which is the equation of the surface curve, $C$ being the constant of integration. To use this let the logarithmic and circular function in the second parenthesis of the second member be designated by $\phi(x)$ or $\phi(d / D)$, namely,

$$
\phi(x)=\phi(d / D)=\frac{1}{6} \log _{0} \frac{x^{2}+x+1}{(x-1)^{2}}-\frac{1}{\sqrt{3}} \operatorname{arccot} \frac{2 x+1}{\sqrt{3}}
$$

Then the above value of $l$ may be written

$$
l=\frac{D x}{i}-D\left(\frac{1}{i}-\frac{\mathrm{c}^{2}}{g}\right) \phi\left(\frac{d}{D}\right)+C
$$

Now let $d_{2}$ be the depth at the dam and let $l$ be measured up-stream from that point to a section where the depth is $d_{1}$. Then, taking the integral between these limits the constant $C$ disappears, and

$$
\begin{equation*}
l=\frac{d_{2}-d_{1}}{i}+D\left(\frac{\mathrm{r}}{i}-\frac{\mathrm{c}^{2}}{g}\right)\left[\phi\left(\frac{d_{1}}{D}\right)-\phi\left(\frac{d_{2}}{D}\right)\right] \tag{140}
\end{equation*}
$$

which is the practical formula for use. In like manner $d_{2}$ may represent a depth at any given section and $d_{1}$ any depth at the distance $l$


Fig. 140a. up the stream.

When $d=D$, the depth of the backwater becomes equal to that of the previous uniform flow, $x$ is unity, and hence $l$ is infinity. The slope $C C$ of uniform flow is therefore an asymptote to the backwater curve. Accordingly the depth $d_{1}$ is always greater than $D$, although practically the difference may be very small for a long distance $l$.

In the investigation of backwater problems by the above formula there are two cases: first, $d_{2}$ and $d_{1}$ may be given and $l$ is to be found and second, $l$ and one of the depths are given and the other depth is to be found. To solve these problems the values of the backwater function $\phi(d / D)$ computed by Bresse are given in Table 140.* The argument of the table is $D / d$, which, being always less than unity, is more convenient for tabular purposes than $d / D$, since the values of the latter range from I to $\infty$. By the help of Table 140 practical problems may be discussed and the following examples will illustrate the method of procedure.

Table 140. Values of the Backwater Function

| $\frac{D}{d}$ | $\phi\left(\frac{d}{D}\right)$ | $\frac{D}{d}$ | $\phi\left(\frac{d}{D}\right)$ | $\frac{D}{d}$ | $\phi\left(\frac{d}{D}\right)$ | $\frac{D}{d}$ | $\phi\left(\frac{d}{D}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | $\infty$ | 0.954 | 0.9073 | 0.845 | 0.5037 | 0.61 | 0.2058 |
| 0.999 | 2.1834 | . $95{ }^{2}$ | . 8931 | . 840 | . 4932 | . 60 | . 1980 |
| . 998 | 1.9523 | . 950 | . 8795 | . 835 | . 4831 | . 59 | . 1905 |
| . 997 | 1.8172 | . 948 | . 8665 | . 830 | . 4733 | . 58 | . 1832 |
| . 996 | 1.7213 | . 946 | . 8539 | . 825 | . 4637 | . 57 | .1761 |
| . 995 | 1. 6469 | . 944 | . 8418 | . 820 | . 4544 | . 56 | . 1692 |
| . 994 | 1.5861 | . 942 | . 8301 | . 815 | . 4454 | . 55 | . 1625 |
| . 993 | 1.5348 | . 940 | -. 8188 | .810 | . 4367 | . 54 | . 1560 |
| . 992 | 1.4902 | . 938 | . 8079 | . 805 | .428I | . 53 | . 1497 |
| . 991 | 1.4510 | . 936 | . 7973 | . 800 | . 4198 | . 52 | . 1435 |
| . 990 | 1.4159 | . 934 | . 7871 | . 795 | . 4117 | .5I | . 1376 |
| . 989 | I. 3841 | . 932 | .7772 | . 790 | . 4039 | . 50 | . 1318 |
| . 988 | I.3551 | . 930 | . 7675 | . 785 | . 3962 | . 49 | . 1262 |
| . 987 | I. 3284 | . 928 | .758I | . 780 | . 3886 | . 48 | . 1207 |
| . 986 | 1. 3037 | . 926 | . 7490 | . 775 | . 3813 | . 47 | .1154 |
| . 985 | 1. 2807 | . 924 | .7401 | .770 | . 3741 | . 46 | .1102 |
| . 984 | 1. 2592 | . 922 | . 7315 | . 765 | . 3671 | . 45 | . 1052 |
| . 983 | I. 2390 | . 920 | .723I | . 760 | . 3603 | . 44 | . 1003 |
| . 982 | I. 2199 | . 918 | . 7149 | . 755 | . 3536 | . 43 | . 0995 |
| . 981 | 1.2019 | .916 | . 7069 | . 750 | . 3470 | . 42 | . 0909 |
| . 980 | 1.1848 | . 914 | . 6990 | . 745 | . 3406 | . 41 | . 0865 |
| . 979 | I. 1686 | .912 | . 6914 | . 740 | . 3343 | . 40 | . 0821 |
| . 978 | 1.1531 | . 910 | . 6839 | . 735 | . 3282 | . 39 | . 0779 |
| . 977 | 1.1383 | . 908 | . 6766 | . 730 | . 3221 | . 38 | . 0738 |
| . 976 | 1.124I | . 906 | . 6695 | . 725 | . 3162 | . 37 | . 0699 |
| . 975 | I.1105 | . 904 | . 6625 | . 720 | . 3104 | . 36 | . 0660 |
| . 974 | 1.0974 | . 902 | . 6556 | . 715 | . 3047 | . 35 | . 0623 |
| . 973 | 1.0848 | . 900 | . 6489 | . 710 | . 2991 | . 34 | . 0587 |
| . 972 | 1.0727 | . 895 | . 6327 | . 705 | . 2937 | . 33 | . 0553 |
| . 971 I | 1.0610 | . 890 | . 6173 | . 70 | . 2883 | . 32 | .0519 |
| . 970 | 1. 0497 | . 885 | . 6025 | . 69 | . 2778 | . 30 | . 0455 |
| . 968 | 1.0282 | . 880 | . 5884 | . 68 | . 2677 | . 28 | . 0395 |
| . 966 | 1.0080 | . 875 | . 5749 | . 67 | . 2580 | . 25 | . 0314 |
| . 964 | 0.9890 | . 870 | . 5619 | . 66 | . 2486 | . 20 | . 0201 |
| . 962 | . 9709 | . 865 | . 5494 | . 65 | . 2395 | . 15 | . 0113 |
| . 960 | . 9539 | . 860 | . 5374 | . 64 | . 2306 | . 10 | . 0050 |
| . 958 | . 9376 | . 855 | . 5258 | . 63 | . 2221 | . 05 | . 0015 |
| . 956 | . 9221 | . 850 | . 5146 | . 62 | . 2138 | . 00 | . 0000 |

* Bresse's Mécanique appliqués (Paris, I868), vol. é, p. 556 .

A stream of 5 feet depth is to be dammed so that the water shall be io feet deep a short distance up-stream from the dam. The uniform slope of its bed and surface is 0.00018 , or a little less than one foot per mile, and its channel is such that the coefficient c is 65 . It is required to find at what distance up-stream the depth of water is 6 feet. Here $D=5, d_{2}=10, d_{1}=6$ feet, $\mathrm{I} / i=529 \mathrm{I}$, and $\mathrm{c}^{2} / \mathrm{g}=$ 131. Now $D / d_{2}=0.5$, for which the table gives $\phi\left(d_{2} / D\right)=0.1318$, and $D / d_{1}=0.833$, for which the table gives $\phi\left(d_{1} / D\right)=0.4792$. These values inserted in $(140)_{2}$ give

$$
l=529 \mathrm{I}(10-6)+5(5291-13 \mathrm{I})\left(0.479^{2}-0.1318\right)
$$

from which $l=30125$ feet $=5.70$ miles. In this case the water is raised one foot at a distance 5.7 miles up-stream from the dam.

The inverse problem, to compute $d_{2}$ or $d_{1}$, when one of these and $l$ are given, can only be solved by repeated trials by the help of Table 140. For example, let $l=30 \mathrm{I} 25$ feet, the other data as above, and let it be required to determine $d_{2}$ so that $d_{1}$ shall be only 5.2 feet, or 0.2 greater than the original depth of 5 feet. Here $D / d_{1}=0.962$, for which the table gives $\phi\left(d_{1} / D\right)=0.9709$. Then $(140)_{2}$ becomes

$$
30125=529 \mathrm{I}\left(d_{2}-5.2\right)+25800\left[0.9709-\phi\left(d_{2} / D\right)\right]
$$

which is easily reduced to the simpler form

$$
3^{2} 590=5^{291} d_{2}-25800 \phi\left(d_{2} / D\right)
$$

Values of $d_{2}$ are now to be assumed until one is found that satisfies this equation. Let $d_{2}=8$ feet, then $\left(D / d_{2}\right)=0.625$ and, from the table, $\phi\left(d_{2} / D\right)=0.2180$; substituting these, the second member becomes 36700 , which shows that the assumed value is too large. Again, take $d_{2}=7$ feet, then $D / d_{2}=0.714$, for which $\phi\left(d_{2} / D\right)=0.3047$, whence the second member is 29200 , showing that 7 feet is too small. If $d_{2}=7.4$ feet, then $D / d_{2}=0.675$ and $\phi\left(d_{2} / D\right)=0.2629$, and with these values the equation is nearly satisfied, but 7.4 is still too small. On trying 7.5 it is found to be too large. The value of $d_{2}$ hence lies between 7.4 and 7.5 feet, which is as close a solution as will generally be required. The height of dam required to maintain this depth may now be computed from Art. 136.

If the slope, width, or depth of the stream changes materially, the above method, in which the distance $l$ is measured from the dam as an origin, cannot be used. In such cases the stream should be di-
vided into reaches, for each of which the slope, width, and depth can be regarded as constant. The formula can then be used for the first reach and the depth of its upper section be determined; then the application can be made to the next reach, and so on in order. For common rivers and for shallow canals it will probably be a good plan to determine $D$ by actual measurement of the area and wetted perimeter of the cross-section, the hydraulic radius computed from these being taken as the value of $D$. Strictly speaking, the coefficient c varies with the slope and with $D$, and its values may be found by Kutter's formula, if it be thought worth the while. Even if this be done, the results of the computations must be regarded as liable to considerable uncertainty. In computing depths for given lengths an uncertainty of Io percent or more in the value of $d_{2}-d_{1}$ should be expected.

The following method of computation is readily applicable to cases of backwater and gives results which are often sufficiently satisfactory. The distance $l$ between two sections does not appear in the formulas, but it is essential that this distance shall be small enough so that the water surface between them may be regarded as a straight line. In some streams the distance apart of sections may be as high as 1000 feet, in others smaller. Let Fig. $140 b$ represent the case of a stream where an obstruction, which is some distance downstream from the station $M$, causes a rise of the original surface.


Fig. 140b At the several stations
$M, N, P, Q, R$, etc., elevations of the original surface above a datum plane are taken. A cross-section of the stream is also made at each station, the levels being extended upward on the banks so that for any water level the area $a$ and the wetted perimeter $p$ may be ascertained from a drawing. At the first station $M$ the elevation of the backwater is known, it being either assumed or computed from Art. 136. The problem then is to determine the elevation of the backwater at each of the stations up-stream from $M$.

Fig. $140 c$ shows on a larger scale the profile between $M$ and $N$ and also the two cross-sections at $M$ which are drawn from the given data. In this diagram the elevations of $M_{1}, M_{2}$, and $N_{1}$ are known, and it is required to find that of $N_{2}$. Let $a_{1}$ and $a_{2}$


Fig. 140c. .
denote the areas of the cross-section at $M$, the first for the original flow and the second for the backwater, and let $p_{1}$ and $p_{2}$ be the corresponding wetted perimeters. Let $h_{1}$ be the known difference of the elevations of $M_{1}$ and $N_{1}$, and $h_{2}$ the unknown difference of the elevations of $M_{2}$ and $N_{2}$. Then the formula

$$
\begin{equation*}
h_{2}=h_{1} \frac{a_{1}^{3} p_{2}}{a_{2}^{3} p_{1}} \tag{140}
\end{equation*}
$$

determines $h_{2}$, and accordingly the elevation of $N_{2}$ is known. This formula expresses the condition that the same quantity of water flows through the cross-sections $a_{1}$ and $a_{2}$, and it is deduced as follows. The mean discharges in these two sections are, from the Chezy formula, $\mathrm{c}_{1} a_{1} \sqrt{r_{1} s_{1}}$ and $\mathrm{c}_{2} a_{2} \sqrt{r_{2} s_{2}}$. Equating these, replacing $r_{1}$ and $r_{2}$ by $a_{1} / p_{1}$ and $a_{2} / p_{2}$, squaring, and making the coefficients $C_{1}$ and $c_{2}$ equal, gives the equation $s_{1} a_{1}{ }^{3} / p_{1}=s_{2} a_{2}{ }^{3} / p_{2}$. Now $s_{1}=h_{1} l$ and $s_{2}=h_{2} l$ where $l$ is the distance between the two sections. Hence $h_{1} a_{1}^{3} / p_{1}=h_{2} a_{2}^{3} / p_{2}$, from which the above formula $(140)_{3}$ at once results.

As an example, take the case of four stations on Coal River, W.Va., data for the original water surface being as follows:

| Station | M | N | $P$ | Q | $R$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Elevation | = 10.05 | II. 53 | 11. 95 | 13.44 | J4.39 |
| Rise | $h_{1}=$ | 1.48 | 0.42 | 1.49 | 0.95 ft . |
| Area | $a_{1}=3034$ | 3012 | 3210 | 2749 | 2340 sq. ft. |
| Perimete | $=255$ | 260 | 280 | 20 | 192 |

and let it be required to find the elevations of the backwater surface when an obstruction down-stream from $M$ raises the water to elevation 12.05 at $M_{2}$. Drawing the water level in the crosssection at $M$, there are found $a_{2}=3533$ square feet and $p_{2}=260$ feet. Then

$$
h_{2}=1.48 \frac{3034^{3} \times 260}{3533^{3} \times 255}=0.95 \text { feet, }
$$

and hence the elevation at $N_{2}$ is $12.05+0.95=13.00$ feet. For this water-level the cross-section for station $N$ gives 3390 square feet area and 264 feet wetted perimeter for the backwater condition. Then the backwater rise at station $P$ is

$$
h_{2}=0.42 \frac{3012^{3} \times 280}{3390^{3} \times 264}=0.30 \text { feet },
$$

which gives 13.30 feet for the elevation of the backwater surface at $P$. The results for the five stations are arranged as follows, the last line showing the required elevations of the backwater surface:

| Station | M | $N$ | $P$ | $Q$ | $R$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Area | $a_{2}=3533$ | 3390 | 3580 | 2940 | 2492 sq. ft. |
| Perimeter | $p_{2}=260$ | 264 | 286 | 9 | 197 ft . |
| Rise | $h_{2}=$ | 0.95 | 0.30 | 1.10 | 0.80 ft . |
| Elevation | $=12.05$ | 13. | 13.30 | 14.40 | 15.20 |

While there are several assumptions and limitations in this method, it does not appear that they introduce more error than that which obtains when the formula $(140)_{2}$ is applied to a stream of irregular section. By the exercise of much judgment in selecting the stations, and by taking the data for a cross-section as the mean of several on both sides of a station, it is believed that the method can be used with much confidence in all cases where extreme conditions do not obtain. If the Chezy coefficients at a station can be found, then the formula $(140)_{3}$ may be written in the more exact form

$$
h_{2}=h_{1} \mathrm{c}_{1}^{2} a_{1}^{3} p_{2} / \mathrm{c}_{2}^{2} a_{2}^{3} p_{1}
$$

$(140)_{4}$
Prob. 140. A stream, having a cross-section of 2400 square feet and a wetted perimeter of 300 feet, has a uniform slope of 2.07 feet per mile, and its channel is such that $\mathrm{c}=70$. It is proposed to build a dam to raise the water 6 feet above the former level, without increasing the width. Compute the rise of the backwater at a distance of one mile up-stream.

Art 141. The Drop-down Surface Curve
When a sudden fall occurs in a stream, the water surface for a long distance above it is concave to the béd, as seen in Fig. $138 b$ or in Fig. 141. This case also occurs when


Fig. 141. the entire discharge of a canal is allowed to flow out through a forebay $F$ to supply a water-power plant. Let $D$ be the original uniform depth of water having its surface parallel to the bed, the slope of both being $i$. Let $d 1$ and $d_{2}$ be two of the depths after the steady non-uniform flow has been established by letting water out at $F$, and let $d_{1}$ be greater than $d_{2}$, the distance between them being $l$. The investigation of the last article applies in all respects to this form of surface curve, and

$$
\begin{equation*}
l=-\frac{d_{1}-d_{2}}{i}+D\left(\frac{1}{i}-\frac{c^{2}}{g}\right)\left[\phi\left(\frac{d_{1}}{D}\right)-\phi\left(\frac{d_{2}}{D}\right)\right] \tag{141}
\end{equation*}
$$

is the equation for practical use, in which c is the coefficient in the Chezy formula $v=\mathrm{c} \sqrt{r s}$, and $g$ is the acceleration of gravity. Table 140 cannot, however, be used for this case because $d / D$ in that table is greater than unity, while here it is less than unity.

The function $\phi(d / D)$ with values of $d / D$ less than unity is here called the "drop-down function," in order to distinguish it from the backwater function of the last article, although the algebraic expression for the two functions is the same. Table 141, due also to Bresse, gives values of this drop-down function for values of the argument $d / D$, ranging from o to I , and by its use approximate solutions of practical problems can be made. For example, take a canal io feet deep, having a coefficient c equal to 80 , and let the slope of its bed be $\mathrm{I} / 5000$ and its surface slope be the same when the water is in uniform flow. Here $D=$ Io feet, $\mathrm{c}^{2} / g=200$, and $\mathrm{I} / i=5000$. Then

$$
l=-5000\left(d_{1}-d_{2}\right)+48000\left[\phi\left(\frac{d_{1}}{D}\right)-\phi\left(\frac{d_{2}}{D}\right)\right]
$$

Now suppose that a break occurs in the bank of the canal out of which rushes more water than that delivered in normal flow when the depth is io feet, and let it be required to find the distance between two points where the depths of water are 8 and 7 feet. Here $d_{1} / D=0.8$, for which

Table 141. Values of the Drop-down Function

| $\frac{d}{D}$ | $\phi\left(\frac{d}{D}\right)$ | $\frac{d}{D}$ | $\phi\left(\frac{d}{D}\right)$ | $\frac{d}{D}$ | $\phi\left(\frac{d}{D}\right)$ | $\frac{d}{D}$ | $\phi\left(\frac{d}{D}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | $\infty$ | 0.954 | 0.8916 | 0.845 | 0.4478 | 0.61 | 0.0454 |
| 0.999 | 2.1831 | . 952 | . 8767 | . 840 | . 4353 | . 60 | . 0325 |
| . 998 | 1.9517 | . 950 | . 8624 | . 835 | . 4232 | . 59 | . 0199 |
| . 997 | 1.8162 | . 948 | . 8487 | . 830 | .4114 | . 58 | +0.0074 |
| . 996 | 1.7206 | . 946 | . 8354 | . 825 | . 3988 | . 57 | -0.0050 |
| . 995 | I. 6452 | . 944 | . 8226 | . 820 | . 3886 | . 56 | - . 0172 |
| . 994 | 1.584I | . 942 | .8102 | . 815 | . 3776 | . 55 | -. 0293 |
| . 993 | 1.5324 | . 940 | . 7982 | .810 | . 3668 | . 54 | - . 0412 |
| . 992 | 1.4876 | . 938 | . 7866 | . 805 | . 3562 | . 53 | -. 0530 |
| .991 | 1.4486 | . 936 | . 7753 | . 800 | . 3459 | . 52 | - . 0647 |
| . 990 | 1.4125 | . 934 | .7643 | . 795 | . 3357 | . 51 | - . 0763 |
| . 989 | I. 3804 | . 932 | . 7537 | . 790 | . 3258 | . 50 | - . 0878 |
| . 988 | I. 351 II | . 930 | . 7433 | . 785 | . 3160 | . 49 | - .0991 |
| . 987 | I.324I | . 928 | . 7332 | . 780 | . 3064 | . 48 | -. 1104 |
| . 986 | I. 2990 | . 926 | . 7234 | . 775 | . 2970 | . 47 | -. 1216 |
| . 985 | I. 2757 | . 924 | . 7138 | . 770 | . 2877 | . 46 | - . 1327 |
| . 984 | 1.2538 | . 922 | . 7045 | . 765 | . 2785 | 45 | -. 1438 |
| . 983 | 1.2323 | . 920 | . 6953 | . 760 | . 2696 | . 44 | -. 1547 |
| . 982 | 1.2139 | . 918 | . 6864 | . 755 | . 2607 | . 43 | -. .1656 |
| .98I | I. 1955 | .916 | . 6776 | . 750 | . 2520 | . 42 | -. .1765 |
| .980 | 1.1781 | .914 | .6691 | ${ }^{745}$ | . 2434 | .41 | -. 1872 |
| . 979 | I.1615 | . 912 | . 6607 | . 740 | . 2350 | . 40 | - . 1980 |
| . 978 | 1.1457 | .910 | . 6525 | . 735 | - . 2260 | . 39 | - . 2086 |
| . 977 | 1.1305 | . 908 | . 6445 | . 730 | . 2184 | . 38 | - . 2192 |
| . 976 | 1.1160 | . 906 | . 6366 | . 725 | . 2102 | . 37 | - . 2298 |
| . 975 | 1.1020 | . 904 | . 6289 | . 720 | . 2022 | . 36 | - . 2403 |
| . 974 | 1.0886 | . 902 | . 6213 | . 715 | . 1943 | . 35 | - . 2508 |
| . 973 | 1.0757 | . 900 | .6138 | . 710 | . 1864 | . 34 | - . 2612 |
| . 972 | 1.0632 | . 895 | . 5958 | . 705 | . 1787 | . 33 | - . 2716 |
| . 971 | 1.0512 | . 890 | . 5785 | . 70 | .1711 | . 32 | - .2819 |
| . 970 | 1.0396 | . 885 | .5619 | . 69 | . 1560 | . 30 | -. 3025 |
| . 968 | 1.0174 | . 880 | . 5459 | . 68 | .1413 | . 28 | -. 3230 |
| . 966 | 0.9965 | . 875 | . 5305 | . 67 | . 1268 | . 25 | -. 3536 |
| . 964 | . 9767 | . 870 | . 5156 | . 66 | .1127 | . 20 | -. 4042 |
| . 962 | . 9580 | . 865 | . 5012 | . 65 | . 0987 | . 15 | - . 4544 |
| . 960 | . 9402 | . 860 | . 4872 | . 64 | . 0851 | . 10 | -. 5046 |
| . 958 | . 9233 | . 855 | . 4737 | . 63 | . 0716 | . 05 | -. 5546 |
| . 956 | . 9071 | . 850 | . 4605 | . 62 | . 0584 | . 00 | -. 6046 |

- 

$\phi\left(d_{1} / D\right)=0.3459$, and $d_{2} / D=0.7$, for which $\phi\left(d_{2} / D\right)=0.1711$. Inserting these values in the equation, there is found $l=7890$ feet.

- In this case there is a certain limiting depth below which the above formula is not valid. This limit is the value of $x$ for which $\delta / / \delta x$ becomes zero or the value of $x$ where the surface curve is vertical and the bore occurs (Art. 139). From (140) $)_{1}$ this happens when

$$
x^{3}=\mathrm{c}^{2} i / g \quad \text { or } \quad d=D\left(\mathrm{c}^{2} i / g\right)^{\frac{1}{3}}
$$

and for the above example this limiting depth is found to be 3.4 feet. Near this limit, however, the velocity becomes large, so that there is much uncertainty regarding the value of the coefficient c .

When a given discharge per second is taken out of a forebay at the end of a canal having its bed on a slope $i$, the above formula must be modified. Let $q$ be the discharge and let $D_{1}$ be the depth at a section where the slope is $s$, then $q$ equals $c b D_{1} \sqrt{D_{1} s}$. If this value of $q$ be substituted in the equation (138) ${ }_{1}$ and then the same reasoning be followed as at the beginning of Art. 140, it will be found that formula (141) will apply to this case if $D_{1}(s / i)^{\frac{1}{3}}$ be used instead of $D$. For example, let $q=3000$ cubic feet per second, $D_{1}=10$ feet, $i=\mathrm{I} / \mathrm{I0} 000, \mathrm{c}=80$, and the width $b=100$ feet. Then

$$
s=q^{2} / \mathrm{c}^{2} b^{2} D_{1}^{3}=1 / 7100 \quad D=D_{1}(s / i)^{\frac{1}{3}}=11.2 \text { feet. }
$$

Now if it be required to find the distance between two points where the depths of water are ro and 9 feet, formula (141) can be directly applied, and accordingly there is found, by the help of Table 141,

$$
l=-10000(10-9)+109800(0.578-0.355)=14400 \text { feet },
$$

and hence a forebay admitting the given discharge will not draw down the water to a depth less than 9 feet if it be located I4 400 feet downstream from the section where the mean depth is io feet.

Navigation canals are often built with the bed horizontal between locks, and here $i=0$. The above formula cannot be applied to this case because the differential equation (138)2 vanishes when $i$ is zero. To discuss it, equation (138) ${ }_{1}$ must be resumed, and, inverting the same,

$$
\frac{\delta l}{\delta d}=-\frac{\mathrm{c}^{2} b^{2} d^{3}}{q^{2}}+\frac{\mathrm{c}^{2}}{g}
$$

The integration of this between the limits $d_{1}$ and $d_{2}$ gives

$$
\begin{equation*}
l=\frac{\mathrm{c}^{2} b^{2}}{4 q^{2}}\left(d_{1}^{4}-d_{2}^{4}\right)-\frac{\mathrm{c}^{2}}{g}\left(d_{1}-d_{2}\right) \tag{141}
\end{equation*}
$$

from which $l$ may be computed when $q$ is known. As an example, take a rectangular trough for which $q=20$ cubic feet per second, $b=5$ feet, $\mathrm{c}=89$, and let $d_{1}=2.00$ feet and $d_{2}=1.9 \mathrm{I}$ feet. Then from the formula $l$ is found to be 317 feet. This is the reverse of the example at the end of Art. 137, where $l$ was given as 333 feet, so that the agreement is very good.

To compare a canal having a level bed with the one previously considered, the same data will be used, namely, $d_{1}=10$ feet, $d_{2}=9$ feet, $b=100$ feet, $\mathrm{c}=80$, and $q=3000$ cubic feet per second. Then from (141)2 there is found

$$
l=1.778\left(10^{4}-9^{4}\right)-200(10-9)=5920 \text { feet, }
$$

and accordingly the water level is drawn down in one-third of the distance of that of the previous case. The quantity of water that can be obtained from a navigation canal is always less than from one having a sloping bed, and it has frequently happened, when such a canal is abandoned for navigation purposes and is used to furnish water for power or for a public supply, that the quantity delivered is very much smaller than was expected.

The method of computation explained at the end of Art. 140 may be used also to determine the drop-down curve. Referring to Fig. $140 b$ the upper curve will be the original one and the lower one that which is obtained by computation. The fprmula $(140)_{3}$ is to be used by taking $h_{1}, a_{1}, p_{1}$ for the upper curve and $h_{2}, a_{2}, p_{2}$ for the lower one. For example, let the data for a station on the upper original curve be $a_{1}=600$ square feet and $p_{1}=80$ feet, $a_{2}=480$ square feet and $p_{2}=66$ feet. Let the elevations of two points on the upper curve be 18.26 and 16.68 feet so that $h_{1}=1.58$ feet, then the fall in the lower curve is

$$
h_{2}=1.58 \frac{600^{3} \times 66}{480^{3} \times 80}=2.57 \text { feet },
$$

and hence when the elevation of the first station on the lower curve is 16.26 feet, the probable elevation of the second station on that curve is 13.69 feet. The fall 2.57 feet is here probably liable to a considerable error, since the application of $(141)_{1}$ to these data gives a much smaller result for $h_{2}$. Experiments are greatly needed in order to test the comparative value of
these two methods of computation, and these, on a small scale, might well be undertaken in the hydraulic laboratory of an engineering college.

Prob. 141a. A canal from a river to a power house is two miles long, its bed is on a slope of $\mathrm{I} / 10000$, and c is 70 . When the water is in uniform flow, the depth $D$ is 6.0 feet, and the discharge is 800 cubic feet per second. If there be a power house which takes 1000 cubic feet per second, find the probable depth of water at the entrance to its forebay.

Prob. 141b. Show that the last formula in Art. 135, when reduced to the metric system, becomes $v=v^{\prime}+6.1 \sqrt{r s}$.

Prob. 141c. A stream 18 r meters wide and 5 meters deep has a discharge of $\mathrm{I}_{3} 18$ cubic meters per second. Find the height of backwater when the stream is contracted by piers and abutments to a width of 96 meters.

Prob. 141d. Which has the greater discharge, a stream I. 2 meters deep and 20 meters wide on a slope of 3 meters per kilometer, or a stream 1. 6 meters deep and 26 meters wide on a slope of 2 meters per kilometer?

Prob. 141e. A stream 2 meters deep is to be dammed so that water shall be 4 meters deep at the dam. Its slope is 0.0002 and its channel is such that the metric value of c is 39 . Compute the distance to a section up-stream where the depth of water is 3.6 meters.

