and the mean velocity is $v = 826/1410 = 0.59$ feet per second. A second gaging of the stream, made a week later, when the water level was 0.59 feet higher, gave for the discharge 1336 cubic feet per second, for the total area 1630 square feet, and for the mean velocity 0.82 feet per second.

In the following tabulation are illustrated both the field notes and the subsequent computations made to determine the discharge of a stream from a current meter gaging.

Distance from Initial Point	Depth of Water	Depth of Point of Observation	Time in Seconds	Meter Revolutions	Velocity At Point	Velocity Mean in Vertical	Velocity Mean in Section	Mean Water Depth	Distance between Sections	Area of Section	Discharge Cubic Feet per Second
0	0	—	—	—	0	0					
		0.4	33	40	2.80		1.08	1.0	3.0	3.0	3.2
3	2	1.6	61	40	1.52	2.16					
		1.2	36	50	3.20		2.40	4.0	2.0	8.0	19.2
5	6	4.8	45	40	2.08	2.64					
		2.0	37	60	3.70		2.95	8.0	5.0	40.0	118.0
10	10	8.0	41	50	2.82	3.26					
		1.0	49	70	3.30		3.05	7.5	5.0	37.5	114.4
15	5	4.0	39	40	2.38	2.84					
		0.6	32	40	2.90		2.60	4.0	3.0	12.0	31.2
18	3	2.4	44	80	1.82	2.36					
							1.18	1.5	2.0	3.0	3.5
20	0	—			0	0					
								Totals	20.0	103.5	289.5

After a number of discharge measurements have been made at a particular gaging station and at a number of different gage heights or water stages it becomes possible to plot for the station a rating curve which will show the discharge of the stream at any given stage. It is also convenient, for purposes of record and comparison, to plot on the same sheet the curves of mean velocity and areas of the cross-sections for each of the gagings. As new measurements of discharge are made they, together with their corresponding velocities and areas, may then be plotted upon this sheet, and any errors or differences such as those due to a change in the stream bed become at once apparent. Such curves are shown in Fig. 131*b*.

After the discharge curve for a station has been established, it becomes possible, by keeping a record of the gage heights at the station, to determine the total quantity of water which passes the station in any given time. Observations on the gage height

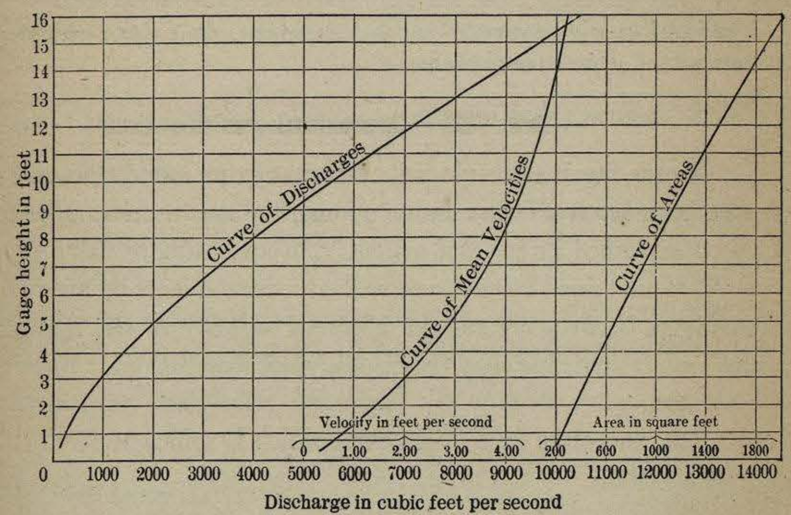

Fig. 131*b*.

may be made from two to three or more times a day, and in cases where the highest accuracy is desired a self-recording gage, such as described in Art. 34, should be installed for the purpose of getting a continuous record of the water height.

When changes in the bed of a stream occur as the result of scouring during freshets, or from the formation of bars or other causes, new rating curves must be constructed, and care should always be taken to see that all of the water flowing down the stream passes the section at which measurements are made. If diversions past the point of gaging occur, or in case two or more channels are found during times of high water, proper allowances or new gagings should be made.

As to the accuracy of the above described methods of gaging the discharge it may be said that with ordinary work, using rod floats, the discrepancies in results obtained under different conditions ought not to exceed 10 percent; and with careful work, using current meters, they may often be of a higher degree of precision. In any event the results derived from such gagings are more reliable than can be obtained by the use of any formula for the discharge of a stream.

Prob. 131. A stream 140 feet wide is divided into seven equal parts, the six soundings being 1.9, 4.0, 4.8, 4.6, 2.7, and 1.0 feet. The seven velocities as found by a current meter are 0.7, 1.6, 2.4, 3.5, 3.0, 1.4, and 0.6 feet per second. Compute the discharge.

Art. 132. Approximate Gagings

When the mean velocity v of a stream can be found, the discharge is known from the relation $q = av$, the area a being measured as explained in the last article. An approximate value of v may be ascertained by one or more float measurements by means of relations between it and the observed velocity of the floats which have been deduced by the discussion of observations. Such measurements are usually less expensive than those explained in Art. 131, and often give information which is sufficient for the inquiry in hand.

The ratio of the mean velocity v to the maximum surface velocity V has been found to usually lie between 0.7 and 0.85, and about 0.8 appears to be a rough mean value. Accordingly,

$$v = 0.8\,V$$

from which, if V be accurately determined, v can be computed with an uncertainty usually less than 20 percent. Many attempts have been made to deduce a more reliable relation between v and V. The following rule derived from the investigations of Bazin makes the relation dependent on the coefficient c, the value of which for the particular stream under consideration is to be obtained from the evidence presented in the last chapter:

$$v = V\Big/\left(1 + \frac{25}{c}\right)$$

It is probable, however, that the relation depends more on the hydraulic radius and the shape of the section than upon the degree of roughness of the channel, which c mainly represents.

The influence of wind upon the surface velocities is so great that these methods of determining v may not give good results

except in calm weather. A wind blowing up-stream decreases the surface velocities, and one blowing down-stream increases them, without materially affecting the mean velocity and discharge of the stream.

The ratio of the mean velocity v_1 in any vertical to its surface velocity V_1 is less variable, for it lies between 0.79 and 0.98, or

$$v_1 = 0.86\,V_1$$

may be used with but an uncertainty of a few per cent. If several velocities V_1, V_2, etc., are determined by surface floats, the mean velocities v_1, v_2, etc., for the several sub-areas a_1, a_2, etc., are known, and the discharge is $q = a_1v_1 + a_2v_2 +$ etc., as before explained.

By means of a sub-surface float, or by a current meter, the velocity V' at mid-depth in any vertical may be measured. The mean velocity v_1 in that vertical is very closely

$$v_1 = 0.98\,V'$$

In this manner the mean velocities in several verticals across the stream may be determined by a single observation at each point, and these may be used, as in Art. 131, in connection with the corresponding areas to compute the discharge.

It was shown by the observations of Humphreys and Abbot on the Mississippi that the velocity V' is practically unaffected by wind, the vertical velocity curves for different intensities of wind intersecting each other at mid-depth. The mid-depth velocity is therefore a reliable quantity to determine and use in approximate gagings, particularly as the corresponding mean velocity v_1 for the vertical rarely varies more than 1 or 2 per cent from the value $0.98\,V'$.

Since the maximum surface velocity is greater than the mean velocity v, and since the velocities at the shores are usually small, it follows that there are in the surface two points at which the velocity is equal to v. If by any means the location of either of these could be discovered, a single velocity observation would directly give the value of v. The position of these points is subject to so much variation in channels of different forms, that no satisfactory method of locating them has yet been devised.

In cases where it is desired to construct an approximate discharge curve and where only a few discharge measurements have been made, the method indicated by Stevens* may be followed. From a cross-section of the stream the values of $a\sqrt{r}$ in the Chezy formula $q = ac\sqrt{rs}$ may be determined for each gage height and a curve plotted. The discharge q then being known for several gage heights, it becomes possible to determine a value for $c\sqrt{s}$. The value of this latter function is nearly a constant, and the desired discharge curve can thus be approximated.

Other methods of making approximate gagings consist in adding a solution of some chemical or salt to the water of the stream to be measured at some point where thorough mixing will occur. If the strength of the chemical solution and the rate of its application are known, and if samples of the water of the stream are taken above the point where the solution is introduced and down-stream after thorough mixing has occurred, the discharge of the stream is then equal to the number of times the chemical solution has been diluted by the water of the stream multiplied by the rate of application of the chemical. For example, if 2 quarts of a solution of common salt containing 10 000 parts per million of chlorine be added each second to the stream and if a sample taken one-half a mile down-stream shows the chlorine to be 20 parts per million then the dilution has been 10 000/20 or 500 and the discharge then is 500 × 2 quarts = 1000 quarts per second. No account has here been taken of the chlorine naturally found in the water of the stream, and this must in all cases be allowed for. Stromeyer † has experimented in this manner with solutions of common salt and sulphuric acid. On small streams he found that the results agreed well with both the measurements of a weir and a Venturi meter, thus leading him to conclude that results correct within 1 percent can be obtained in this manner. It is doubtful, however, if such accuracy could be had in large streams.

Benzenberg,‡ in gaging the flow in a portion of the sewer system of Milwaukee where the sewer lay in a tunnel below the hydraulic gradient, injected a quantity of red eosine into the water at one end of the tunnel and observed its appearance at the other. He found that the color in the water was never distributed over a length

* Engineering News, July 18, 1907.
† Proceedings, Institution of Civil Engineers, vol. 160.
‡ Transactions American Society of Civil Engineers, December, 1893.

greater than 7 to 9 feet, and thus the mean velocity was determined with great accuracy. This experiment was of interest also in indicating the relatively small extent to which the particles of water in a given cross-section, such as that of a sewer, become separated from each other, even during a one-half mile journey.

Prob. 132. A stream 60 feet wide is divided into three sections, having the areas 32, 65, and 38 square feet, and the surface velocities near the middle of these are found to be 1.3, 2.6, and 1.4 feet per second. What is the approximate mean velocity of the stream and its discharge?

Art. 133. Comparison of Gaging Methods

This chapter, together with those preceding, furnishes many methods by which the quantity of water flowing through an orifice, pipe, or channel may be determined. A few remarks will now be made by way of summary and comparison.

The method of direct measurement in a tank is always the most accurate, but except for small quantities is expensive, and for large quantities is impracticable. Next in reliability and convenience come the methods of gaging by orifices and weirs. An orifice one foot square under a head of 25 feet will discharge about 24 cubic feet per second, which is as large a quantity as can usually be profitably passed through a single opening. A weir 20 feet long with a depth of 2.0 feet will discharge about 200 cubic feet per second, which may be taken as the maximum quantity that can be conveniently thus gaged. The number of weirs may be indeed multiplied for larger discharges, but this is usually forbidden by the expense of construction. Hence, for larger quantities of water indirect measurements must be adopted.

The formulas deduced for the flow in pipes and channels in Chaps. 8 and 9 enable an approximate estimation of their discharge to be determined when the coefficients and data which they contain can be closely determined. The remarks in Art. 128 indicate the difficulty of ascertaining these data for streams, and show that the value of the formulas lies in their use in cases of investigation and design rather than for precise gagings. For pipes an accurately rated water meter is a convenient method of measuring the discharge, while for conduits it will often be found difficult to devise an accurate and economical plan for precise determinations, unless the conditions are such that the discharge may be made to pass over a weir or to be retained in a large reservoir, the capacity of which is known for every tenth of a foot in depth. For large aqueducts, and for canals and streams, the usually available methods are those explained in this chapter. In the case of the Catskill Aqueduct under construction in 1912 a number of Venturi meters of capacities up to 770 cubic feet per second have been introduced (Art. 39).

Surface floats are not to be recommended except for rude determinations, because they are affected by wind and because the deduction of mean velocities from them is in many cases subject to much uncertainty. Nevertheless many cases arise in practice where the results found by the use of surface floats are sufficiently precise to give valuable information concerning the flow of streams. The double float for sub-surface velocity is used in deep and rapid rivers, where a current meter cannot be well operated on account of the difficulty of anchoring a boat. In addition to its disadvantages already mentioned may be noted that of expense, which becomes large when many observations are to be taken.

The method of determining the mean velocities in vertical planes by rod floats is very convenient in canals and channels which are not too deep or too shallow. The precision of a velocity determination by a rod float is always much greater than that of one taken by the double float, so that the former is to be preferred when circumstances will allow. In cases where the velocity is rapid, or where there are no bridges over the stream, rod floats may often give results more reliable than can be obtained by any other method.

Current-meter observations are those which now generally take the highest rank for precision in streams where the conditions are not abnormal. The first cost of the outfit is greater than that required for rod floats, but if much work is to be done, it will prove the cheaper. The main objection is the difficulty of use

in cases of high velocities and to the errors which may be introduced from the lack of proper rating; this is required to be done at intervals, since it is found that the relation between the velocity and the recorded number of revolutions may change during use.

In the execution of hydraulic operations which involve the measurement of water a method is to be selected which will give the highest degree of precision with given expenditure, or which will secure a given degree of precision at a minimum expense. Any one can build a road, or a water-supply system; but the art of engineering teaches how to build it well, and at the least cost of construction and maintenance. Similarly the science of hydraulics teaches the laws of flow and records the results of experiments, so that when the discharge of a conduit is to be measured or a stream is to be gaged, the engineer may select that method which will furnish the required information in the most satisfactory manner and at the least expense.

Prob. 133. Consult Humphreys and Abbot's Physics and Hydraulics of the Mississippi River (Washington, 1862 and 1876), and find two methods of measuring the velocity of a current different from those described in the preceding pages.

Art. 134. Variations in Discharge

When the stage of water rises and falls, a corresponding increase or decrease occurs in the velocity and discharge. The relation of these variations to the change in depth may be approximately ascertained in the following manner, the slope of the water surface being regarded as remaining uniform: Let the stream be wide, so that its hydraulic radius is nearly equal to the mean depth d; then

$$v = c\sqrt{ds} = cs^{\frac{1}{2}}d^{\frac{1}{2}}$$

Differentiating this with respect to v and d gives

$$\delta v/v = \tfrac{1}{2}\,\delta d/d$$

Here the first member is the relative change in velocity when the depth varies from d to $d \pm \delta d$, and the equation hence shows that the relative change in velocity is one-half the relative change in depth. For example, a stream 3 feet deep, and with a mean velocity of 4 feet per second, rises so that the depth is 3.3 feet;

then $dv = 4 \times \frac{1}{2} \times 0.3/3 = 0.2$, and the velocity of the stream becomes $4 + 0.2 = 4.2$ feet per second.

In the same manner the variation in discharge may be found. Let b be the breadth of the stream, then

$$q = cbd\sqrt{ds} = cbs^{\frac{1}{2}}d^{\frac{3}{2}}$$

and by differentiating with respect to q and d,

$$\delta q/q = \tfrac{3}{2}\,\delta d/d$$

Hence the relative change in discharge is $1\frac{1}{2}$ times that of the relative change in depth. This rule, like the preceding, supposes that δb is very small, and will not apply to large variations in the depth of the water.

The above conclusions may be expressed as follows: If the mean depth changes 1 percent, the velocity changes 0.5 percent, and the discharge changes 1.5 percent. They are only true for streams with such cross-sections that the hydraulic radius may be regarded as proportional to the depth, and even for such sections are only exact for small variations in d and v. They also assume that the slope s remains the same after the rise or fall as before; this will be the case if a condition of permanency is established, but, as a rule, while the stage of water is rising the slope is increasing, and while falling the slope is decreasing.

Gages for reading the stages of water are now set up on many rivers, and daily observations are taken. Such a gage is usually a vertical board graduated to feet and tenths and set if possible with its zero below the lowest known water level. Another form is the box-and-chain gage, which consists of a box fastened on a bridge with a graduated scale within it and a chain that can be let down to the water level; the length of the chain being known, the gage height can then be read from the scale if its zero is set so that the reading will be zero when the end of the chain just touches the water surface when it is at zero height. Such observations of the daily stage of a river are of great value in planning engineering constructions, and they are now made at many

stations by the United States government through the Department of Agriculture and the Geological Survey Bureau.

When several measurements of the discharge of a stream have been made for different stages of water, a curve may be drawn to show the law of variation of discharge (Art. 131), and from this curve the discharge corresponding to any given stage of water may be approximately ascertained. Fig. 131*b* shows a typical discharge curve. Fig. 134 shows the actual discharge curve for the Lehigh River at Bethlehem, Pa., the ordinates being the

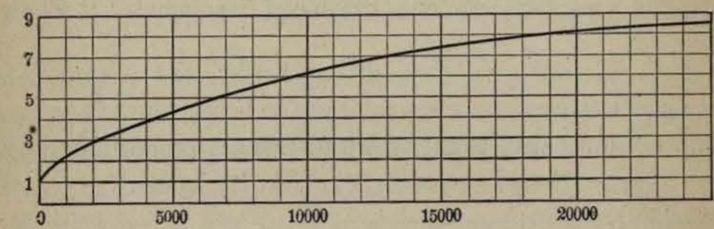

Fig. 134.

heights of the water level as read on the gage, and the abscissas being the discharges of the river in cubic feet per second; this is only a part of the discharge curve for that river, as the water has been known to rise to 22.5 feet and the corresponding discharge was over 100 000 cubic feet per second. Each station on a river has its own distinctive discharge curve, for the local topography determines the heights to which the water level will rise.

Prob. 134. A stream of 4 feet mean depth delivers 800 cubic feet per second. What will be the discharge when the depth is decreased to 3.87 feet? If the stream is 100 feet wide, what will be the velocity when the depth is 4.12 feet?

Art. 135. Transporting Capacity of Currents

The fact that the water of rapid streams transports large quantities of earthy matter, either in suspension or by rolling it along the bed of the channel, is well known, and has already been mentioned in Art. 120. It is now to be shown that the diameters of bodies which can be moved by the pressure of a current vary as the square of its velocity, and that their weights vary as the sixth power of the velocity.

When water causes sand or pebbles to roll along the bed of a channel, it must exert a force approximately proportional to the square of the velocity and to the area exposed (Art. 27), or if d is the diameter of the body and C a constant, the force which is required to move it horizontally is

$$F = Cd^2v^2$$

But if motion just occurs, this force is also proportional to the weight of the body, because the frictional resistance of one body upon another varies as the normal pressure or weight. And as the weight of a sphere varies as the cube of the diameter, it follows that

$$d^3 = Cd^2v^2 \quad \text{or} \quad d = Cv^2$$

Now since d varies as v^2, the weight of the body, which is proportional to d^3, must vary as v^6; which proves the proposition enunciated above. Hence an increase in velocity causes far greater increase in transporting capacity.

Since the weight of sand and stones when immersed in water is only about one-half their weight in air, the frictional resistances to their motion are slight, and this helps to explain the circumstance that they are so easily transported by currents of moderate velocity. It is found by observation that a pebble about one inch in diameter is rolled along the bed of a channel when the velocity is about $3\frac{1}{2}$ feet per second; hence, according to the above theoretical deduction, a velocity five times as great, or $17\frac{1}{2}$ feet per second, will carry along stones of 25 inches diameter. This law of the transporting capacity of flowing water is only an approximate one, for the recorded experiments seem to indicate that the diameters of moving pebbles on the bed of a channel do not vary quite as rapidly as the square of the velocity. The law, moreover, is applicable only to bodies of similar shape, and cannot be used for comparing round pebbles with flat spalls. The following table gives the velocities on the bed or bottom of the channel which are required to move the materials stated. The corresponding mean velocities in the last column are derived from the empirical formula deduced by Darcy,

$$v = v' + 11\sqrt{rs}$$

in which v' is the bottom and v the mean velocity. The bottom or transporting velocities were deduced by Dubuat from experiments in small troughs, and hence are probably slightly less than the velocities which would move the same materials in channels of natural earth.

	Bottom velocity	Mean velocity
Clay, fit for pottery,	0.3	0.4
Sand, size of anise-seed,	0.4	0.5
Gravel, size of peas,	0.6	0.8
Gravel, size of beans,	1.2	1.6
Shingle, about 1 inch in diameter,	2.5	3.5
Angular stones, about $1\frac{1}{2}$ inches,	3.5	4.5

The general conclusion to be derived from these figures is that ordinary small, loose earthy materials will be transported or rolled along the bed of a channel by velocities of 2 or 3 feet per second. It is not necessarily to be inferred that this movement of the materials is of an injurious nature in streams with a fixed regimen, but in artificial canals the subject is one that demands close attention. The velocity of the moving objects after starting has been found to be usually less than half that of the current.*

In a silt-bearing stream there is a certain critical velocity V_0 at which all silt already in suspension is carried on without being deposited and at which no further silt is scoured from the sides and bottom. This velocity, according to the investigation of Kennedy,† is given by $V_0 = md^{0.64}$ where d is the depth of the stream and m is 0.82 for light sandy silt, 0.99 for sandy loam, and 1.07 for coarse silt. Kennedy also found that the amounts of silt carried in the same stream varied with the square root of the fifth power of the velocities, so that if x and x_0 are amounts carried at velocities V and V_0 then $x = x_0\,(V/V_0)^{\frac{5}{2}}$. When V is greater than V_0, then $x - x_0$ is the amount of scour due to the change of velocity; when V is less than V_0, then $x_0 - x$ is the amount of deposit due to the change of velocity.

Prob. 135. In the early history of the earth the moon was half its present distance from the earth's center, and the tides were about eight times

* Herschel, on the erosive and abrading power of water, in Journal Franklin Institute, 1878, vol. 75, p. 330.

† Proceedings, Institution of Civil Engineers, vol. 119, 1894–95, p. 281.