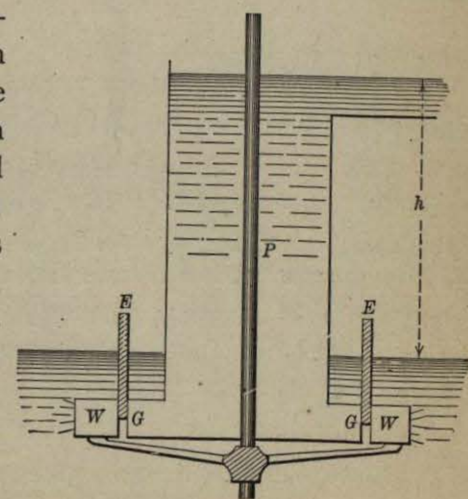

Fig. 174a.

that, as in the reaction wheel, the discharge will be influenced by the speed of rotation. The general arrangement of the moving part is, however, like that of an impulse wheel, the vanes being set between two annular frames, which are attached

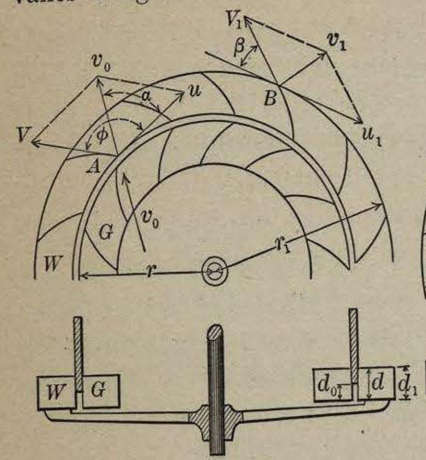

Fig. 174b.

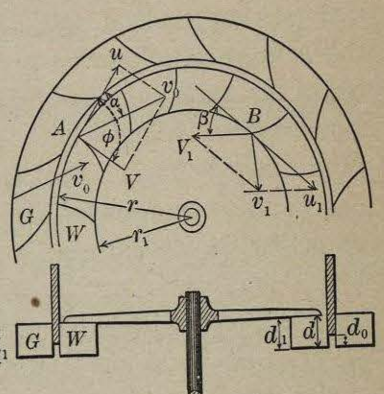

Fig. 174c.

by arms to a central axis. In Fig. 174a is a vertical section showing an outward-flow wheel W to which the water is brought by guides G from a fixed penstock P. Between the guides and the wheel there is an annular space in which slides

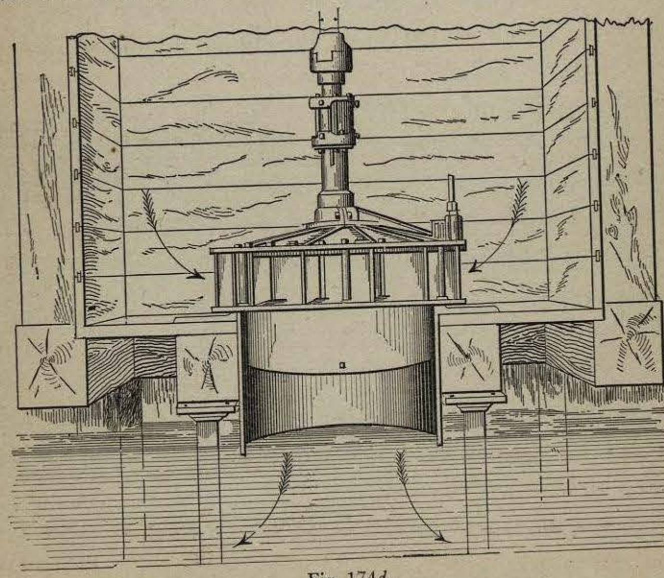

Fig. 174d.

an annular vertical gate E; this serves to regulate the quantity of water, and when it is entirely depressed, the wheel stops. Many other forms of gates are, however, used in the different styles of turbines found in the market.

In Figs. 174b and 174c are given horizontal and vertical sections of both the outward- and the inward-flow types, showing the arrangement of guides and vanes. The fixed guide passages which lead the water from the penstock are marked G, while the moving wheel is marked W. It is seen that the water is introduced around the entire circumference of the wheel, and hence the quantity supplied, and likewise the power, is far greater than in the impulse wheels of the last chapter.

In order that the static pressure may be transmitted into the wheel it is placed under water, as in Fig. 174a, or the exit orifices are partially closed by gates, or the air is prevented from entering them by some other device.

In Fig. 174d a Leffel turbine of the inward-flow type is illustrated, the arrows showing the direction of the water as it enters and leaves. The wheel itself is not visible, it being within the inclosing case through which the water enters by the spaces between the guides. In Fig. 174e is shown a view of a Hunt turbine, which is also of the inward- and downward-flow type. In both cases the guides are seen with the small shaft for moving the gates, these being partly raised in Fig. 174e. The flange at the base of the guides serves to support the weight of the entire apparatus upon the floor of the inclosing penstock, which is filled with water to the level of

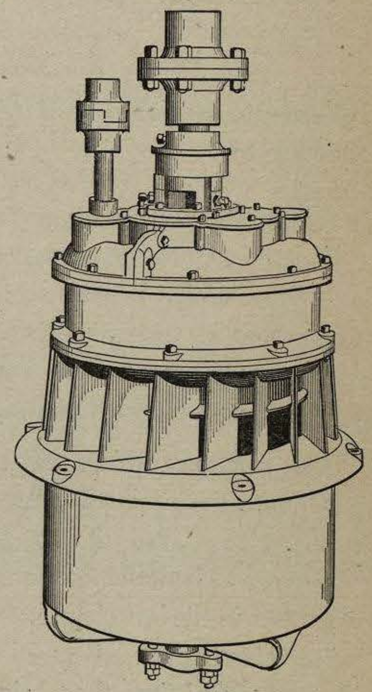

Fig. 174e.

the head bay. The cylinder below the flange, commonly called a draft-tube, carries away the water from the wheel, and the level of the tail water should stand a little higher than its lower rim in order to prevent the entrance of air and thus insure that the wheel may act as a reaction turbine. Iron penstocks are frequently used instead of wooden ones, and for the pure outward- and inward-flow types the wheel is often placed below the level of the tail race.

Turbines are sometimes placed vertically on a horizontal shaft. Fig. 174*f* shows twin Eureka turbines thus arranged in

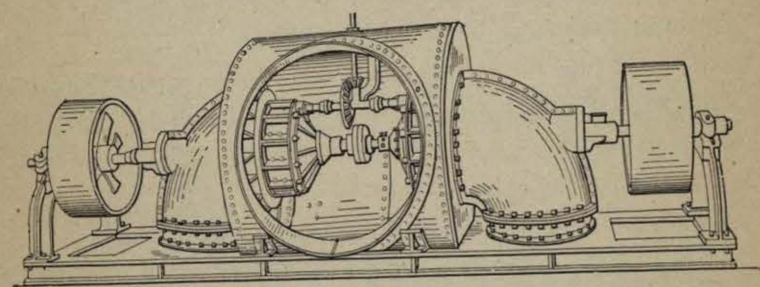

Fig. 174*f*.

an inclosing iron casing. The water enters through a large pipe attached to the cylinder opening, and having filled the cylindrical casing, it passes through the guides, turns the wheels, and escapes by the two elbows. Large twin vertical turbines furnishing 1200 horse-powers have been installed at Niagara Falls by the James Leffel Company.

All reaction turbines will act as impulse turbines when from any cause the passages between the vanes, or buckets, as they are generally called, are not filled with water. In this case the theory of their action is exactly like that of the impulse wheels described in the last chapter. In Arts. 175–178 reaction turbines of the simple outward- and inward-flow types will be discussed, the downward-flow type being reserved for special description in Art. 179.

Prob. 174. Consult Engineering Record, Feb. 5, 1898, and describe methods of regulating the speed of turbines.

Art. 175. Flow through Reaction Turbines

The discharge through an impulse turbine, like that for an impulse wheel, depends only on the area of the guide orifices and the effective head upon them, or $q = av = a\sqrt{2gh}$. In a reaction turbine, however, the discharge is influenced by the speed of revolution, as in the reaction wheel, and also by the areas of the entrance and exit orifices. To find an expression for this discharge let the wheel be supposed to be placed below the surface of the tail water, as in Fig. 175. Let h be the total head between the upper water level and that in the tail race, H_1 the pressure-head on the exit orifices, and H the pressure-head at the gate opening as indicated by a piezometer supposed to be there inserted. Let u_1 and u be the velocities of the wheel at the exit and entrance circumference, which have radii r_1 and r (Fig. 174*b*). Let V_1 and V be the relative velocities of exit and entrance, and v_0 be the absolute velocity of the water as it leaves the guides and enters the wheel; the entering velocity v_0 may be less or greater than $\sqrt{2gh}$, depending upon the value of the pressure-head H. Let a_1, a, and a_0 be the areas of the orifices normal to the directions of V_1, V, and v_0. Now, neglecting all losses of friction between the guides, the theorem of Art. 31, that pressure-head plus velocity-head equals the total head, gives the equation

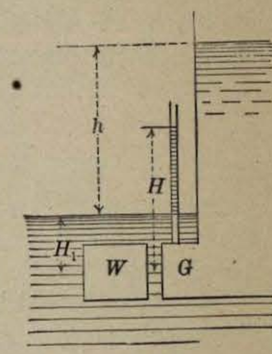

Fig. 175.

$$H + \frac{v_0^2}{2g} = h + H_1$$

Also, neglecting the friction and foam in the buckets, the corresponding theorem of Art. 162 gives

$$H_1 + \frac{V_1^2}{2g} - \frac{u_1^2}{2g} = H + \frac{V^2}{2g} - \frac{u^2}{2g}$$

Adding these equations, the pressure-heads H_1 and H disappear, and there results the formula

$$V_1^2 - V^2 + v_0^2 = 2gh + u_1^2 - u^2 \qquad (175)_1$$

Now, since the buckets are fully filled, the same quantity of water, q, passes in each second through each of the areas a_1, a, and a_0, and hence the three velocities through these areas have the respective values,

$$V_1 = \frac{q}{a_1}, \qquad V = \frac{q}{a}, \qquad v_0 = \frac{q}{a_0}$$

Introducing these values into the formula $(175)_1$, solving for q, and multiplying by a coefficient c to account for losses in leakage and friction, the discharge per second is

$$q = c\sqrt{\frac{2gh + u_1^2 - u^2}{\frac{1}{a_1^2} - \frac{1}{a^2} + \frac{1}{a_0^2}}} \qquad (175)_2$$

This is the formula for the flow through a reaction turbine when the gate is fully raised. The reasoning applies to an inward-flow as well as to an outward-flow wheel. In an outward-flow turbine u_1 is greater than u, and consequently the discharge increases with the speed; in an inward-flow turbine u_1 is less than u, and consequently the discharge decreases as the speed increases.

The value of the coefficient c will usually vary with the head, and also with the size of the areas a_1, a, and a_0. When a turbine has been tested by the methods of Arts. 147–150, and the areas have been measured, the values of c for different speeds may be computed. For example, take the outward-flow Boyden turbine, tests of which at full gate are given in Art. 150. The measured dimensions and angles of this wheel are as follows:

Outer radius of wheel	$r_1 = 3.3167$ feet
Inner radius of wheel	$r = 2.6630$ feet
Outer radius of guide case	$r_0 = 2.5911$ feet
Outer depth of buckets	$d_1 = 0.722$ feet
Inner depth of buckets	$d = 0.741$ feet
Outer area of buckets	$a_1 = 4.61$ square feet
Inner area of buckets	$a = 12.12$ square feet
Outer area of guide orifices	$a_0 = 4.76$ square feet
Exit angle of buckets	$\beta = 13.5$ degrees
Entrance angle of buckets	$\phi = 90$ degrees
Entrance angle of guides	$\alpha = 24$ degrees

Number of buckets 52, number of guides, 32

Inserting in the above formula the values of a_1, a, and a_0, placing for $u_1^2 - u^2$ its value $(\frac{1}{30}\pi N)^2 (r_1^2 - r^2)$, where N is the number of revolutions per minute, it reduces to

$$q = 3.44c\sqrt{2gh + 0.04\ 8N^2}$$

From this the value of c may be computed for each of the seven experiments, and the following tabulation shows the results, the first four columns giving the number of the experiment, the observed head, number of revolutions per minute, and discharge in cubic feet per second. The fifth column gives the theoretic discharge computed from the above formula, taking the coefficient as unity, and the last column is derived by dividing the observed discharge q by the theoretic discharge Q. The discrepancy of 5 or 6 percent is smaller than might be expected, since the formula does not consider frictional resistances.

No.	h	N	q	Q	c
21	17.16	63.5	117.01	123.1	0.950
20	17.27	70.0	118.37	125.2	0.945
19	17.33	75.0	119.53	126.8	0.943
18	17.34	80.0	121.15	128.4	0.944
17	17.21	86.0	122.41	130.0	0.942
16	17.21	93.2	124.74	132.5	0.941
15	17.19	100.0	127.73	134.9	0.947

A satisfactory formula for the discharge through a turbine when the gate is partly depressed is difficult to deduce, because the loss of head which then results can only be expressed by the help of experimental coefficients similar to those given in Art. 92 for the sliding gate in a water pipe, and the values of these for turbines are not known. It is, however, certain that for each particular gate opening the discharge is given by

$$q = m\sqrt{2gh + u_1^2 - u^2}$$

in which m depends upon the areas of the orifices and the height to which the gate is raised. For instance, in the tests of the above Boyden turbine the mean value of m for full gate opening is 3.25, but when the gate was only six-tenths open, its value was 2.81, and when the gate was two-tenths open, its value was 1.36. Each form and size of reaction turbine has its own values of m, depending upon the area of its orifices, and when these have been determined, a turbine may be used as a water meter to measure the discharge with a fair degree of precision.

Prob. 175. Consult Francis' Lowell Hydraulic Experiments, pp. 67–75, and compute the coefficient m for experiments 30 and 31 on the center-vent Boott turbine.