CHAPTER 10

THE FLOW OF RIVERS

Art. 128. General Considerations

Steady flow in a river channel occurs when the same quantity of water passes each section in each unit of time; here the mean velocities in different sections vary inversely as the areas of those sections. Uniform flow is that particular case of steady flow where the sections considered are equal in area. Uniform flow and some other cases of steady flow will be mainly considered in this chapter. Non-steady flow occurs when the stage of a river is rising or falling, and Art. 134 treats of this case.

No branch of hydraulics has received more detailed investigation than that of the flow in river channels, and yet the subject is but imperfectly understood. The great object of all these investigations has been to devise a simple method of determining the mean velocity and discharge without the necessity of expensive field operations. In general it may be said that this end has not yet been attained, even for the case of uniform flow. Of the various formulas proposed to represent the relation of mean velocity to the hydraulic radius and the slope, none has proved to be of general practical value except the empirical one of Chezy given in the last chapter, and this is often inapplicable on account of the difficulty of measuring the slope s and determining the coefficient c. The fundamental equations for discussing the laws of variation in the mean velocity v and in the discharge q are

$$v = c\sqrt{rs} \qquad q = a \cdot c\sqrt{rs}$$

where a is the area of the cross-section and r its hydraulic radius, and all the general principles of the last chapter are to be taken as directly applicable to uniform flow in natural channels.

Kutter's formula for the value of c is probably the best in the present state of science, although it is now generally recognized that it gives too large values for small slopes. In using it the coefficients for rivers in good condition may be taken from Table 120, but for bad regimen n is to be taken at 0.03, and for wild torrents at 0.04 or 0.05. It is, however, too much to expect that a single formula should accurately express the mean velocity in small brooks and large rivers, and the general opinion now is that efforts to establish such an expression will not prove successful. In the present state of the science no engineer can afford in any case of importance to rely upon a formula to furnish anything more than a rough approximation to the discharge in a given river channel, but actual field measurements of its velocity must be made.

When these formulas are used to determine the discharge of a river, a long straight portion or reach should be selected where the cross-sections are as nearly as possible uniform in shape and size. The width of the stream is then divided into a number of parts and soundings taken at each point of division. The data are thus obtained for computing the area a and the wetted perimeter p, from which the hydraulic depth r is derived. To determine the slope s a length l is to be measured, at each end of which bench-marks are established whose difference of elevation is found by precise levels. The elevations of the water surfaces below these benches are then to be simultaneously taken, whence the fall h in the distance l becomes known. As this fall is often small, it is very important that every precaution be taken to avoid error in the measurements, and that a number of them be taken in order to secure a precise mean. Care should be observed that the stage of water is not varying while these observations are being made, and for this and other purposes a permanent gage board must be established. It is also very important that the points upon the water surface which are selected for comparison should be situated so as to be free from local influences such as eddies, since these often cause marked deviations from the normal surface of the stream. If hook gages can be used for referring the water levels to the benches, probably the most accurate results can be obtained. It has been observed that the surface of a swiftly flowing stream is not a plane, but a cylinder, which is concave to the bed, its highest elevation being where the velocity is greatest, and hence the two points of reference should be located similarly with respect to the axis of the current. In spite of all precautions, however, the relative error in h will usually be large in the case of slight slopes, unless l be very long, which cannot often occur in streams under conditions of uniformity.

Owing to the uncertainty of determinations of discharge made in the manner just described, the common practice is to gage the stream by velocity observations, to which subject, therefore, a large part of this chapter will be devoted. The methods given are equally applicable to conduits and canals, and in Art. 133 will be found a summary which briefly compares the various processes.

Prob. 128. Which has the greater discharge, a stream 2 feet deep and 85 feet wide on a slope of 1 foot per mile, or a stream 3 feet deep and 40 feet wide on a slope of 2 feet per mile?

Art. 129. Velocities in a Cross-section

The mean velocity v is the average of all the velocities of all the small sections or filaments in a cross-section (Art. 112). Some of these individual velocities are much smaller, and others materially larger, than the mean velocity. Along the bottom of the stream, where the frictional resistances are the greatest, the velocities are the least; along the center of the stream they are the greatest. A brief statement of the general laws of variation of these velocities will now be made.

In Fig. 129 there is shown at A a cross-section of a stream with contour curves of equal velocity; here the greatest velocity is seen to be near the deepest part of the section a short distance below the surface. At B is shown a plan of the stream with arrows roughly representing the surface velocities; the greatest of these is seen to be near the deepest part of the channel, while the others diminish toward the banks, the curve showing the law of variation resembling a parabola. At C is shown by arrows the variation of velocities in a vertical line, the smallest being

at the bottom and the largest a short distance below the surface; concerning this curve there has been much contention, but it is commonly thought to be a parabola whose axis is horizontal. These are the general laws of the variation of velocity throughout the cross-section; the particular relations are of a complex character, and vary so greatly in channels of different kinds that it is difficult to formulate them, although many attempts to do so have been made. Some of these formulas which connect the mean velocity with particular velocities, such as the maximum surface velocity, mid-depth velocity in the axis of the stream, etc., will be given in Art. 132.

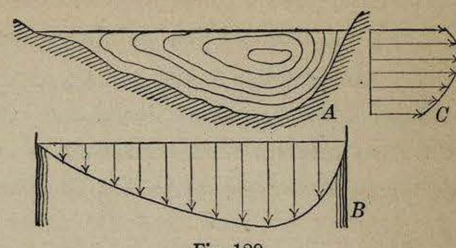

Fig. 129.

Humphreys and Abbot deduced in 1861 for the Mississippi River * an equation of the mean curve of mean velocities in a vertical line, namely,

$$V = 3.261 - 0.7922(y/d)^2$$

in which V is the velocity at any distance y above or below the horizontal axis of the parabolic curve and d is the depth of the water, the axis being at the distance $0.297d$ below the surface. The depth of the axis was found, however, to vary greatly with the wind, an up-stream wind of force 4 depressing it to mid-depth, and a down-stream wind of force 5.3 elevating it to the surface.

In a straight channel having a bed of a uniform nature the deepest part is near the middle of its width, while the two sides are approximately symmetrical. In a river bend, however, the deepest part is near the outer bank, while on the inner side the water is shallow; the cause of this is undoubtedly due to the centrifugal force of the current, which, resisting the change in direction, creates currents which scour away the outer bank or prevent deposits from forming there. It is well known to all

* Physics and Hydraulics of the Mississippi River, edition of 1876, p. 243.

that rivers of the least slope have the most bends; perhaps this is due to the greater relative influence of such cross currents. (See Art. 156.)

The theory of the flow of water in channels, like that of flow in pipes, is based upon the supposition of a mean velocity which is the average of all the parallel individual velocities in the cross-section. But in fact there are numerous sinuous motions of particles from the bottom to the surface which also consume a portion of the lost head. The influence of these sinuosities is as yet but little understood; when in the future this becomes known, a better theory of flow in channels may be possible.

Prob. 129. Show that the above formula for velocities in a vertical can be put into the form

$$V = 3.19 + 0.471\,(x/d) - 0.792\,(x/d)^2$$

in which x is the depth below the surface.

Art. 130. Velocity Measurements

One of the methods for measuring the discharge of streams which has been extensively used is by observing the velocity of flow by the help of floats. Of these there are three kinds, surface floats, double floats, and rod floats. Surface floats should be sufficiently submerged so as to thoroughly partake of the motion of the upper filaments, and should be made of such a form as not to readily be affected by the wind. The time of their passage over a given distance is determined by two observers at the ends of a base on shore by stop-watches; or only one watch may be used, the instant of passing each section being signaled to the time-keeper. If l be the length of the base, and t the time of passage in seconds, the velocity of the float is $v = l/t$. When there are many observations, the numerical work of division is best done by taking the reciprocals of t from a table and multiplying them by l, which for convenience may be an even number, such as 100 or 200 feet.

A sub-surface float consists of a small surface float connected by a fine cord or wire with the large real float, which is weighted so as to remain submerged and keep the cord reasonably taut. The surface float should be made of such a form as to offer but slight resistance to the motion, while the lower float is large, it being the object of the combination to determine the velocity of the lower one alone. This arrangement has been extensively used, but it is probable that in all cases the velocity of the large float is somewhat affected by that of the upper one, as well as by the friction of the cord. In general the use of these floats is not to be encouraged, if any other method of measurement can be devised.

The rod float is a hollow cylinder of tin, which can be weighted by dropping in pebbles or shot so as to stand vertically at any depth. When used for velocity determinations, they are weighted so as to reach nearly to the bottom of the channel, and the time of passage over a known distance determined as above explained. It is often stated that the velocity of a rod float is the mean velocity of all the filaments in contact with it. Theoretically this is not the case, but the rod moves a little slower. However, in practice a rod cannot reach quite to the bed of the stream, and Francis has deduced the following empirical formula for finding the mean velocity V_m of all the filaments between the surface and the bed from the observed velocity V_r of the rod:

$$V_m = V_r(1.012 - 0.116\sqrt{d'/d})$$

in which d is the total depth of the stream and d' the depth of water below the bottom of the rod.* This expression is probably not a valid one, unless d' is less than about one-quarter of d; usually it will be best to have d' as small as the character of the bed of the channel will allow.

The log formerly used by seamen for ascertaining the speed of vessels may be often conveniently used as a surface float when rough determinations only are required, it being thrown from a boat or bridge. The cord of course must be previously stretched when wet, so that its length may not be altered by the immersion; if graduated by tags or knots in divisions of six feet, the log may be allowed to float for one minute, and then the number of divi-

* Lowell Hydraulic Experiments, 4th Edition, p. 195.

sions run out in this time will be ten times the velocity in feet per second.

The determination of particular velocities in streams by means of floats appears to be simple, but in practice many uncertainties are found to arise, owing to wind, eddies, local currents, etc., so that a number of observations are required to obtain a precise mean result.

For conduits, canals, and for many rivers the use of a current meter will often be found to be more satisfactory and less expensive if many observations are required. Comparisons between the results of float and rod gagings have been made by Murphy.* These comparisons include those made at the Cornell University laboratory between the weir and the current meter in 1900.

Other current indicators less satisfactory for work in streams are the Pitot tube and the hydrometric pendulum, shown in Fig. 130*a*. The former has not been found valuable for river measurements, although it has proved to be an instrument of great precision for other classes of work (Art. 41), and the latter, although used by some of the early hydraulicians, has long been discarded as giving only rough indications. The same may be said of the hydrometric balance, in which weights measure the intensity of the pressure of the current, and of the torsion balance, in which the pressure of the current on a submerged plate causes the tightening of a spring. These instruments were used only for measurements of velocities in small channels, and they are now mere curiosities.

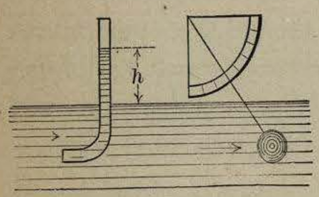

Fig. 130*a*.

The current meter, described in Art. 40, is generally operated from a bridge or cable in the case of a small stream, but it must be often operated from an anchored boat in large rivers. In the latter case precise measurements of surface velocities may be difficult on account of the eddies around the boat. Even when operated from a bridge, it is not easy to obtain successful results when the velocity exceeds 4 or 5 feet per second, and special

* Water Supply and Irrigation Paper No. 95, Washington, 1904.

expedients are necessary to keep the meter in position. However, the current meter, accurately rated, will in general do better work than can be done by floats.

In using the current meter for the determination of velocity four principal methods are used on the work of the U. S. Geological Survey; these have been reviewed by Hoyt.* In the first a vertical velocity curve is determined by placing the meter at regular vertical intervals from the surface of the water to the bottom of the stream and observing the velocity at each such interval. The points so selected are usually from 10 to 20 percent of the water depth apart. On plotting the velocities obtained, a curve results which graphically indicates the variations in the velocity as they are dependent on the depth. The average velocity in the vertical can be determined by averaging all of the observations, or more accurately by ascertaining the area fixed by the curve and the axis of ordinates and then dividing this area by the depth of the water in the vertical. Thus in Fig. 130*b* the mean velocity is the area ABC divided by the depth 9.5 feet.

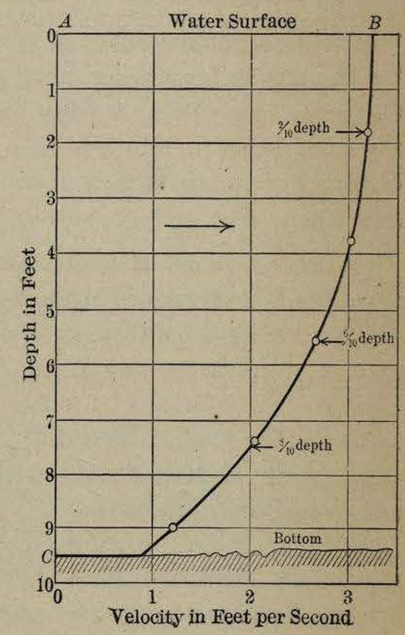

Fig. 130*b*.

In the second of these four methods the velocities at distances below the surface of 0.2 and 0.8 of the depth are determined and the mean taken as the average velocity in the vertical. Many observations have proven that this method is correct, and theoretically it is based on the mathematical fact that if the velocity curve be a parabola, then the mean ordinate will be the average of these at points whose abscissas are 0.2114 and 0.7886.

The third of these methods consists in observing the velocity

* Transactions American Society of Civil Engineers, 1910, vol. 66.

at a distance below the surface equal to 0.6 of the water depth. This procedure is also based on the assumption that the velocity curve is a parabola whose axis is parallel to the water surface and lies below it from 0 to 0.3 of the water depth. Mathematically, therefore, the mean ordinate which represents the mean velocity lies between the points whose abscissas are 0.58 and 0.67 of the water depth.

In the fourth method the mean velocity is determined by observing the velocity at a point from 0.5 to 1.0 feet below the water surface and applying a coefficient determined by observation. This coefficient ranges from 0.78 to 0.98, and Hoyt* recommends the following. For average streams in moderate freshets 0.90; during floods from 0.90 to 0.95, and for ordinary stages of flow from 0.85 to 0.90.

In the following tabulation are shown the results obtained in 476 vertical velocity curves* on 34 rivers in various parts of the United States. The depths of these streams ranged from 1.6 to 27.5 feet and the observed velocities from 0.25 to 9.59 feet per second. The figures given are the coefficients by which the average velocities determined by the various methods should be multiplied in order to obtain the mean velocity as determined from the vertical velocity curve in the first method above described.

Method	Coefficient		
	Maximum	Minimum	Mean
2	1.03	0.95	0.99
3	0.98	0.79	0.87
4	1.03	0.97	1.00

In the 476 velocity curves above referred to it was found that the point of mean velocity occurred at from 58 to 71 percent of the water depth below the surface, and that the average of all the curves showed it to be at 0.62 of the depth.

In cases where the stream to be measured is frozen over it has been found that the best work is done by the vertical velocity curve method, though the 2 and 8 tenths depth method also gives good results. A résumé of studies of the flow under ice by Murphy † indicates that

* Transactions American Society of Civil Engineers, 1910, vol. 66.
† U. S. Water Supply and Irrigation Paper No. 95, 1904.

the maximum velocity is to be found at from 35 to 40 percent of the water depth below the under surface of the ice and that the mean velocity occurs at two points, the first from 0.08 to 0.13 and the second from 0.68 to 0.74 of the water depth below the under surface of the ice.

The so-called integration method of determining the average velocity in a vertical consists in moving the meter at a uniform rate from the surface to the bottom and back again. Each point is thus passed over twice, and if all other conditions are the same, the average velocity indicated should be the mean velocity in the vertical. This method has, since 1900, come to be practically superseded by those before described.

Prob. 130. A rod float runs a distance of 100 feet in 42 seconds, the depth of the stream being 6 feet, while the foot of the rod is 6 inches above the bottom. Compute the mean velocity in the vertical.

Art. 131. Gaging the Discharge

For a very small stream the most precise method of finding the discharge is by means of a weir constructed for that purpose. Streams of considerable size often have dams built across them, and these may also be used like weirs with the help of the coefficients given in Art. 69, if there be no leakage through the dam. When there are no dams, the method now to be explained is generally employed. In all cases the first step should be to set up a vertical board gage, graduated to feet and tenths, and locate its zero with respect to the datum plane used in the vicinity, so that the stage of water may at any time be determined by reading the gage.

The place selected for the gaging should be one where the channel is free from obstructions and as nearly as possible free from bends and curves for some distance both up and down stream. One or more sections at right angles to the direction of the current are to be established, and soundings taken at intervals across the stream upon them, the water gage being read while this is done. The distances between the places of soundings are measured either upon a cord stretched across the stream or by other methods known to surveyors. The data are thus obtained for determining the areas a_1, a_2, a_3, etc., shown upon Fig.

131a, and the sum of these is the total area a. Levels should be run out upon the bank beyond the water's edge, so that in case of a rise of the stream the additional areas can be deduced. If a current meter is used, but one section is needed; if floats are used, at least two are required, and these must be located at a place where the channel is of as uniform size as possible.

Fig. 131a.

The mean velocities v_1, v_2, v_3, etc., are next to be determined for each of the sub-areas. With a current meter this may be done by starting at one side of a subdivision, and lowering it at a uniform rate until the bottom is nearly reached, then moving it a few feet horizontally and raising it to the surface, then moving it a few feet horizontally and lowering it, and thus continuing until the sub-area has been covered. The velocity then deduced from the whole number of revolutions during the time of immersion is the mean velocity for the sub-area. Or, by using any one of the methods for determining the mean velocity in the vertical as described in Art. 130 the mean velocity may be determined. When rod floats are used, they are started above the upper section, and the times of passing to the lower one noted, as explained in Art. 130, the velocity deduced from a float at the middle of a sub-area being taken as the mean for that area. It will be found that the rod floats are more or less affected by wind, the direction and intensity of which should always be recorded in the field notes.

The discharge of the stream is the sum of the discharges through the several sub-areas, or

$$q = a_1v_1 + a_2v_2 + a_3v_3 + \text{ etc.}$$

and if this be divided by the total area a, the mean velocity for the entire section is determined.

If d_1, d_2, d_3, etc., are the depths in feet on the several verticals in Fig. 131a, and if v_1, v_2, v_3, etc., represent the mean velocities in feet per second in these verticals, while i is the constant interval in feet between them, then the discharge in cubic feet per second will be given by the formula

$$Q = \frac{i}{6}[d_1v_1 + (d_1 + d_2)(v_1 + v_2) + d_2v_2] + \text{ etc.}$$

For most cases, however, sufficient accuracy will be given by the expression

$$Q = i\left[\left(\frac{d_1 + d_2}{2}\right)\left(\frac{v_1 + v_2}{2}\right)\right] + \text{ etc.}$$

and this is the method which has been adopted by the U. S. Geological Survey. It permits of ready computation, while at the same time it does not require absolute uniformity in the interval i. Stevens * has compared the various methods and formulas which have been used for the computation of the discharge in such cases.

The following notes give the details of a gaging of the Lehigh River, near Bethlehem, Pa., made at low water in 1885 by the use of rod floats. The two sections were 100 feet apart, and each was divided into 10 divisions of 30 feet width. In the second column are given the soundings in feet taken at the upper section, in the third the mean of the two areas in square feet, in the fourth the times of passage of the floats in seconds, in the fifth the velocities in feet per second, which were obtained by dividing 100 feet by the times, and in the last are the products a_1v_1, a_2v_2, which are the discharges for the subdivisions a_1, a_2, etc. The total discharge is found to be 826 cubic feet per second,

Subdivisions	Depths	Areas	Times	Velocities	Discharges
	0.0				
1		55.5	380	0.263	14.6
	3.0				
2		148.5	220	0.454	67.4
	6.0				
3		201.7	185	0.540	108.9
	7.1				
4		217.5	120	0.833	181.2
	7.0				
5		210.0	145	0.690	144.9
	7.0				
6		186.0	150	0.667	124.1
	5.3				
7		150.8	165	0.606	91.4
	4.3				
8		114.0	200	0.500	57.0
	3.0				
9		84.0	320	0.313	26.3
	2.2				
10		42.0	430	0.233	9.8
	0.0				
		$a = 1410.0$			$q = 825.6$

* Engineering News, June 25, 1908.