ured upon and from the water level in the tail race, be called h_1 and if the velocity in the pipe be v, then

$$h = h_1 + v^2/2g$$

is the effective head acting on the wheel. It is here supposed that the turbine has a draft tube leading below the water level in the tail race; if this is not the case, h_1 should be measured upward from the lowest part of the exit orifices.

Prob. 148. A pressure gage at the entrance of a nozzle registers 116 pounds per square inch, and the coefficient of velocity of the nozzle is 0.98. Compute the effective velocity-head of the issuing jet.

Art. 149. Measurement of Effective Power

The effective work and horse-power delivered by a water-wheel or hydraulic motor is often required to be measured. Water power may be sold by means of the weight W, or quantity q, furnished under a certain head, leaving the consumer to provide his own motor; or it may be sold directly by the number of horse-power. In either case tests must be made from time to time in order to insure that the quantity contracted for is actually delivered and is not exceeded. It is also frequently required to measure effective work in order to ascertain the power and efficiency of the motor, either because the party who buys it has bargained for a certain power and efficiency, or because it is desirable to know exactly what the motor is doing in order to improve if possible its performance.

The test of a hydraulic motor has for its object: first, the determination of the effective energy and power; second, the determination of its efficiency; and third, the determination of that speed which gives the greatest power and efficiency. If the wheel be still, there is no power; if it be revolving very fast, the water is flowing through it so as to change but little of its energy into work: and in all cases there is found a certain speed which gives the maximum power and efficiency. To execute these tests, it is not at all necessary to know how the motor is constructed or the principle of its action, although such knowledge is very

valuable, and is in fact indispensable to enable the engineer to suggest methods by which its operation may be improved.

A method in which the effective work of a small motor may be measured is to compel it to exert all its power in lifting a weight. For this purpose the weight may be attached to a cord which is fastened to the horizontal axis of the motor, and around which it winds as the shaft revolves. The wheel then expends all its power in lifting this weight W_1 through the height h_1 in t_1 seconds, and the work performed per second then is $k = W_1 h_1 / t_1$. This method is rarely used in practice on account of the difficulty of measuring t_1 with precision.

The usual method of measuring the effective work of a hydraulic motor is by means of the friction brake or power dynamometer invented by Prony about 1780. In Fig. 149 is illustrated a simple method of applying the apparatus to a vertical shaft, the upper diagram being a plan and the lower an elevation. Upon the vertical shaft is a fixed pulley, and against this are seen two rectangular pieces of wood hollowed so as to fit it, and connected by two bolts. By turning the nuts on these bolts while the pulley is revolving, the friction can be increased at pleasure, even so as to stop the motion; around these bolts between the blocks are two spiral springs (not shown in the diagram) which press the blocks outward when the nuts are loosened. To one of these blocks is attached a cord which runs horizontally to a small movable pulley over which it passes, and supports a scale-pan in which weights are placed. This cord runs in a direction opposite to the motion of the shaft, so that when the brake is tightened, it is prevented from revolving by the tension caused

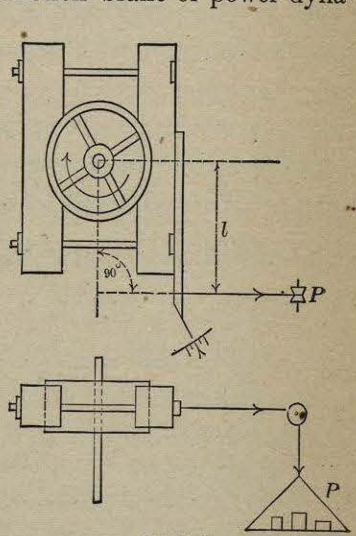

Fig. 149.

by the weights. The direction of the cord in the horizontal plane must be such that the perpendicular let fall upon it from the center of the shaft, or its lever-arm, is constant; this can be effected by keeping the small pointer on the brake at a fixed mark established for that purpose.

To measure the work done by the wheel, the shaft is disconnected from the machinery which it usually runs, and allowed to revolve, transforming all its work into heat by the friction between the revolving pulley and the brake, which is kept stationary by tightening the nuts, and at the same time placing sufficient weights in the scale-pan to hold the pointer at the fixed mark. Let n be the number of revolutions per second as determined by a counter attached to the shaft, P the tension in the cord which is equal to the weight of the scale-pan and its loads, l the lever-arm of this tension with respect to the center of the shaft, r the radius of the pulley, and F the total force of friction between the pulley and the brake. Now in one revolution the force F is overcome through the distance $2\pi r$, and in n revolutions through the distance $2\pi rn$. Hence the effective work done by the wheel in one second is

$$k = F \cdot 2\pi rn = 2\pi n \cdot Fr$$

The force F acting with the lever-arm r is exactly balanced by the force P acting with the lever-arm l; accordingly the moments Fr and Pl are equal, and hence the work done by the wheel in one second is

$$k = 2\pi nPl \qquad (149)_1$$

If P is in pounds and l in feet, the effective horse-power of the wheel is given by

$$\overline{hp} = 2\pi nPl/550$$

As the number of revolutions in one second cannot be accurately read, it is usual to record the counter readings every minute or half-minute; if N be the number of revolutions per minute,

$$\overline{hp} = 2\pi NPl/33\,000 \qquad (149)_2$$

It is seen that this method is independent of the radius of the pulley, which may be of any convenient size; for a small motor the brake may be clamped directly upon the shaft, but for a large

one a pulley of considerable size is needed, and a special arrangement of levers is used instead of a cord.

The efficiency of the motor is now found by dividing the effective work per second by the theoretic work per second. Let K be this theoretic work, which is expressed by Wh, where W and h are determined by the methods of Arts. 147 and 148; then

$$e = k/K \quad \text{or} \quad e = \overline{hp}/\overline{HP}$$

The work measured by the friction brake is that delivered at the circumference of the pulley, and does not include that power which is required to overcome the friction of the shaft upon its bearings. The shaft or axis of every water-wheel must have at least two bearings, the friction of which consumes probably about 2 or 3 percent of the power. The hydraulic power and efficiency of the wheel, regarded as a user of water, are hence 2 or 3 percent greater than the values computed from above formulas. For example, let $P=12.5$ pounds, $l=14.31$ feet, and $N=635$, then 21.6 horse-powers are in total delivered by the wheel, of which about 0.6 horse-power is consumed in shaft friction.

There are in use various forms and varieties of the friction brake, but they all act upon the principle and in the manner above described. For large wheels they are made of iron, and almost completely encircle the pulley; while a special arrangement of levers is used to lift the large weight P.* If the work transformed into friction be large, both the brake and the pulley may become hot, to prevent which a stream of cool water is allowed to flow upon them. To insure steadiness of motion, it is well that the surface of the pulley should be lubricated, which for a wooden brake is well done by the use of soap. It is important that the connection of the cord to the brake should be so made that the lever-arm l increases when the brake moves slightly with the wheel; if this is not done, the equilibrium will be unstable and the wheel will be apt to cause the brake to revolve with it.

Prob. 149. Find the power and efficiency of a motor when the theoretic energy is 1.38 horse-power, which makes 670 revolutions per minute, the weight on the brake being 2 pounds 14 ounces and its lever-arm 1.33 feet.

* Thurston, in Transactions American Society of Mechanical Engineers, 1886, vol. 8, p. 359.

Art. 150. Tests of Turbine Wheels

The following description of a test of a 6-inch Eureka turbine, made in 1888 at the hydraulic laboratory of Lehigh University, may serve to exemplify the methods of the preceding articles. The water was measured by a weir from which it ran into a vertical penstock 15.98 square feet in horizontal cross-section. This plan of having the weir above the wheel is not a good one, but it was here adopted on account of lack of room below the turbine. When a constant head was maintained in the penstock, the quantity of water flowing through the wheel was the same as that passing the weir; if, however, the head in the penstock fell x feet per minute, the flow through the wheel in cubic feet per minute was $60q + 15.98x$, in which q is the discharge per second over the weir. As the supply of water was very limited, the wheel could not be run to its fully capacity. The level of water in the penstock was read upon a head gage consisting of a glass tube behind which a graduated scale was fixed, the zero of which was a little above the water level in the tail race. The latter level was read upon a fixed graduated scale having its zero in the same horizontal plane as the first; these readings were hence essentially negative. The head upon the wheel is then found by adding the readings of the two gages.

The vertical shaft of the turbine, being about 15 feet long, was supported by four bearings, and to a small pulley upon its

Time on April 13, 1888	Depth on Weir Crest Feet	Penstock Gage Feet	Tail-race Gage Feet	Revolutions in One Minute	Weight on Brake Pounds	Remarks
$3^h\ 17^m$	0.288	11.25	−0.21	635	2.5	Length of weir,
18	0.289	11.17	0.20	625	2.5	$b = 1.909$ feet.
19	0.289	11.13	0.21	635	2.5	Length of lever-
20	0.288	11.10	0.21		2.5	arm on brake,
						$l = 1.431$ feet.
$3^h\ 22^m$	0.287	10.81	−0.20	535	3.0	Gate of wheel ¾
23	0.287	10.69	0.20	540	3.0	open during all ex-
24	0.287	10.62	0.21	535	3.0	periments.
25	0.286	10.57	0.21		3.0	

upper end was attached the friction dynamometer, as described in the last article. The number of revolutions was read from a counter placed in the top of this shaft. The observations were taken at minute intervals, electric bells giving the signals, so that precisely at the beginning of each minute simultaneous readings were taken by observers at the weir, at the head gage, at the tail gage, and at the counter, the operator at the brake continually keeping it in equilibrium with the resisting weight in the scale-pan by slightly tightening and loosening the nuts as required. The above shows notes of all the observations of two sets of tests, each lasting three minutes, the weight in the scale-pan being different in the two sets.

The following are the results of the computations made from the above notes for each minute interval. The second column is derived from formula $(63)_1$, using the coefficient corresponding to the given length of weir and depth on crest. The third column is obtained by taking the differences of the observed readings of the penstock head gage. The fourth column gives the discharge

Interval of Time	Discharge over Weir Cubic Feet per Minute	Fall in Penstock Feet	Flow through Wheel Cubic Feet per Minute	Head on Wheel Feet	Theoretic Horse-power of the Water	Effective Horse-power of the Wheel	Efficiency of the Wheel Percent
17^m to 18^m	58.49	+0.08	59.77	11.41	1.290	0.433	33.6
18 to 19	58.66	+0.04	59.30	11.36	1.274	0.426	33.4
19 to 20	58.49	+0.03	58.97	11.32	1.262	0.433	34.3
22^m to 23^m	58.05	+0.13	60.13	10.95	1.245	0.437	35.1
23 to 24	58.05	+0.07	59.17	10.86	1.215	0.441	36.3
24 to 25	57.88	+0.05	58.68	10.80	1.198	0.437	36.5

Q through the wheel found as above explained. The fifth column is the mean head h on the wheel during the minute, as derived from the observed readings of head and tail gage. The sixth column is found by formula $(146)_2$, using for W its value $\frac{1}{60}wQ$, in which w is taken at 62.4 pounds per cubic foot. The seventh column is computed from formula $(149)_2$; and the last column is found by dividing the numbers in the seventh by those in the sixth column.

These results show that the mean efficiency of the wheel and shaft under the conditions stated was about 35 percent; this low figure being due to the circumstance that the gate was not fully opened. It is also seen that the mean efficiency of the second set is 2.2 percent greater than that of the first set; this is due to the lower speed, and with still lower speeds the efficiency was found to be lower, so that a speed of about 535 revolutions per minute gives the maximum efficiency.

The work of Francis on the experiments made by him at Lowell, Mass., will always be a classic in American hydraulic literature, for the methods therein developed for measuring the theoretic power of a waterfall and the effective power utilized by the wheel are models of careful and precise experimentation.* In determining the speed of the wheel he used a method somewhat different from that above explained, namely, the counter attached to the shaft was connected with a bell which struck at the completion of every 50 revolutions; the observer at the counter had then only to keep his eye upon the watch, and to note the time at certain designated intervals, say at every sixth stroke of the bell. The number of revolutions per second was then obtained by dividing the number of revolutions in the interval by the number of seconds, as determined by the watch. This method requires a stop-watch in order to do good work, unless the observer be very experienced, as an error of one second in an interval of one minute amounts to 1.7 percent.

At Holyoke, Mass., there is a permanent flume for testing turbines arranged with a weir which can be varied up to lengths of 20 feet, so as to test the largest wheels which are constructed. As the expense of fitting up the apparatus for testing a large turbine at the place where it is to be used is often great, it is sometimes required in contracts that the wheel shall be sent to a place where a special outfit for such work exists. The wheel is mounted in the testing flume, and there, by the methods explained in the

* Lowell Hydraulic Experiments, 1st Edition, 1855; 4th, 1883.

preceding articles, it is run at different speeds in order to determine the speed which gives the maximum efficiency as well as the effective power developed at each speed. As the efficiency of a turbine varies greatly with the position of the gate which admits the water to it, tests are made with the gate fully opened and at various partial openings. The results thus obtained are not only valuable in furnishing full information concerning the effective power and efficiency of the wheel, but they also convert the turbine into a water meter, so that when running under the same head as during the tests, the quantity of water which passes through it per second can at any time be closely ascertained by noting the number of revolutions per second.

The following gives the results of the tests of an 80-inch outward-flow Boyden turbine, made at Holyoke in 1885, the gate being fully opened in each experiment. The heads in the second column were derived from the head and tail race gages, these being arranged so

Number	Head in Feet	Revolutions per Minute	Discharge Cubic Feet per Second	Horse-power	Efficiency Percent
21	17.16	63.5	117.01	172.57	75.85
20	17.27	70.0	118.37	177.41	76.60
19	17.33	75.0	119.53	178.63	76.11
18	17.34	80.0	121.15	178.32	74.92
17	17.21	86.0	122.41	178.57	74.81
16	17.21	93.2	124.74	176.44	72.54
15	17.19	100.0	127.73	167.94	67.51

that one observer could directly read the difference. The numbers in the third column were found by dividing the total number of revolutions during the experiment by its length in minutes; those in the fourth by the weir formula $(63)_1$; those in the fifth by $(149)_2$ from the records of the friction dynamometer; and those in the last column were computed by $(146)_3$. It is seen that the discharge always increased with the speed of the wheel, and the reason for this is explained in Art. 166. The maximum efficiency of 76.6 percent occurred at 70 revolutions per minute; and for 100 revolutions per minute the efficiency was lowered to 67.7 percent, notwithstanding that the quantity of water passing through the wheel was much greater.

Prob. 150. Compute the theoretic horse-power and the efficiency for the above experiments, Nos. 15 and 21, on the large Boyden outward-flow turbine.

Art. 151. Facts concerning Water Power

The number of horse-powers generated by water-wheels and turbines and used in manufacturing establishments in the United States was 1 130 431 in 1870, 1 225 379 in 1880, 1 263 343 in 1890, and 1 727 258 in 1900; these figures do not include the electric power derived from water. In 1908* the total development was 5 356 680 horse-powers in 52 827 wheels and turbines. Since 1890 there has been a large development of water power in connection with electric light and trolley service, and this development promises to attain great proportions during the twentieth century. It has been estimated that the rivers of the United States can furnish about 212 000 000 horse-powers, so that the possibilities for the future are almost unlimited.

Water power is sometimes sold by what is called the "mill power," which may be roughly supposed to be such a quantity as the average mill requires, but which in any particular case must be defined by a certain quantity of water under a given head. Thus at Lowell the mill power is 30 cubic feet per second under a head of 25 feet, which is equivalent to 85.2 theoretic horse-power. At Minneapolis it is 30 cubic feet per second, under 22 feet head, or 75 theoretic horse-power. At Holyoke it is 38 cubic feet per second under 20 feet head, or 86.4 theoretic horse-power. This seems an excellent way to measure power when it is to be sold or rented, as the head in any particular instance is not subject to much variation; or if so liable, arrangements must be adopted for keeping it nearly constant, in order that the machinery in the mill may be run at a tolerably uniform rate of speed. Thus nothing remains for the water company to measure except the water used by the consumer. The latter furnishes his own motor, and is hence interested in securing one of high efficiency, that he may derive the greatest power from the water for which he pays. The perfection of American turbines is undoubtedly largely due

* Water Supply and Irrigation Paper, No. 234.

to this method of selling power, and the consequent desire of the mill owners to limit their expenditure for water. The turbine itself, when tested and rated, becomes a meter by which the company can at any time determine the quantity of water that passes through it.

A common method of selling the power which is generated by turbines is by the nominal horse-power of the wheel as stated in the catalogue of the manufacturer. The seller fixes a price per annum for one horse-power on this basis, and the buyer furnishes his own wheel. By this method no controversy can arise regarding the amount of water used, for the purchaser has the right to use all that can pass through the turbine. The head to be used for finding the nominal horse-power is the mean head which can be utilized by the wheel, and this must be agreed upon in advance between the parties.

The power of electric generators is usually expressed in kilowatts. One English horse-power in 0.746 kilowatts, and one metric horse-power is 0.736 kilowatts. One kilowatt is 1.340 English horse-powers or 1.360 metric horse-powers. The efficiency of a good electric generator is about 95 percent, so that it delivers 95 percent of the work imparted to it by the turbine wheel; if the efficiency of this wheel is 75 percent, the combined efficiency of the hydraulic and electric plant is 71 percent. Electric power is usually sold by the kilowatt-hour, this being measured by a wattmeter.

The available power of natural waterfalls is very great, but it is probably exceeded by that which can be derived from the tides and waves of the ocean. Twice every day, under the attraction of the sun and moon, an immense weight of water is lifted, and it is theoretically possible to derive from this a power due to its weight and lift. Continually along every ocean beach the waves dash in roar and foam, and energy is wasted in heat which by some device might be utilized in power. The expense of deriving power from these sources is indeed greater than that of the water wheel under a natural fall, but the time may come when the profit will exceed the expense, and then it will certainly be done. Coal and wood and oil may become exhausted, but as long as the force of gravitation exists, and the ocean remains upon which it can act, power, heat, and light can be generated in unlimited quantities.

Prob. 151*a*. Deduce the simple and useful rule that one inch of rainfall per hour is, very nearly, equivalent to one cubic foot per second per acre.

Prob. 151*b*. Find the theoretic horse-power of a plant where 1200 cubic feet of water per second is used under a total head of 49.5 feet. If the efficiency of the approaches is 99 per cent, the efficiency of the turbines 76 percent, and the efficiency of the dynamos 96 percent, what power in kilowatts is delivered?

Prob. 151*c*. What is the theoretic metric horse-power of a plant where 112 cubic meters of water per second are used under a head of 23.5 meters? If the efficiencies of the approaches, turbines, and electric generators are 98.5, 74.3, and 97.5 percent, respectively, compute the number of metric horse-powers delivered, and also the power in kilowatts.

Prob. 151*d*. When a turbine is tested by a friction dynamometer, show that its power in kilowatts is $0.00103NPl$, if P be the load on the brake in kilograms, l its lever-arm in meters, and N the number of revolutions per minute. When $N = 200$, $P = 250$ kilograms, and $l = 2.01$ meters, what electric power is delivered by a dynamo attached to the turbine when the efficiency of the dynamo is 97.2 percent?

Prob. 151*e*. The hectare-meter is a convenient unit for estimating large quantities of water in irrigation and water-supply work. Show that one hectare-meter is 10 000 cubic meters. Show that 100 centimeters of rainfall falling in one month is, very nearly, 0.004 cubic meters per second per hectare.

CHAPTER 12

DYNAMIC PRESSURE OF WATER

Art. 152. Definitions and Principles

The pressures exerted by moving water against surfaces which change its direction or check its velocity are called dynamic, and they follow very different laws from those which govern the static pressures that have been discussed and used in the preceding chapters. A static pressure due to a certain head may cause a jet to issue from an orifice; but this jet in impinging upon a surface may cause a dynamic pressure less than, equal to, or greater than that due to the head. A static pressure at a given point in a mass of water is exerted with equal intensity in all directions; but a dynamic pressure is exerted in different directions with different intensities. In the following chapters the words "static" and "dynamic" will generally be prefixed to the word "pressure," so that no confusion may result.

The dynamic pressure exerted by a stream flowing with a given velocity against a surface at rest is evidently equal to that produced when the surface moves in still water with the same velocity. This principle was applied in Art. 40 in rating the current meter, the vanes of which move under the impulse of the impinging water. The dynamic pressure exerted upon a moving body by a flowing stream depends upon the velocity of the body relative to the stream.

The "impulse" of a jet or stream of water is defined as the dynamic pressure which it is capable of producing in the direction of its motion when its velocity is entirely destroyed in that direction. This can be done by deflecting the jet normally sidewise by a fixed surface; when the surface is smooth, so that no energy is lost in frictional resistances, the actual velocity remains un-