and dividing this by the theoretic energy, the efficiency is

$$e = 1 - (v_1/v)^2$$

This is the same as the general formula (163) if $h' = 0$; that is, if losses in foam and friction are disregarded, and if the wheel is set at the level of the tail race. It is now required to state the conditions which will render these losses and also the velocity v_1 as small as possible. The reasoning will be general and applicable to both outward and inward-flow wheels.

At the point A where the water enters the wheel let the parallelogram of velocities be drawn, the absolute velocity of entrance being resolved into its two components, the velocity u of the wheel at that point, and the velocity V relative to the vane; let the approach angle between u and v be called α, and the entrance angle between u and V be called ϕ. At the point B where the water leaves the wheel let V_1 be its velocity relative to the vane, and u_1 the velocity of the wheel at that point; then their resultant is v_1, the absolute velocity of exit. Let the exit angle between V_1 and the reverse direction of u_1 be called β. The directions of the velocities u and u_1 are of course tangential to the circumferences at the points A and B. Let r and r_1 be the radii of these circumferences; then the velocities of revolution are directly as the radii, or $ur_1 = u_1r$.

In order that the water may enter the wheel without shock and foam, the relative velocity V should be tangent to the vane at A, so that the water may smoothly glide along it. This will be the case if the wheel is run at such speed that the parallelogram at A can be formed, or when the velocities u and v are proportional to the sines of the angles opposite them in the triangle Auv. The velocity v_1 will be rendered very small by running the impulse wheel at such speed that the velocities u_1 and V_1 are equal, since then the parallelogram at B becomes a rhombus and the diagonal v_1 is very small. Hence

$$\frac{u}{v} = \frac{\sin(\phi - \alpha)}{\sin\phi} \quad \text{and} \quad u_1 = V_1 \qquad (168)_1$$

are the two conditions of maximum hydraulic efficiency.

Now, referring to the formula (160), which expresses the relation between the velocities of rotation and the relative velocities of the water for revolving vanes, it is seen that if $u_1 = V_1$, then also $u = V$. But u cannot equal V unless $\phi = 2\alpha$, and then $u = v/2\cos\alpha$, which is the advantageous velocity of the circumference at A. Therefore the two conditions above reduce to

$$\phi = 2\alpha \quad \text{and} \quad u = \frac{v}{2\cos\alpha} \qquad (168)_2$$

which show how the wheel should be built and what speed it should have to secure the greatest efficiency. When this speed obtains, the absolute velocity v_1 is

$$v_1 = 2u_1 \sin\tfrac{1}{2}\beta = 2u\frac{r_1}{r}\sin\tfrac{1}{2}\beta = v\frac{r_1}{r}\frac{\sin\frac{1}{2}\beta}{\cos\alpha}$$

and the corresponding hydraulic efficiency is

$$e = 1 - \left(\frac{r_1}{r}\frac{\sin\frac{1}{2}\beta}{\cos\alpha}\right)^2 \qquad (168)_3$$

by the discussion of which proper values of the approach angle α and the exit angle β can be derived.

This formula shows that both the approach angle α and the exit angle β should be small in order to give high efficiency, but they cannot be zero, as then no water could pass through the wheel; values of from 15° to 30° are usual in practice. It also shows that β is more important than α, and if β be small, α may sometimes be made 40° or 45°. It likewise shows that for given values of α and β the inward-flow wheel, in which r_1 is less than r, has a higher efficiency than the outward-flow wheel.

The condition $V_1 = u_1$ renders the absolute exit velocity v_1 very small, but it does not give its true minimum. This will be obtained by making $V_1 = u_1\cos\beta$, so that the direction of v_1 is normal to that of V_1, and thus $v_1 = u_1\sin\beta$. The discussion of water wheels and turbines under this condition of the true minimum leads to very complex formulas, and hence in this book, as in many others, the simpler condition $V_1 = u_1$ is used.

Prob. 168. Compute the maximum efficiency of an outward-flow impulse wheel when $r_1 = 3$ feet, $r = 2$ feet, $\alpha = 45°$, $\phi = 90°$, $\beta = 30°$, and find the number of revolutions per minute required to secure such efficiency when the velocity of the entering stream is $v = 100$ feet per second.

Art. 169. Downward-flow Impulse Wheels

In the impulse wheels thus far considered the water leaves the vanes in a horizontal direction. Another form used less frequently is that of a horizontal wheel driven by water issuing from an inclined nozzle so that it passes downward along the vanes without approaching or receding from the axis. Figure 169 shows an outline plan of such an impulse wheel and a development of a part of a cylindrical section. Let v be the velocity of the entering stream, u that of the wheel at the point where it strikes the vanes, and v_1 the absolute velocity of the departing water. At the entrance A the direction of v makes with that of u the approach angle α, and the direction of the relative velocity V makes with that of u the entrance angle ϕ. The water then passes over the vane, and, neglecting the influence of friction and gravity, it issues at B with the same relative velocity V, making the exit angle β with the plane of motion.

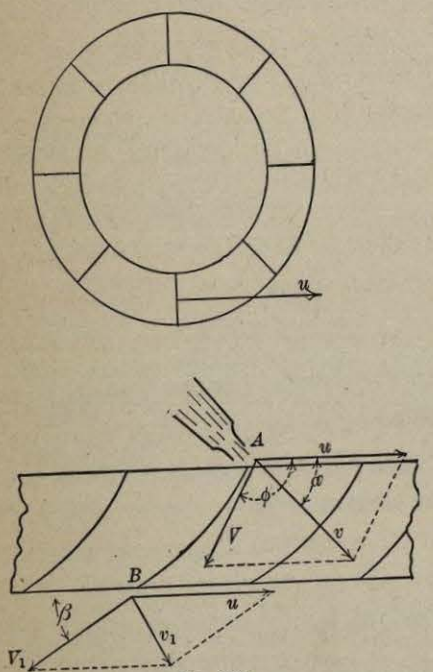

Fig. 169.

The condition that impact and foam shall be avoided at A is fulfilled by making the relative velocity V tangent to the vane, and the condition that the absolute velocity v_1 shall be small is fulfilled by making the velocities u and V equal at B. Hence, as in the last article, the best construction is to make $\phi = 2\alpha$,

and the best speed of the wheel is $u = v/2\cos\alpha$. Also by the same reasoning the efficiency under these conditions is

$$e = 1 - (\sin\tfrac{1}{2}\beta/\cos\alpha)^2$$

which shows that α, and especially β, should be a small angle to give a high numerical value of e. For instance, if both these angles are 30°, the efficiency is 0.92, but if $\alpha = 45°$ and $\beta = 10°$, the efficiency is 0.94.

Although these wheels are but little used, there seems to be no hydraulic reason why they should not be employed with a success equal to or greater than that attained by vertical impulse wheels. It will be possible to arrange several nozzles around the circumference and thus to secure a high power with a small wheel. The fall of the water through the vertical distance between A and B will also add slightly to the power of the wheel, and if this be taken into account, the above values of advantageous velocity and efficiency will be modified, both being slightly increased, as the following investigation shows.

Let h_1 be the vertical fall between A and B; then the theoretic energy of the water with respect to B is

$$K = W\left(h_1 + \frac{v^2}{2g} - \frac{v_1^2}{2g}\right).$$

and the hydraulic efficiency of the wheel is

$$e = 1 - \frac{v_1^2}{v^2 + 2gh_1}$$

Here the relative velocity V_1 at B is greater than V, or

$$V_1^2 = V^2 + 2\,gh_1$$

and since u should equal V_1, this equation becomes, after inserting for V its value in terms of u, v, and α,

$$u = \frac{v}{2\cos\alpha}\left(1 + \frac{2gh_1}{v^2}\right)$$

which gives the advantageous velocity of the wheel. Since

$$v_1 = 2u\sin\tfrac{1}{2}\beta,$$

the above expression for the theoretic hydraulic efficiency reduces to

$$e = 1 - \left(1 + \frac{2gh_1}{v^2}\right)\left(\frac{\sin\frac{1}{2}\beta}{\cos\alpha}\right)^2$$

For this case the approach angle ϕ must be a little greater than 2α, and its value can be found by

$$\cot\phi = \cot\alpha - \frac{v^2 + 2gh_1}{v^2 \sin 2\alpha}$$

and by using this angle ϕ, losses due to impact will be avoided when the wheel is run at the advantageous speed. For example, if $v = 50$ feet per second, and $h_1 = 1$ foot, and $\alpha = 30°$, the value of ϕ is about 63° instead of 60° as the simpler condition requires, while the increase in the advantageous speed is about 2 percent over the former value.

Prob. 169. A wheel like Fig. 169 is driven by water which issues from a nozzle with a velocity of 100 feet per second. If the diameter is 3 feet, the efficiency 0.90, and the approach angle $\alpha = 45°$, find the best value of the entrance and exit angles and the best speed.

Art. 170. Nozzles for Impulse Wheels

Impulse wheels are driven by the dynamic pressure of water issuing from nozzles attached to the end of a pipe which conducts the water from a reservoir. It is shown in Art. 101 that the greatest velocity is secured when the diameter of the nozzle is as small as possible and that the greatest discharge occurs when there is no nozzle. To secure the greatest power, however, there is a certain diameter of nozzle which will now be determined, and it is advisable for economical reasons to use a nozzle of this size and adjust the speed of the wheel thereto.

Let h be the hydrostatic head on the nozzle, l the length, and d the diameter of the pipe, and D the diameter of the nozzle. Let all the resistances except that due to friction in the pipe and nozzle be neglected; then from Art. 101 the velocity of the jet from the nozzle is

$$V = \sqrt{\frac{2gh}{f(l/d)(D/d)^4 + (1/c_1)^2}}$$

in which f is the friction factor for the pipe and c_1 is the coefficient of velocity for the nozzle. Let w be the weight of a cubic foot of water; then the theoretic energy of the jet per second is

$$K = w \cdot \tfrac{1}{4}\pi D^2 V \cdot \frac{V^2}{2g} = \frac{\pi w}{8g}\left(\frac{2gc_1^2 h d^5 D^{\frac{4}{3}}}{fc_1^2 l D^4 + d^5}\right)^{\frac{3}{2}}$$

and the value of D which renders this a maximum is, by the usual method of differentiation, ascertained to be

$$D = (d^5/2fc_1^2 l)^{\frac{1}{4}} \qquad (170)_1$$

and for a nozzle of this size the velocity of the jet is

$$V = 0.816\, c_1 \sqrt{2gh}$$

or, since c_1 is about 0.97, the velocity of the jet when leaving the nozzle is about 80 percent of the theoretic velocity due to the head on the nozzle.

As an example let a pipe be 1200 feet long and 1 foot in diameter; then, taking for f the mean value 0.02 and using $c_1 = 0.97$, there is found $D = 0.39$ feet, and hence a nozzle $4\frac{5}{8}$ inches in diameter is required to give the maximum power. This result may be revised, if thought necessary, by finding the velocity in the pipe and thus getting a better value of f from Table 90a. If the head be 100 feet, this velocity is found to be 9.2 feet per second, whence $f = 0.018$, and on repeating the computation there is found $D = 0.40$ feet $= 4.8$ inches. If the pipe be 12 000 feet long, the advantageous diameter of the nozzle will be found to be much smaller, namely, $2\frac{1}{4}$ inches.

When there is more than one nozzle at the end of the pipe, the above investigation must be modified. Let there be two nozzles with the diameters D_1 and D_2, each having the coefficient c_1. Then the discharge $\frac{1}{4}\pi d^2 v$ through the pipe equals the discharge $\frac{1}{4}\pi(D_1^2 V_1 + D_2^2 V_2)$. But the velocities V_1 and V_2 are equal if the tips of the nozzles are on the same elevation, and hence $d^2 v$ equals $(D_1^2 + D_2^2)V$, where V is the velocity of flow from each nozzle. Now, referring to Art. 101 and to the proof of (170), it is seen that it applies to this case provided D^2 be replaced by $D_1^2 + D_2^2$, and accordingly

$$D_1^2 + D_2^2 = (d^5/2fc_1^2 l)^{\frac{1}{2}} \qquad (170)_2$$

is the formula for determining the sizes of the two nozzles which will furnish the maximum power; if D_1 be assumed, the value of D_2 can be computed. The area of the circle of diameter D found from $(170)_1$ is equal to the sum of the areas of the two circles found from $(170)_2$. If there be three or more nozzles, the sum of their areas is equal to that corresponding to the diameter D as computed from

$(170)_1$. For example, let there be a pipe 1200 feet long and one foot in diameter to which three nozzles of equal size are attached. The diameter found above for one nozzle is 4.80 inches, and the corresponding area is 18.10 square inches; hence the area of the cross-section of the tip of each of the three nozzles is 6.03 square inches, which corresponds to a diameter of 2.77 inches.

Prob. 170. A pipe 15 000 feet long and 18 inches in diameter runs from a mountain reservoir to a power plant, where the water is to be delivered through two nozzles against a hurdy-gurdy wheel. If the diameter of one nozzle is 2 inches, find the diameter of the other in order that the maximum power may be developed. If the head on the nozzles is 623 feet and the efficiency of the wheel 79 percent, compute the horse-power that may be expected.

Art. 171. Special Forms of Wheels

Numerous varieties of the water wheels above described have been used, but the variation lies in mechanical details rather than in the introduction of any new hydraulic principles. In order that a wheel may be a success it must furnish power as cheap as or cheaper than steam or other motors, and to this end compactness, durability, and low cost of installation and maintenance are essential.

A variety of the overshot wheel, called the back-pitch wheel, has been built, in which the water is introduced on the back instead of on the front of the wheel. The buckets are hence differently arranged from those of the usual form, and the wheel revolves also in an opposite direction. One of the largest overshot wheels ever constructed is at Laxey, on the east coast of the Isle of Man. It is $72\frac{1}{2}$ feet in diameter, about 10 feet in width, and furnishes about 150 horse-power, which is used for pumping water out of a mine.

A breast wheel with very long curved vanes extending over nearly a fourth of the circumference has been used for small falls, the water entering directly from the penstock without impulse, so that the action is that of weight alone. This form is made of iron and gives a high efficiency.

Undershot wheels with curved floats for use in the open current of a river have been employed, but in order to obtain much

power they require to be large in size, and hence have not been able to compete with other forms. The great amount of power wasted in all rivers should, however, incite inventors to devise wheels that can economically utilize it. Currents due to the movement of the tides also afford opportunity for the exercise of inventive talent.

The conical wheel, or danaïde, is an ancient form of downward-flow impulse wheel, in which the water approaches the axis as it descends, and thus its relative motion is decreased by the centrifugal force. The theory of this is almost precisely the same as that of an inward-flow impulse wheel, and there seems to be no hydraulic reason why it should not give a high efficiency. Another form of danaïde has two or more vertical vanes attached to an axis, which are inclosed in a conical case to prevent the lateral escape of the water.

A water-pressure engine is a hydraulic motor which moves under the static pressure of water acting against a piston or a revolving disk. The piston forms are reciprocating in motion like the steam-engine and operate in the same way, the water entering and leaving through ports which are opened and closed by a link motion connected with the piston-rod. The other forms give rotary motion directly from the revolving vanes or disks. The piston engine has been employed in Germany to a considerable extent to drive pumps for draining mines, but the rotary engine has not been widely used, and it cannot be advantageously arranged to deliver a high power. On account of the incompressibility of water, special devices for regulating the opening and closing of the valves are necessary.

Numerous other special devices for utilizing the energy of water by means of water wheels have been invented, but they do not introduce any new hydraulic principle. The efficiency of these special forms is often low on account of the imperfections of the apparatus, but it should be borne in mind that high efficiency is only obtained after trials extending over much time, such trials enabling the imperfections to be discovered and removed. The formulas for hydraulic efficiency deduced in the preceding pages do not include losses due to friction, and these may often amount to 10 or 20 percent of the theoretic energy, so that due allowance for them should be made in estimating the power which a proposed design may deliver.

Power may be obtained from the ocean waves, which are constantly rising and falling, by a suitable arrangement of wheels and levers, and some inventions in this direction have given fair promise of success. One in operation on the coast of England about 1890 consisted of a large buoy which rose and fell with the waves on a fixed vertical shaft fastened in the rock bottom. As the buoy moved up and down it operated a system of levers and wheels which drove an air-compressor, and this in turn ran a dynamo that generated electric power. The rise of the ocean tide also affords opportunity for impounding water which may be used to generate power when the tide falls. Plants for this purpose are to be located along tidal rivers where opportunities for impounding occur, the wheels being idle during the rise of the tide, and in operation during its fall. Owing to this intermittent generation of power, it will be necessary to provide for its storage, so that industries using it may be in continuous operation.

Prob. 171*a*. A wheel using 10.5 cubic meters of water per minute under an effective head of 23.4 meters has an efficiency of 75 percent. What metric horse-power does it deliver? What is its power in kilowatts?

Prob. 171*b*. A breast wheel has $c_1 = 0.95$, $h_0 = 1.3$ meters, and $\alpha = 12°$. If its diameter is 3.5 meters, compute the most advantageous number of revolutions per minute.

Prob. 171*c*. An inward-flow impulse wheel has $\phi = 104°$, $\alpha = 52°$, and $\beta = 12°$, its inner diameter being 0.82 meters and its outer diameter 1.22 meters. If this wheel uses 0.86 cubic meters of water per second under an effective head of 7.9 meters, compute its efficiency and its probable effective horse-power.

Prob. 171*d*. A pipe 3200 meters long and 40 centimeters in diameter delivers water through two nozzles against a hurdy-gurdy wheel. When the diameter of one nozzle is 5 centimeters, find the diameter of the other nozzle in order that the energy of the two jets may be a maximum. If the head on the nozzles is 107 meters and the efficiency of the wheels is 81 percent, compute the horse-power which the wheels will deliver.

CHAPTER 14

TURBINES

Art. 172. The Reaction Wheel

The reaction wheel, invented by Barker about 1740, consists of a number of hollow arms connected with a hollow vertical shaft, as shown in Fig. 172. The water issues from the ends of the arms in a direction opposite to that of their motion, and by the dynamic pressure due to its reaction the energy of the water is transformed into useful work. Let the head of water CC in the shaft be h; then the pressure-head BB which causes the flow from the arms is greater than h, on account of the centrifugal force due to the rotation of the wheel. Let u_1 be the absolute velocity of the exit orifices, and V_1 be the velocity of discharge relative to the wheel; then, as shown in Art. 29, and also in Art. 162,

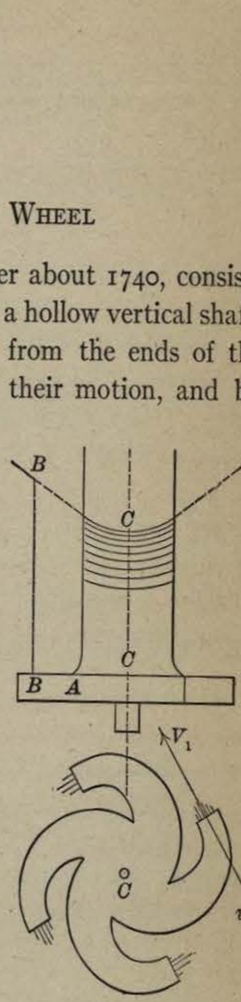

Fig. 172.

$$V_1 = \sqrt{2gh + u_1^2}$$

The absolute velocity v_1 of the issuing water now is

$$v_1 = V_1 - u_1 = \sqrt{2gh + u_1^2} - u_1$$

It is seen at once that the efficiency can never reach unity unless $v_1 = 0$, which requires that $V_1 = u_1$. This, however, can only occur when $u_1 = \infty$, since the above formula shows that V_1 must be greater than u_1 for any finite values of h and u_1. To deduce an expression for the efficiency the work of the wheel