which it can act, power, heat, and light can be generated in unlimited quantities.

Prob. 151*a*. Deduce the simple and useful rule that one inch of rainfall per hour is, very nearly, equivalent to one cubic foot per second per acre.

Prob. 151*b*. Find the theoretic horse-power of a plant where 1200 cubic feet of water per second is used under a total head of 49.5 feet. If the efficiency of the approaches is 99 per cent, the efficiency of the turbines 76 percent, and the efficiency of the dynamos 96 percent, what power in kilowatts is delivered?

Prob. 151*c*. What is the theoretic metric horse-power of a plant where 112 cubic meters of water per second are used under a head of 23.5 meters? If the efficiencies of the approaches, turbines, and electric generators are 98.5, 74.3, and 97.5 percent, respectively, compute the number of metric horse-powers delivered, and also the power in kilowatts.

Prob. 151*d*. When a turbine is tested by a friction dynamometer, show that its power in kilowatts is $0.00103NPl$, if P be the load on the brake in kilograms, l its lever-arm in meters, and N the number of revolutions per minute. When $N = 200$, $P = 250$ kilograms, and $l = 2.01$ meters, what electric power is delivered by a dynamo attached to the turbine when the efficiency of the dynamo is 97.2 percent?

Prob. 151*e*. The hectare-meter is a convenient unit for estimating large quantities of water in irrigation and water-supply work. Show that one hectare-meter is 10 000 cubic meters. Show that 100 centimeters of rainfall falling in one month is, very nearly, 0.004 cubic meters per second per hectare.

CHAPTER 12

DYNAMIC PRESSURE OF WATER

Art. 152. Definitions and Principles

The pressures exerted by moving water against surfaces which change its direction or check its velocity are called dynamic, and they follow very different laws from those which govern the static pressures that have been discussed and used in the preceding chapters. A static pressure due to a certain head may cause a jet to issue from an orifice; but this jet in impinging upon a surface may cause a dynamic pressure less than, equal to, or greater than that due to the head. A static pressure at a given point in a mass of water is exerted with equal intensity in all directions; but a dynamic pressure is exerted in different directions with different intensities. In the following chapters the words "static" and "dynamic" will generally be prefixed to the word "pressure," so that no confusion may result.

The dynamic pressure exerted by a stream flowing with a given velocity against a surface at rest is evidently equal to that produced when the surface moves in still water with the same velocity. This principle was applied in Art. 40 in rating the current meter, the vanes of which move under the impulse of the impinging water. The dynamic pressure exerted upon a moving body by a flowing stream depends upon the velocity of the body relative to the stream.

The "impulse" of a jet or stream of water is defined as the dynamic pressure which it is capable of producing in the direction of its motion when its velocity is entirely destroyed in that direction. This can be done by deflecting the jet normally sidewise by a fixed surface; when the surface is smooth, so that no energy is lost in frictional resistances, the actual velocity remains unaltered, but the velocity in the original direction has been rendered null. In Art. 27 it is shown that the theoretic force of impulse of a stream of cross-section a and velocity v is

$$F = W\frac{v}{g} = wq\frac{v}{g} = 2wa\,\frac{v^2}{2g} \qquad (152)$$

in which W and q are the weight and volume delivered per second, and w is the weight of one cubic unit of water. This equation shows that the dynamic pressure that may be produced by impulse is equal to the static pressure due to twice the head corresponding to the velocity v. It would then be expected, when two equal orifices or tubes are placed exactly opposite, as in Fig. 125, and a loose plate is placed vertically against one of them, that the dynamic pressure upon the plate caused by the impulse of the jet issuing from A under the head h would balance the static pressure caused by the head $2h$. This conclusion has been confirmed by experiment, for a tube A which has a smooth inner surface and rounded inner edges so that its coefficient of discharge is unity.

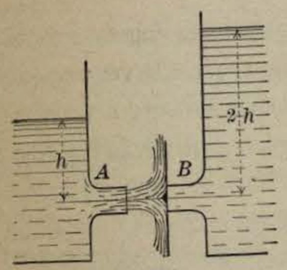

Fig. 152.

The reaction of a jet or stream is the backward dynamic pressure, in the line of its motion, which is exerted against a vessel out of which it issues, or against a surface away from which it moves. This is equal and opposite to the impulse, and the equation above given expresses its value and the laws which govern it. The expression for the reaction or impulse F in (152) may be also proved as follows: The definition by which forces are compared with each other is, that forces are proportional to the accelerations which they can produce. The weight W, if allowed to fall, acquires the acceleration g; the force F which can produce the acceleration v is hence related to W and g by the equation $F/W = v/g$, and accordingly $F = W \cdot v/g$.

The forces of impulse and reaction do not really exist in a stream flowing with constant velocity and direction, although F indicates the force that was exerted in putting the stream into motion and the force that is required to stop it. When the direction of the stream is changed by opposing obstacles, the impulse and reaction produce dynamic pressure; if, in making this change, the absolute velocity is retarded, energy is converted into work. Impulse and reaction are of practical value, because the resulting dynamic pressures may be utilized for the production of work. For this purpose water is made to impinge upon moving vanes, which alter both its direction and velocity, thus producing a dynamic pressure P, which overcomes in each second an equal resisting force through the space u. The work done per second is then $k = Pu$, and it is the object in designing a hydraulic motor to make this work as large as possible; for this purpose, the most advantageous values of P and u are to be selected.

The word "impact" is sometimes popularly used to designate impulse or pressure, but in hydraulics it refers to those cases where energy is lost in eddies and foam, as when a jet impinges into water or upon a rough plane surface. Impact is not defined in algebraic terms, but the energy lost in impact may be so defined and computed. When the energy of a stream of water is to be utilized, losses due to impact should be avoided. Whenever impact occurs, kinetic energy is transformed into heat.

Prob. 152. When a jet is one inch in diameter, how many gallons per second must it deliver in order that its impulse may be 100 pounds?

Art. 153. Experiments on Impulse and Reaction

A simple device by which the dynamic pressure P exerted upon a surface by the impulse and reaction of a jet that glides over it can be directly weighed is shown in Fig. 153a. It consists merely of a bent lever supported on a pivot at O, and having a plate A attached at the lower end of the vertical arm upon which the stream impinges, while weights applied at the end of the other arm measure the dynamic pressure produced by the impulse. By means of an apparatus of this nature, experiments have been made by Bidone, Weisbach, and others, the results of which will now be stated.

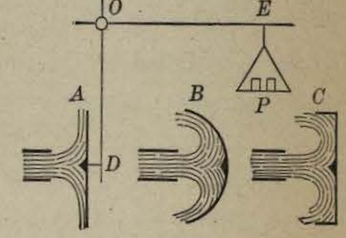

Fig. 153a.

When the surface upon which the stream impinges is a plane normal to the direction of the stream, as shown at A, the dynamic pressure P closely agrees with that given by the theoretic formula for F in the last article, namely,

$$P = W\frac{v}{g} = 2wa\frac{v^2}{2g} \qquad (153)$$

being about 2 percent greater according to Bidone, and about 4 percent less according to Weisbach. The actual value of P was found to vary somewhat with the size of the plate, and with its distance from the end of the tube from which the jet issued.

When the surface upon which the stream impinges is curved, as at B, or so arranged that the water is turned backward from the surface, the value of the dynamic pressure P was found to be always greater than the theoretic value, and that it increased with the amount of backward inclination. When a complete reversal of the original direction of the water was obtained, as at C, it was found that P, as measured by the weights, was nearly double the value of that against the plane. This is explained by stating that as long as the direction of the flow is toward the surface the dynamic pressure of its impulse is exerted upon it, but when the water flows backward away from the surface, the dynamic pressure due to both impulse and reaction is then exerted upon it. The sum of these is

$$P = F + F = 2W\frac{v}{g} = 4wa\frac{v^2}{2g}$$

which agrees with the results experimentally obtained.

An experiment by Morosi * shows clearly that the dynamic pressure against a surface may be increased still further by the device shown in Fig. 153*b*, where the stream is made to perform two complete reversals upon the surface. He found that in this case the value of the dynamic pressure was 3.32 times as great as that against a plane, for $P = 3.32\,F$, whereas theoretically the 3.32 should be 4. In this case, as in those preceding, the water in passing over the surface loses energy in friction and foam, so that

* Ruhlman's Hydromechanik (Hannover, 1879), p. 586.

its velocity is diminished, and it should hence be expected that the experimental values of the dynamic pressures would be less than the theoretic values, as in general they are found to be.

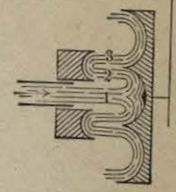

Fig. 153*b*.

While the experiments here briefly described thoroughly confirm the results of theory, they further show it is the change in direction of the velocity when in contact with the surface which produces the dynamic pressure. If the stream strikes normally against a plane, the direction of its velocity is changed 90°, and this is the same as the entire destruction of the velocity in its original direction, so that the dynamic pressure P should agree with the impulse F. This important principle of change in direction will be theoretically exemplified later.

The dynamic pressure which is produced by the direct reaction of a stream of water when issuing from a vertical orifice in the side of a vessel was measured by Ewart with the apparatus shown in Fig. 153*c*, which will be readily understood without a detailed description. The discussion of these experiments made by Weisbach * shows that the measured values of P were from 2 to 4 percent less than the theoretic value F as given by (153), so that in this case, also, theory and observation are in accordance.

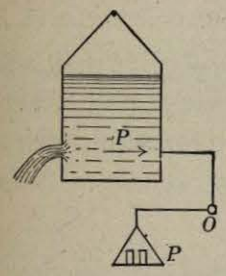

Fig. 153*c*.

An experiment by Unwin,† illustrated in Fig. 153*d*, is very interesting, as it perhaps explains more clearly than formula (152) why it is that the dynamic pressure due to impulse is double the static pressure. Two vessels having converging tubes of equal size were placed so that the jet from A was directed exactly into B. The head in A was kept uniform at $20\frac{1}{2}$ inches,

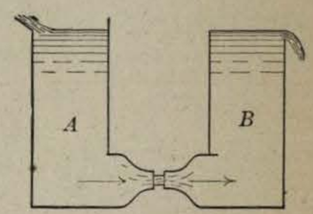

Fig. 153*d*.

* Theoretical Mechanics, Coxe's translation, vol. 1, p. 1004.
† Encyclopedia Britannica, 9th Edition, vol. 12, p. 467.

when it was found that the water in B continued to rise until a head of 18 inches was reached. All the water admitted into A was thus lifted in B by the impulse of the jet, with a loss of $2\frac{1}{2}$ inches of head, which was caused by foam and friction. If such losses could be entirely avoided, the water in B might be raised to the same level as that in A. In the case shown in the figure where the water overflows from B, the impulse of the jet has not only to overcome the static pressure due to the head h, but also to furnish the dynamic pressure equivalent to a second head h in order to raise the water through that height. But the level in B can never rise higher than in A, for the velocity-head of the jet cannot be greater than that of the static head which generates it.

Prob. 153. Accepting as an experimental fact that the force of impulse or reaction is double the static pressure, show that the theoretic velocity of flow is $\sqrt{2gh}$.

Art. 154. Surfaces at Rest

Let a jet of water whose cross-section is a impinge in permanent flow with the uniform velocity v upon a surface at rest. Let the surface be smooth, so that no resisting force of friction exists, and let the stream be prevented from spreading laterally. The

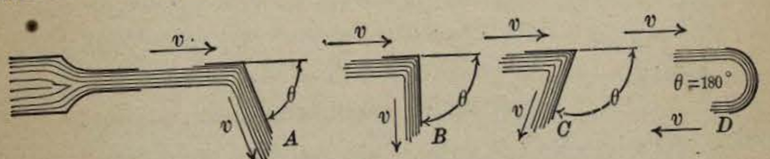

Fig. 154a.

water then passes over the surface, and leaves it with the original velocity v, producing upon it a dynamic pressure whose value depends upon its change of direction. At B in Fig. 154a the stream is deflected normal to its original direction, and at D a complete reversal is effected. Let θ be the angle between the initial and final directions, as shown. It is required to determine the dynamic pressure exerted upon the surface in the same direction as that of the jet. In the above figures, as in those that follow, the stream is supposed to lie in a horizontal plane, so that no

acceleration or retardation of its velocity will be produced by the action of gravity.

The stream entering upon the surface exerts its impulse F in the same direction as that of its motion; leaving the surface, it exerts its reaction F in opposite direction to that of its motion. Let P be the dynamic pressure thus produced in the direction of the initial motion, F_1 the component of the reaction F in the same direction. Then

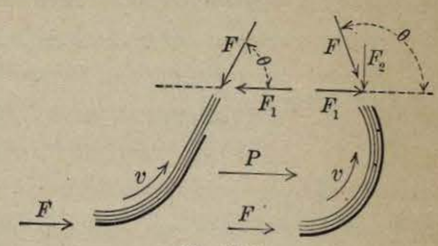

Fig. 154b.

$$P = F - F_1 = F(1 - \cos\theta)$$

and inserting for F its value as given by (152),

$$P = (1 - \cos\theta)W\frac{v}{g} \qquad (154)_1$$

which is the formula for the dynamic pressure in the direction of the impinging jet. If in this $\theta = 0°$, the stream glides along the surface without changing its direction, and P becomes zero; if θ is 90°, the resulting dynamic pressure is

$$P = F = W\frac{v}{g}$$

and if θ becomes 180°, a complete reversal of direction is obtained, and the resulting dynamic pressure that is exerted by the jet against the surface is

$$P = 2F = 2W\frac{v}{g}$$

These theoretic conclusions agree with the experimental results described in the last article. In the deduction of $(154)_1$ the angle θ has been regarded as less than 90°, but the same formula results if θ be considered greater than 90°, since then the sign of the reaction F_1 is positive.

The resultant dynamic pressure exerted upon the surface is found by combining by the parallelogram of forces the impulse F

and the equal reaction F. In Fig. 154b it is seen that this resultant bisects the angle $180 - \theta$, and that its value is

$$P' = 2F\cos\tfrac{1}{2}(180 - \theta) = 2\sin\tfrac{1}{2}\theta \cdot W\frac{v}{g}$$

It is usually, however, more important to ascertain the pressure in a given direction than the resultant. This can be found by taking the component of the resultant in that direction, or by taking the algebraic sum of the components of the initial impulse and the final reaction.

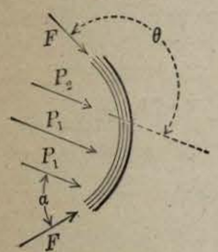

Fig. 154c.

To find the dynamic pressure P in a direction which makes an angle α with the entering and the angle θ with the departing stream, the components in that direction are

$$P_1 = F\cos\alpha \qquad P_2 = -F\cos\theta$$

and the algebraic sum of these two components is

$$P = F(\cos\alpha - \cos\theta) = (\cos\alpha - \cos\theta)W\frac{v}{g} \qquad (154)_2$$

This becomes equal to F when $\alpha = 0$ and $\theta = 90°$, as at B in Fig. 154a, and also when $\alpha = 90°$ and $\theta = 180°$. When $\alpha = 0°$ and $\theta = 180°$ the entering and departing streams are parallel, as at D in Fig. 154a, so that the value of P is $2F$, which in this case is the same as the resultant pressure:

The formulas here deduced are entirely independent of the form of the surface, and of the angle with which the jet enters upon it. It is clear, however, if, as in the planes in Fig. 154a, this angle is such as to allow shock to occur, that foam and changes in cross-section may result which will cause energy to be dissipated in heat. These losses will diminish the velocity v and decrease the theoretic dynamic pressure. These effects cannot be formulated, but it is a general principle, which is confirmed by experiment, that they may be largely avoided by allowing the jet to impinge tangentially upon the surface.

In all the foregoing formulas the weight W which impinges upon the surface per second is the same as that which issues from the orifice or nozzle that supplies the stream, or

$$W = wq = wav$$

To find W it is hence necessary to use the methods of the preceding chapters to determine either the discharge q or the mean velocity v.

Prob. 154. If F is 10 pounds, $\alpha = 0°$, and $\theta = 60°$, show that the pressure parallel to the direction of the jet is 5 pounds, that the pressure normal to that direction is 8.66 pounds, and that the resultant dynamic pressure is 10 pounds.

Art. 155. Immersed Bodies

When a body is immersed in a flowing stream, or when it is moved in still water, so that filaments are caused to change their direction, a dynamic pressure is exerted by the stream or overcome

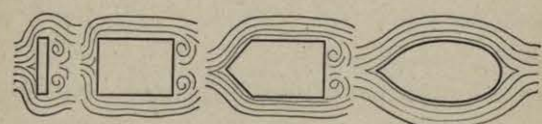

Fig. 155.

by the body. It is to be inferred from what has preceded that the dynamic pressure in the direction of the motion is proportional to the force of impulse of a stream which has a cross-section equal to that of the body, or

$$P = m \cdot wa\frac{v^2}{2g}$$

in which m is a number depending upon the length and shape of the immersed portion, and whose value is 2 for a jet impinging normally upon a plane.

Experiments made upon small plates held normally to the direction of the flow show that the value of m lies between 1.25 and 1.75, the best determinations being near 1.4 and 1.5. It is to be expected that the dynamic pressure on a plate in a stream would be less than that due to the impulse of a jet of the same cross-section, as the filaments of water near the outer edges are crowded sideways in the latter case and hence do not impinge with full normal effect, and the above results confirm this supposition. The few experiments on record were made with small plates, mostly less than 2 square feet in area, and they seem to indicate that the value of the coefficient m is greater for large surfaces than for small ones.

The determination of the dynamic pressure upon the end of an immersed cylinder or prism is difficult because of the resisting friction of the sides; but it is well ascertained to be less than that upon a plane of the same area, and within certain limits to decrease with the length. For a conical or wedge-shaped body the dynamic pressure is less than that upon the cylinder, and it is found that its intensity is much modified by the shape of the rear surface of the body.

When a body is so shaped as to gradually deflect the filaments of water in front, and to allow them to gradually close in again upon the rear, the impulse of the front filaments upon the body is balanced by the reaction of those in the rear, so that the resultant dynamic pressure is zero. The forms of boats and ships should be made so as to obtain this result as nearly as possible, and then the propelling force has only to overcome the frictional resistance of the surface upon the water. A body so shaped is said to have a "fair form" (Art. 183).

The dynamic pressure produced by the impulse of ocean waves striking upon piers or lighthouses is often very great. The experiments of Stevenson on Skerryvore Island, where the waves probably acted with greater force than usual, showed that during the summer months the mean dynamic pressure per square foot was about 600 pounds, and during the winter months about 2100 pounds, the maximum observed value being 6100 pounds. At the Bell Rock lighthouse the greatest value observed was about 3000 pounds per square foot. The observations were made by allowing the waves to impinge upon a circular plate about 6 inches in diameter, and the pressure produced was registered by the compression of a spring. Such high unit-pressures do not probably act upon large areas of masonry which are exposed to wave action.*

Prob. 155. Compute the probable dynamic pressure upon a surface 1 foot square when immersed in a current whose velocity is 9 feet per second, the direction of the current being normal to the surface.

* Cooper on Ocean Waves, in Transactions American Society Civil Engineers, 1896, vol. 36, p. 150.

Art. 156. Curved Pipes and Channels

The dynamic pressures discussed in the preceding article have been those caused by jets, or isolated streams, of water. There is now to be considered the case of dynamic pressures caused by streams flowing in pipes, conduits, or channels of any kind; these are sometimes called limited or bounded streams, the boundary being the surface whose cross-section is the wetted perimeter. When such a stream is straight and of uniform section, and all its filaments move with the same velocity v, the impulse, or the pressure which it can produce, is the quantity F given by the general expression in Art. 152; under these conditions it exerts no dynamic pressure, but if a body be immersed and

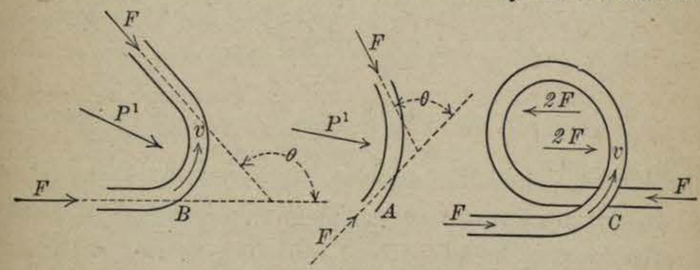

Fig. 156*a*.

held stationary, pressure is produced upon it. If its direction changes in an elbow or bend, pressure upon the bounding surface is produced; if its cross-section increases or decreases, pressure is developed or absorbed.

The resultant dynamic pressure P' upon a curved pipe which runs full of water with the uniform velocity v depends upon the angle θ between the initial and final directions, and must be the same as that produced upon a surface by an impinging jet which passes over it without change in velocity. The formula of Art. 154 then directly applies, and

$$P' = 2\sin\tfrac{1}{2}\theta \cdot F = 2\sin\tfrac{1}{2}\theta \cdot W\frac{v}{g}$$

if $\theta = 0°$, there is no bend, and $P' = 0$; if $\theta = 180°$, the direction of flow is reversed, and $P' = 2F$. If the direction is twice reversed, as at C in Fig. 156*a*, the value of θ is 360°, and the re-