CHAPTER 13

WATER WHEELS

ART. 163. CONDITIONS OF HIGH EFFICIENCY

A hydraulic motor is an apparatus for utilizing the energy of a waterfall. It generally consists of a wheel which is caused to revolve either by the weight of water falling from a higher to a lower level, or by the dynamic pressure due to the change in direction and velocity of a moving stream. When the water enters at only one part of the circumference, the apparatus is called a water wheel; when it enters around the entire circumference, it is called a turbine. In this chapter and the next these two classes of motors will be discussed in order to determine the conditions which render them most efficient. Overshot wheels, which move under the weight of water caught in their buckets, and undershot wheels, which move under the impact of a flowing stream, are forms that have been used for many centuries. Impulse wheels, which owe their motion to a jet of water striking their vanes with high velocity, were perfected in the eighteenth century.

The efficiency e of a motor ought, if possible, to be independent of the amount of water used, or if not, it should be the greatest when the water supply is low. This is very difficult to attain. It should be noted, however, that it is not the mere variation in the quantity of water which causes the efficiency to vary, but it is the losses of head which are consequent thereon. For instance, when water is low, gates must be lowered to diminish the area of orifices, and this produces sudden changes of section which diminish the effective head h. A complete theoretic expression for the efficiency will hence not include W, the weight of water supplied per second, but it should, if possible, include the losses of energy or head which result when W varies. The actual efficiency of a motor can only be determined by tests with the friction brake (Art. 149); the theoretic efficiency, as deduced from formulas like those of the last chapter, will as a rule be higher than the actual, because it is impossible to formulate accurately all the sources of loss. Nevertheless the deduction and discussion of formulas for theoretic efficiency are very important for the correct understanding and successful construction of all kinds of hydraulic motors.

When a weight of water W falls in each second through the height h, or when it is delivered with the velocity v, its theoretic energy per second is

$$K = Wh \qquad \text{or} \qquad K = W\frac{v^2}{2g}$$

The actual work per second equals the theoretic energy minus all the losses of energy. These losses may be divided into two classes: first, those caused by the transformation of energy into heat; and second, those due to the velocity v_1 with which the water reaches the level of the tail race. The first class includes losses in friction, losses in foam and eddies consequent upon sudden changes in cross-section or from allowing the entering water to dash improperly against surfaces; let the loss of work due to this be Wh', in which h' is the head lost by these causes. The second loss is due merely to the fact that the departing water carries away the energy $W \cdot v_1^2/2g$. The work per second imparted by the water to the wheel then is

$$k = W\left(h - h' - \frac{v_1^2}{2g}\right)$$

and dividing this by the theoretic energy the efficiency is,

$$e = 1 - \frac{h'}{h} - \left(\frac{v_1}{v}\right)^2 \qquad (163)$$

in which v is the velocity due to the head h. This formula, although very general, must be the basis of all discussions on the theory of water wheels and motors. It shows that e can only become unity when $h' = 0$ and $v_1 = 0$, and accordingly the two following fundamental conditions must be fulfilled in order to secure high efficiency:

1. The water must enter and pass through the wheel without losing energy in friction and foam.
2. The water must reach the level of the tail race without absolute velocity.

These two requirements are expressed in popular language by the well-known maxim "the water should enter the wheel without shock and leave without velocity." Here the word "shock" means that method of introducing the water upon the wheel which produces foam and eddies.

The friction of the wheel upon its bearings is included in the lost work when the power and efficiency are actually measured as described in Art. 149. But as this is not a hydraulic loss it should not be included in the lost work k' when discussing the wheel merely as a user of water, as will be done in this chapter. The amount lost in shaft and journal friction in good constructions may be estimated at 2 or 3 percent of the theoretic energy, so that in discussing the hydraulic losses the maximum value of e will not be unity, but about 0.98 or 0.97. This will usually be rendered considerably smaller by the friction of the wheel upon the air or water in which it moves, and which will here not be regarded. The efficiency given by (163) is called the hydraulic efficiency to distinguish it from the actual efficiency as determined by the friction brake.

Prob. 163. A wheel using 70 cubic feet of water per minute under a head of 12.4 feet has an efficiency of 63 percent. What effective horse-power does it deliver?

Art. 164. Overshot Wheels

In the overshot wheel the water acts largely by its weight. Figure 164 shows an end view or vertical section, which so fully illustrates its action that no detailed explanation is necessary. The total fall from the surface of the water in the head race or flume to the surface in the tail race is called h, and the weight of water delivered per second to the wheel is called W. Then the theoretic energy per second imparted to the wheel is Wh. It is required to determine the conditions which will render the effective work of the wheel as near to Wh as possible.

The total fall may be divided into three parts: that in which the water is filling the buckets, that in which the water is descending in the filled buckets, and that which remains after the buckets are emptied. Let the first of these parts be called h_0, and the last h_1. In falling the distance h_0 the water acquires a velocity v_0 which is approximately equal to $\sqrt{2gh_0}$, and then, striking the buckets, this is reduced to u, the tangential velocity of the wheel, whereby a loss of energy in impact occurs. It then descends through the distance $h - h_0 - h_1$, acting by its weight alone, and finally, dropping out of the buckets, reaches the level of the tail race with a velocity which causes a second loss of energy. Let h' be the head lost in entering the buckets, and let v_1 be the velocity of the water as it reaches the level of the tail race. Then the hydraulic efficiency of the wheel is given by the general formula (163), or

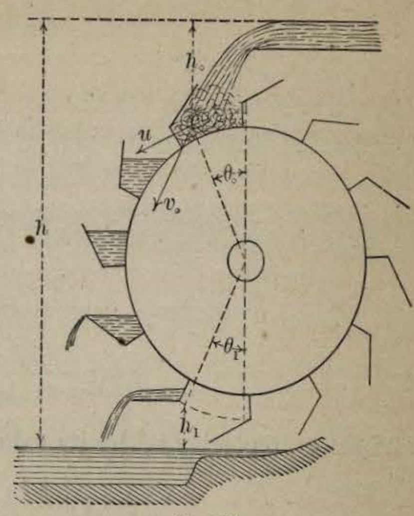

Fig. 164.

$$e = 1 - \frac{h'}{h} - \frac{v_1^2}{v^2}$$

and to apply it, the values of h' and v_1 are to be found. In this equation v is the velocity due to the head h, or $v = \sqrt{2gh}$.

The head lost in impact when a stream of water with the velocity v_0 is enlarged in section so as to have the smaller velocity u, is, as proved in Art. 76,

$$h' = \frac{(v_0 - u)^2}{2g} = \frac{v_0^2 - 2\,v_0 u + u^2}{2g}$$

The velocity v_1 with which the water reaches the tail race depends upon the velocity u and the height h_1. Its kinetic energy as it leaves the buckets is $W \cdot u^2/2g$, the potential energy of the fall h_1 is Wh_1, and the resultant kinetic energy as it reaches the tail race is $W \cdot v_1^2/2g$; hence the value of v_1 is

$$v_1 = \sqrt{u^2 + 2\,gh_1}$$

Inserting these values of h' and v_1 in the formula for e, and placing for v^2 its equivalent $2gh$, there is found

$$e = 1 - \frac{v_0^2 - 2v_0u + 2u^2 + 2gh_1}{2gh}$$

The value of u which renders e a maximum is found by equating the first derivative to zero, which gives

$$u = \tfrac{1}{2}v_0$$

or the velocity of the wheel should be one-half that of the entering water. Inserting this value, the hydraulic efficiency corresponding to the advantageous velocity is

$$e = 1 - \frac{\frac{1}{2}v_0^2 + 2gh_1}{2gh}$$

and lastly, replacing v_0^2 by its value $2gh_0$, it becomes

$$e = 1 - \frac{1}{2}\frac{h_0}{h} - \frac{h_1}{h} \qquad (164)$$

which is the maximum efficiency of the overshot wheel.

This investigation shows that one-half of the entrance fall h_0 and the whole of the exit fall h_1 are lost, and it is hence plain that in order to make e as large as possible both h_0 and h_1 should be as small as possible. The fall h_0 is made small by making the radius of the wheel large; but it cannot be made zero, for then no water would enter the wheel; it is generally taken so as to make the angle θ_0 about 10 or 15 degrees. The fall h_1 is made small by giving to the buckets a form which will retain the water as long as possible. As the water really leaves the wheel at several points along the lower circumference, the value of h_1 cannot usually be determined with exactness.

The practical advantageous velocity of the overshot wheel, as determined by the method of Art. 149, is found to be about $0.4v_0$, and its efficiency is found to be high, ranging from 70 to 90 percent. In times of drought, when the water supply is low, and it is desirable to utilize all the power available, its efficiency is the highest, since then the buckets are but partly filled and h_1 becomes small. Herein lies the great advantage of the overshot wheel; its disadvantage is in its large size and the expense of construction and maintenance.

The number of buckets and their depth are governed by no laws except those of experience. Usually the number of buckets is about $5r$ or $6r$, if r is the radius of the wheel in feet, and their radial depth is from 10 to 15 inches. The breadth of the wheel parallel to its axis depends upon the quantity of water supplied, and should be so great that the buckets are not fully filled with water, in order that they may retain it as long as possible and thus make h_1 small. The wheel should be set with its outer circumference at the level of the tail water.

Prob. 164. Estimate the horse-power and efficiency of an overshot wheel which uses 1080 cubic feet of water per minute under a head of 26 feet, the diameter of the wheel being 23 feet, and the water entering 15° from the top and leaving 12° from the bottom.

Art. 165. Breast Wheels

The breast wheel is applicable to small falls, and the action of the water is partly by impulse and partly by weight. As represented in Fig. 165 water from a reservoir is admitted through an orifice upon the wheel under the head h_0 with the velocity v_0; the water being then confined between the vanes and the curved breast acts by its weight through a distance h_2, which is approximately equal to $h - h_0$, until finally it is released at the level of the tail race and departs with the velocity u, which is the same as that of the circumference of the wheel. The total energy of the water being Wh, the work of the wheel is eWh, if e be its efficiency.

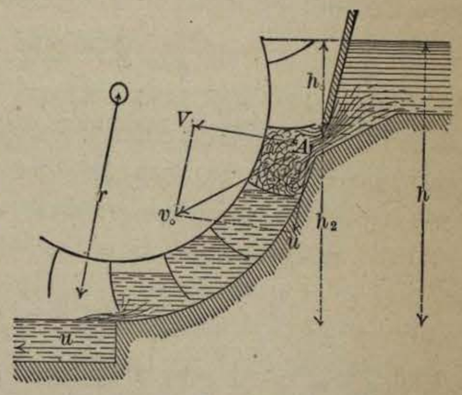
Fig. 165.

The reasoning of the last article may be applied to the breast wheel, h_1 being made equal to zero, and the expression there deduced for e may be regarded as an approximate value of its theoretic efficiency. It appears, then, that e will be the greater the smaller the fall h_0; but owing to leakage between the wheel and the curved breast, which cannot be theoretically estimated, and which is less for high velocities than for low ones, it is not desirable to make v_0 and h_0 small. The efficiency of the breast wheel is hence materially less than that of the overshot, and usually ranges from 50 to 80 percent, the lower values being for small wheels.

Another method of determining the theoretic efficiency of the breast wheel is to discuss the action of the water in entering and leaving the vanes as a case of impulse. Let at the point of entrance Av_0 and Au be drawn parallel and equal to the velocities v_0 and u, the former being that of the entering water and the latter that of the vanes. Let α be the angle between v_0 and u, which may be called the angle of approach. Then the dynamic pressure exerted by the water in entering upon and leaving the vanes is, from Art. 158,

$$P = W\frac{v_0 \cos\alpha - u}{g}$$

and the work performed by it per second is

$$k_0 = W\frac{(v_0 \cos\alpha - u)u}{g}$$

This expression has its maximum value when

$$u = \tfrac{1}{2}v_0 \cos\alpha$$

which gives the advantageous velocity of the wheel circumference, and the corresponding work of the dynamic pressure is

$$k_0 = W\frac{v_0^2 \cos^2\alpha}{4g}$$

Adding this to the work Wh_2 done by the weight of the water, the total work of the wheel when running at the advantageous velocity is found to be

$$k = W\left(\frac{v_0^2 \cos^2\alpha}{4g} + h_2\right)$$

or, if v_0^2 be replaced by its value $c_1^2 \cdot 2gh_0$, where c_1 is the coefficient of velocity for the stream as it leaves the orifice of the reservoir,

$$k = W(\tfrac{1}{2}c_1^2 \cos^2\alpha \cdot h_0 + h_2)$$

whence the maximum hydraulic efficiency of the wheel is

$$e = \tfrac{1}{2}c_1^2 \cos^2\alpha \cdot \frac{h_0}{h} + \frac{h_2}{h} \qquad (165)$$

If in this expression h_2 be replaced by $h - h_0$, and if $c_1 = 1$ and $\alpha = 0°$, this reduces to the same value as found for the overshot wheel. The angle α, however, cannot be zero, for then the direction of the entering water would be tangential to the wheel, and it could not impinge upon the vanes; its value, however, should be small, say from 10° to 25°. The coefficient c_1 is to be rendered large by making the orifice of the discharge with well-rounded inner corners so as to avoid contraction and the losses incident thereto. The above formulas cannot be relied upon in practice to give close values of k and e, on account of losses by foam and leakage along the curved breast, which of course cannot be algebraically expressed.

Prob. 165. A breast wheel is 10.5 feet in diameter, and has $c_1 = 0.93$, $h_0 = 4.2$ feet, and $\alpha = 12$ degrees. Compute the most advantageous number of revolutions per minute.

Art. 166. Undershot Wheels

The common undershot wheel has plane radial vanes, and the water passes beneath it in a direction nearly horizontal. It may then be regarded as a breast wheel where the action is entirely by impulse, so that in the preceding equations h_2 becomes 0, h_0 becomes h, and α will be 0°. The theoretic efficiency then is $e = \frac{1}{2}c_1^2$. In the best constructions the coefficient c_1 is nearly unity, so it may be concluded that the maximum efficiency of the undershot wheel is about 0.5. Experiments show that its actual efficiency varies from 0.20 to 0.40, and that the advantageous velocity is about $0.4v_0$ instead of $0.5v_0$. The lowest efficiencies are obtained from wheels placed in an unlimited flowing current, as upon a scow anchored in a stream; and the highest from those where the stream beneath the wheel is confined by walls so as to prevent the water from spreading laterally.

The Poncelet wheel, so called from its distinguished inventor, has curved vanes, which are so arranged that the water leaves them tangentially, with its absolute velocity less than that of the velocity of the wheel. If in Fig. 165 the fall h_2 be very small, and the vanes be curved more than represented, it will exhibit the main features of the Poncelet wheel. The water entering with the absolute velocity v_0 takes the velocity u of the vane and the velocity V relative to the vane. Passing then under the wheel, its dynamic pressure performs work; and on leaving the vane its relative velocity V is probably nearly the same as that at entrance. Then if V be drawn tangent to the vane at the point

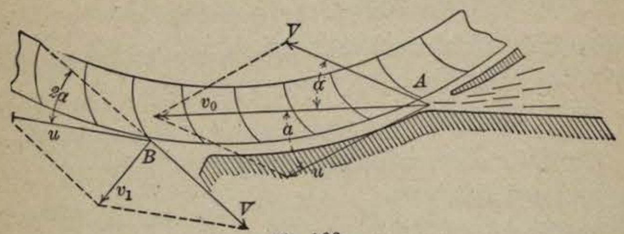

Fig. 166.

of exit, and u tangent to the circumference, their resultant will be v_1, the absolute velocity of exit, which will be much less than u. Consequently the energy carried away by the departing water is less than in the usual forms of breast and undershot wheels, and it is found by experiment that the efficiency may be as high as 60 percent.

In Fig. 166 is shown a portion of a Poncelet wheel. At A the water enters the wheel through a nozzle-like opening with the absolute velocity v_0 and at B it leaves with the absolute velocity v_1. In the figure A and B have the same elevation. At A the entering stream makes the approach angle α with the circumference of the wheel and the same angle with the vane, so that the relative velocity V is equal to the velocity of the outer circumference u. If h be the head on A, the theoretic work of the water is Wh, and the work of the wheel is

$$k = W\frac{v_0^2 - v_1^2}{2g}$$

and the efficiency, neglecting friction and leakage, is

$$e = \frac{v_0^2 - v_1^2}{2gh}$$

Now, let c_1 be the coefficient of velocity of the entrance orifice,

then $v_0 = c_1\sqrt{2gh}$. From the parallelograms of velocity at A and B, there are found

$$u = \frac{v_0}{2\cos\alpha} \qquad v_1 = 2u\sin\alpha = v_0\tan\alpha$$

and for this velocity u the efficiency of the wheel is

$$e = c_1^2(1 - \tan^2\alpha) \tag{166}$$

If $c_1 = 1$ and $\alpha = 0$, the efficiency becomes unity. In the best constructions c_1 may be made from 0.95 to 0.98, but α cannot be a very small angle, since then no water could enter the wheel. If $\alpha = 30°$ and $c_1 = 0.95$ the efficiency is 0.60, which is probably a higher value than usually attained in practice. If the velocity be greater or less than $\frac{1}{2}v_0/\cos\alpha$, the efficiency will be lowered on account of shock and foam at A.

Prob. 166. Estimate the horse-power that can be obtained from an undershot wheel with plane radial vanes placed in a stream having a mean velocity of 5 feet per second, the width of the wheel being 15 feet, its diameter 8 feet, and the maximum immersion of the vanes being 1.33 feet. How many revolutions per minute should this wheel make in order to furnish the maximum power?

Art. 167. Vertical Impulse Wheels

A vertical wheel like Fig. 166, but having smaller vanes against which the water is delivered from a nozzle, is often called an impulse wheel, or a "hurdy-gurdy" wheel. The Pelton wheel, the Cascade wheel, and other forms can be purchased in several sizes and are convenient on account of their portability. Figure 167a shows an outline sketch of such a wheel with the vanes somewhat exaggerated in size. The simplest vanes are radial planes as at A, but these give a low efficiency. Curved vanes, as at B, are generally used, as these cause the water to turn backward, opposite to the direction of the motion, and thus to leave the wheel with a low absolute velocity (Art. 159). In the plan

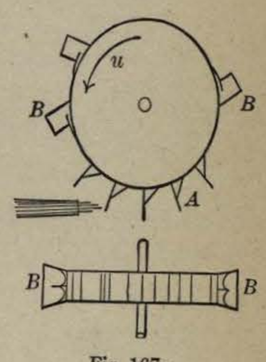

Fig. 167a.

of the wheel it is seen that the vanes may be arranged so as also to turn the water sidewise while deflecting it backward. The experiments of Browne * show that with plane radial vanes the highest efficiency was 40.2 percent, while with curved vanes or cups 82.5 percent was attained. The velocity of the vanes which gave the highest efficiency was in each case almost exactly one-half the velocity of the jet.

The Pelton wheel is used under high heads, and also being of small size it has a high velocity. The effective head is that measured at the entrance of the nozzle by a pressure gage, corrected for velocity of approach and the loss in the nozzle by formula $(83)_1$. These wheels are wholly of iron, and are provided with a casing to prevent the spattering of the water. Fig. 167*b* shows a form with three nozzles, by which three streams are applied at different parts of the circumference, in order to obtain a greater power than by a single nozzle, or to obtain a greater speed by using smaller nozzles. For an effective head of 100 feet and a single nozzle the following quantities are given by the manufacturers:

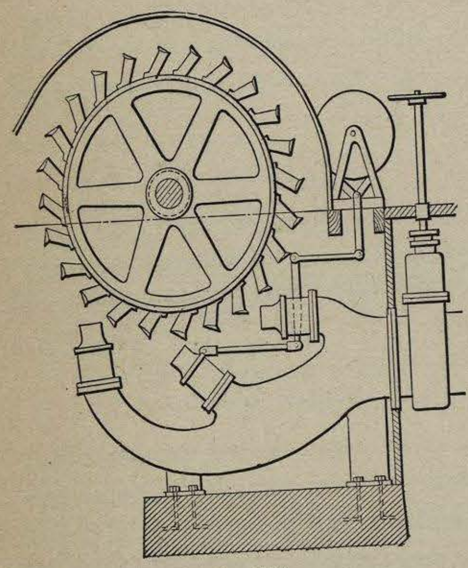

Fig. 167*b*.

Diameter in feet,	1	2	3	4	6
Cubic feet per minute,	8.29	44.19	99.52	176.7	398.1
Revolutions per minute,	726	363	242	181	121
Horse-powers,	1.40	7.49	16.84	29.93	67.3

and these figures imply an efficiency of 85 percent.

The general theory of these vertical impulse wheels is the same as that given for moving vanes in Art. 158. Owing to the high

* Bowie's Treatise on Hydraulic Mining (New York, 1885), p. 193.

velocity, more or less shock occurs at entrance, and as the angle of exit β cannot be made small, the water leaves the vanes with more or less absolute velocity. The advantageous velocity of the vanes or cups is between 40 and 50 percent of that of the entering jet.

Prob. 167. The diameter of a hurdy-gurdy wheel is 12.5 feet between centers of vanes, and the impinging jet has a velocity of 58.5 feet per second and a diameter of 0.182 feet. The efficiency of the wheel is 44.5 percent, when making 62 revolutions per minute What effective horse-power does it furnish?

Art. 168. Horizontal Impulse Wheels

When a wheel is placed with its plane horizontal and is driven by a stream of water from a nozzle, it is called a horizontal impulse wheel. There are two forms, known as the outward-flow and the inward-flow wheel. In the former, shown in Fig. 168*a*, the water enters the wheel upon the inner and leaves it upon the

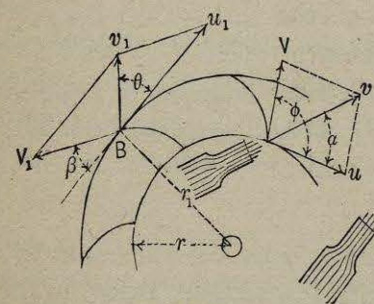

Fig. 168*a*.

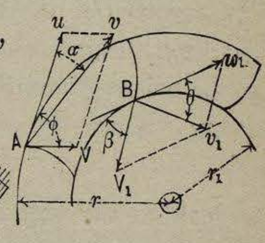

Fig. 168*b*.

outer circumference; in the latter, shown in Fig. 168*b*, the water enters upon the outer and leaves upon the inner circumference. The water issuing from the nozzle with the velocity v impinges upon the vanes, and in passing through the wheel alters both its direction and its absolute velocity, thus transforming its energy into useful work. The energy of the entering water is $W \cdot v^2/2g$ and that of the departing water is $W \cdot v_1^2/2g$. Neglecting frictional resistances, the work imparted to the wheel by the water is

$$k = W\left(\frac{v^2}{2g} - \frac{v_1^2}{2g}\right)$$