comparison because it is probable that no channel in neat cement has ever been constructed having a hydraulic radius as great as 7 feet, but it serves to show that these empirical formulas differ widely when applied to unusual cases. For the present, at least, the formula of Kutter appears to receive the most general acceptance, but undoubtedly the time will come when it will be replaced by a more satisfactory one. An actual gaging of the discharge by the method of Art. 131 will always give more reliable information than can be obtained from any formula.

For a hydraulic radius of 3.28 feet Kutter's formula for c reduces to the convenient expression

$$c = 1.811/n \quad \text{whence} \quad v = \frac{1.811}{n}\sqrt{rs}$$

and this may be used for approximate computations when r lies between 2 and 6 feet. Here n is the roughness factor, the values of which are given in Art. 118. When $r = 3.28$ feet, Bazin's formula gives $c = 136$ for brickwork, while Kutter's gives $c = 140$; for canals in good order Bazin's formula gives $c = 69$, while Kutter's gives $c = 72$. The comparison is very satisfactory, and so close an agreement is not generally to be expected when computations are made from different formulas. The formula of Bazin is largely used in France and England, and that of Kutter in other countries.

Prob. 122. Solve Problem 118 by the use of Bazin's coefficients.

Art. 123. Masonry Conduits

Masonry conduits or aqueducts for conveying water have been used since the days of ancient Rome. In cases where large quantities of water are to be carried on small slopes and where the topography of the country is at a suitable elevation they offer the most economical means for its conveyance. The Sudbury and Wachusett aqueducts for the supply of Boston, the Jersey City aqueduct for the supply of that city, the old Croton and the New Croton aqueducts for the supply of New York City are among the largest and longest which have yet been constructed.

In 1912 there are being built the Catskill aqueduct also for New York City and the Los Angeles aqueduct for the city of Los Angeles in California. Large portions of these aqueducts are in tunnels on the hydraulic gradient, and in the case of the Catskill aqueduct of a total of 110 miles of main conduit nearly 30 percent is in rock tunnel from 300 to 1100 feet below the surface. These tunnels are circular in cross-section, and their diameters range from 11 to 15 feet.

Relatively few experiments for determining the coefficients of flow have been made on these aqueducts. From their gagings of the Sudbury aqueduct, Fteley and Stearns * determined a formula for mean velocity. The cross-section of this aqueduct, which is laid on a slope of 0.0002, consists of a part of a circle 9.0 feet in diameter, having an invert of 13.22 feet radius, whose span is 8.3 feet and depression 0.7 feet, the axial depth of the conduit being 7.7 feet. It is lined with brick, having cement joints $\frac{1}{4}$ of an inch thick. The flow was allowed to occur with different depths, for each of which the discharge was determined by weir measurement. A discussion of the results led to the conclusion that in the portion with the brick lining the coefficient c had the value $127r^{0.12}$ when r is in feet, and hence results the exponential formula

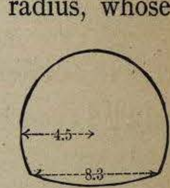

Fig. 123a.

$$v = 127\, r^{0.12}\sqrt{rs} = 127\, r^{0.62}s^{0.50}$$

In a portion of this conduit where the brick lining was coated with pure cement, the coefficient was found to be from 7 to 8 percent greater than $127r^{0.12}$. In another portion where the brick lining was covered with a cement wash laid on with a brush, the coefficient was from 1 to 3 percent greater. For a long tunnel in which the rock sides were ragged, but with a smooth cement invert it was found to be about 40 percent less.

Gagings on the New Croton Aqueduct † showed that the mean velocity when the aqueduct was new could be represented by the

* Transactions American Society of Civil Engineers, 1883, vol. 13, p. 114.
† Engineering Record, 1895, vol. 32, p. 223.

expression $v = 124r^{0.56}\sqrt{s}$. This aqueduct is constructed of brick laid in close mortar joints. Its cross-section is shown in Fig. 126*b*. It is 13.53 feet in height by 13.6 feet in maximum width. The radius of its invert is 18.5 feet, the span of the invert chord is 12.0 feet, and the depression of the invert below the chord is 1.0 foot. Its slope is 0.0003.

Gagings on various portions of the aqueduct of the Jersey City Water Supply Company,* a cross-section of which is shown in Fig. 123*b*, gave, when the aqueduct was new, values of the coefficient c in the Chezy formula of from 122 to 145, while the average value of n in Kutter's formula was 0.0127. The value of the mean velocity in this conduit is closely given by the expression $v = 131r^{0.50}s^{0.50}$, where s is the observed slope of the water surface. This slope during the experiments varied from 0.00011 to 0.00036, the aqueduct being laid on a slope of 0.000095. This conduit is of concrete which was cast against smooth wooden forms, the invert being made of screeded and troweled concrete.

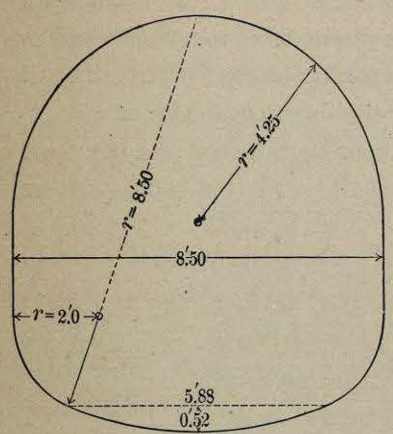

Fig. 123*b*.

Owing to the fouling of such conduits as the result of vegetable growths and the deposition of materials from the water, a diminution in capacity of from 10 to 20 percent with age may be expected, and accordingly corresponding allowances should be made in the design.

It is to be noted that Kutter's formula (Art. 118) indicates that c steadily increases with the hydraulic radius if n and the slope be constant. The results of the experiments above quoted, however, indicate that c becomes constant and has a maximum value

* By courtesy of Jersey City Water Supply Company, Paterson, N. J.

of not far from 140 for values of the hydraulic radius of 3 feet and upward.

In an aqueduct of masonry constructed so that the water will flow in it with a free surface it will be found that the slope of the water surface is seldom if ever parallel to the bottom of the aqueduct. This, of course, is as it should be, since the expression for the slope is $s = Q^2/a^2c^2r$. Here both a and r vary with Q, and it seldom happens that the value of c realized in the completed structure is the same as that assumed in the original design. Since the slope of the water surface is not parallel to that of the bottom of the aqueduct, there results a condition of steady non-uniform flow, and the formula of Art. (137) must be employed whenever precise determinations of the value of c are to be made from the results of experiments.

Prob. 123. Compute the mean velocity in the New Croton Aqueduct when it is flowing one-half full.

Art. 124. Other Formulas for Channels

Many attempts have been made to express the mean velocity and discharge in a channel by the formulas

$$v = Cr^x s^y \qquad q = aCr^x s^y$$

where x and y are derived from the data of observations by processes similar to those explained in Art. 42. As a rule these attempts have not proved successful except for special classes of conduits, as the exponents of r and s vary with different values of r and with different degrees of roughness. For conduits having the same kind of surface a formula of this kind may be established which will give good results. The values $x = \frac{2}{3}$ and $x = \frac{3}{4}$ are frequently advocated, y being not far from $\frac{1}{2}$; with such values C is found to vary less for certain classes of surfaces than the c of the Chezy formula, and this seems to be the only strong argument in favor of exponential formulas.

Among the many exponential formulas which have been advocated, those derived by Foss may be cited. For surfaces corresponding to Kutter's values of n less than 0.017 he finds*

$$r^{\frac{11}{6}} = Cr^{\frac{4}{3}}s \qquad \text{or} \qquad v = C^{\frac{6}{11}}r^{\frac{8}{11}}s^{\frac{6}{11}}$$

* Journal of Association of Engineering Societies, 1894, vol. 13, p. 295.

in which C has the following values:

for n =	0.009	0.010	0.011	0.012	0.013	0.015	0.017
C =	23 000	19 000	15 000	12 000	10 000	8 000	6 000

For surfaces corresponding to Kutter's values of n greater than 0.018, his formula is

$$v^2 = Cr^{\frac{4}{3}} s \qquad \text{or} \qquad v = C^{\frac{1}{2}} r^{\frac{2}{3}} s^{\frac{1}{2}}$$

and the values of C for this case are

for n =	0.020	0.025	0.030	0.035
C =	5000	3000	2000	1000

For circular sections running full he also proposes the formula $s = 0.0065 q^{\frac{11}{6}}/d^5$. These formulas are open to objection on account of the great range in the values of C.

Tutton*, as the result of a study of many experiments, proposed the formula $v = Cr^{(1.17-m)} s^m$, where s and r represent the slope and hydraulic radius as in the Chezy formula. The values of m ranged from 0.48 for tarred iron pipes to 0.58 for pipes of lead, tin, and zinc, the average for all cases being $m = 0.54$. Using this value, the formula became

$$v = Cr^{0.63} s^{0.54}$$

for which the value of C was given as from 127 to 153 for new cast-iron pipes, from 83 to 98 for lap-riveted iron pipes, from 127 to 153 for wooden pipes, and about 188 for lead, tin, and zinc pipes.

Williams and Hazen † have discussed experiments on both pipes and open channels, and have proposed an exponential formula that is equivalent to

$$v = 1.318\, cr^{0.63} s^{0.54}$$

in which c has different values for different surfaces and sections, but its range of values is less than that of the c of the Chezy formula. The values of c and c are the same when r is 1 foot and s is 0.001. The greater the roughness of the surface, the smaller is c; in general, c is supposed to vary but little for different values of r. The following shows the range of the mean values of c found from the records of experiments with different surfaces:

* Transactions Engineers' Society of Western New York, April, 1896.

† Hydraulic Tables, New York, 1910.

For coated new cast-iron pipes,	from 111 to 146
For tuberculated cast-iron pipes,	from 16 to 112
For riveted pipes,	from 97 to 142
For wooden stave pipes,	from 113 to 129
For new wrought-iron pipes,	from 113 to 124
For fire hose, rubber lined,	from 116 to 140
For masonry aqueducts,	from 118 to 145
For brick sewers,	from 102 to 141
For plank aqueducts, unplaned,	from 113 to 120
For masonry sluiceways,	from 34 to 75
For canals in earth,	from 33 to 71

The authors of this formula suggest that in computations for pipe capacity c be taken as 100 for cast-iron, 95 for riveted steel, 120 for wooden, 110 for vitrified pipes, 100 for brick sewers, and 120 for first-class masonry conduits.

The circumstance that values of C in some of the exponential formulas of this article have a smaller range of values than the c of the Chezy formula is sometimes cited as an argument in their favor. While this is a good argument, the fact must not be overlooked that probably the true theoretic formula for mean velocity in a pipe or channel is of the form noted in the first paragraph of Art. 110.

In conclusion, it may be noted that when the velocity is very low, the Chezy formula is not valid. In such a case the velocity does not vary with the square root of the slope, but with its first power, the same conditions obtaining as in pipes (Art. 110). A glacier moving in its bed at the rate of a few feet per year has a velocity directly proportional to its slope. Water flowing in a channel with a velocity less than one-quarter of a foot per second follows the same law, and the formulas of this chapter cannot be applied. The formula for this case is $v = Cr^2 s$, but values of C are not known. It is greatly to be desired that series of experiments should be made for determining values of C.

Prob. 124. Compute the fall of the water surface in a length of 1000 feet for a ditch where $v = 3.62$ feet per second, $r = 2.75$ feet, and $n = 0.025$; first by Williams and Hazen's formula, and second, by formula (122) and Bazin's coefficients.

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Art. 125. Losses of Head

The only loss of head thus far considered is that due to friction, but other sources of loss may often exist. As in the flow in pipes, these may be classified as losses at entrance, losses due to curvature, and losses caused by obstructions in the channel or by changes in the area of cross-section.

When water is admitted to a channel from a reservoir or pond through a rectangular sluice, there occurs a contraction similar to that at the entrance into a pipe, and which may be often observed in a slight depression of the surface, as at D in Fig. 125*a*. At this point, therefore, the velocity is greater than the mean velocity v, and a loss of energy or head results from the subsequent expansion, which is approximately measured by the difference of the depths d_1 and d_2, the former being taken at the entrance of the channel, and the latter below the depression where the uniform flow is fully established. According to the experiments of Dubuat, made late in the eighteenth century, the loss of head for this case is

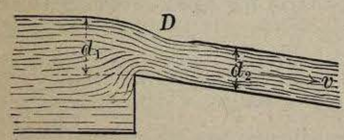

Fig. 125*a*.

$$d_1 - d_2 = m\frac{v^2}{2g}$$

in which m ranges between 0 and 2 according to the condition of the entrance. If the channel be small compared with the reservoir, and both the bottom and side edges of the entrance be square, m may be nearly 2; but if these edges be rounded, m may be very small, particularly if the bottom contraction is suppressed. The remarks in Chap. 5 regarding suppression of the contraction apply also here, and it is often important to prevent losses due to contraction by rounding the approaches to the entrance. Screens are sometimes placed at the entrance to a channel in order to keep out floating matter; if the cross-section of the channel is n times that of the meshes of the screen, the loss of head, according to $(76)_2$, is $(n-1)^2v^2/2g$.

The loss of head due to bends or curves in the channel is small if the curvature be slight. Undoubtedly every curve offers a resistance to the change in direction of the velocity, and thus requires an additional head to cause the flow beyond that needed to overcome the frictional resistances. Several formulas have been proposed to express this loss, but they all seem unsatisfactory, and hence will not be presented here, particularly as the data for determining their constants are very scant. It will be plain that the loss of head due to a curve increases with its length, as in pipes (Art. 91). When a channel turns with a right angle, as in Fig. 125*b*, the loss of head may be taken as equal to the velocity-head, since the experiments of Weisbach on such bends in pipes indicate that value. In this case there is a contraction of the stream after passing the corner, and the subsequent expansion of section and the resulting impact causes the loss of head.

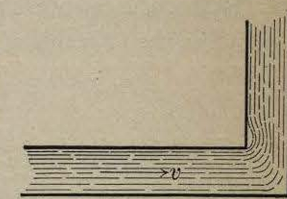

Fig. 125*b*.

The losses of head caused by sudden enlargement or by sudden contraction of the cross-section of a channel may be estimated by the rules deduced in Arts. 76 and 77. In order to avoid these losses changes of section should be made gradually, so that energy may not be lost in impact. Obstructions or submerged dams may be regarded as causing sudden changes of section, and the accompanying losses of head are governed by similar laws. The numerical estimation of these losses will generally be difficult, but the principles which control them will often prove useful in arranging the design of a channel so that the maximum work of the water can be rendered available. But as all losses of head are directly proportional to the velocity-head $v^2/2g$, it is plain that they can be rendered inappreciable by giving to the channel such dimensions as will render the mean velocity very small. This may sometimes be important in a short conduit or flume which conveys water from a pond or reservoir to a hydraulic motor, particularly in cases where the supply is scant, and where all the available head is required to be utilized.

If no losses of head exist except that due to friction, this can be computed from (113) if the velocity v and the coefficient c be known. For since the value of s is v^2/c^2r and also h/l, where h is the fall expended in overcoming friction, h may be found from

$$h = ls = lv^2/c^2r \qquad (125)$$

but this computation will usually be liable to much error.

As an example of the computations which sometimes occur in practice the following actual case will be discussed. From a canal

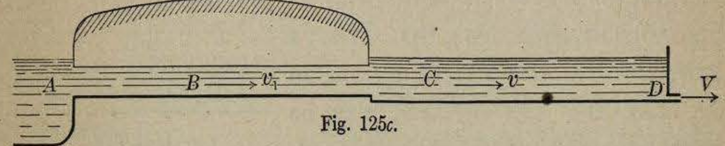

Fig. 125c.

A water is carried through a cast-iron pipe B to an open wooden forebay C, where it passes through the orifice D and falls upon an overshot wheel. At the mouth of the pipe is a screen, the area between the meshes being one-half that of the cross section of the pipe. The pipe is 3 feet in diameter and 32 feet long. The forebay is of unplaned timber, 5 feet wide and 38 feet long, and it has three right-angled bends. The orifice is 5 inches deep and 40 inches wide, with standard sharp edges on top and sides and contraction suppressed on lower side so that its coefficient of contraction is about 0.68 and its coefficient of velocity about 0.98. The water level in the canal being 3.75 feet above the bottom of the orifice, it is required to find the loss of head between the points A and D.

The total head on the center of the orifice is $3.75 - 0.208 = 3.542$ feet. Let v_1 be the mean velocity in the pipe, v that in the forebay, and V that in the contracted section beyond the orifice. The area of the cross-section of the pipe is 7.07 square feet; that of the forebay, taking the depth of water as 3.7 feet, is 18.5 square feet, and that of the contracted section of the jet issuing from the orifice is 0.945 square feet. It will be convenient to express all losses of head in terms of the velocity-head $v^2/2g$, and hence the first operation is to express v_1 and V in terms of v, or $v_1 = 2.62v$ and $V = 19.6v$. Starting with the screen, the loss of head due to expansion of section after the water passes through it is, by Art. 76,

$$h' = \frac{(2v_1 - v_1)^2}{2g} = 6.9\frac{v^2}{2g}$$

The loss of head in friction in the pipe, using 0.02 for the friction factor, is, by Art. 90,

$$h' = f\frac{l}{d}\frac{v_1^2}{2g} = 1.4\frac{v^2}{2g}$$

The loss of head in the expansion of section from the pipe to the forebay is, by Art. 76,

$$h' = \frac{(v_1 - v)^2}{2g} = 2.6\frac{v^2}{2g}$$

The loss of head in friction in the forebay, taking c from Table 122 for the hydraulic radius 1.5 feet and degree of roughness $m = 0.16$, is then found to be

$$h' = \frac{lv^2}{c^2r} = 0.1\frac{v^2}{2g}$$

The loss of head in the three right-angled bends of the forebay is estimated, as above noted, by

$$h' = 3.0\frac{v^2}{2g}$$

The loss of head on the edges of the orifice is, by Art. 56,

$$h' = 0.041\frac{V^2}{2g} = 15.9\frac{v^2}{2g}$$

Now the total head is expended in these lost heads and in the velocity-head of the jet issuing from the orifice, or

$$3.542 = 29.9\frac{v^2}{2g} + \frac{V^2}{2g} = 417\frac{v^2}{2g}$$

from which the value of $v^2/2g$ is found to be 0.00851 feet. Finally the total loss of head or fall in the free surface of the water before reaching the orifice is

$$(29.9 - 15.9)\frac{v^2}{2g} = 14.0 \times 0.00851 = 0.119 \text{ feet},$$

and therefore the water surface at D is 0.119 feet lower than that at A, and the pressure-head on the center of the orifice is 3.433 feet. This is the result of the computations, but on making measurements with an engineer's level the water surface at D was found to be 0.125 feet lower than that at A; the error of the computed result is therefore 0.006 feet.

Prob. 125. Compute from the above data the velocities v, v_1, and V, and the discharge through the orifice. Show that the head lost in passing through the screen was 0.059 feet, which is about one-half of the total.

Art. 126. Velocities in a Cross-section

For a circular conduit running full and under pressure the velocities in different parts of the section vary similarly to those in pipes (Art. 86). When it is partly full, so that the water flows with a free surface, the air resistance along that surface is much smaller than that along the wetted perimeter, and hence the surface velocities are greater than those near the perimeter. Fig. 126*a* illustrates the variation of velocities in a cross-section of the

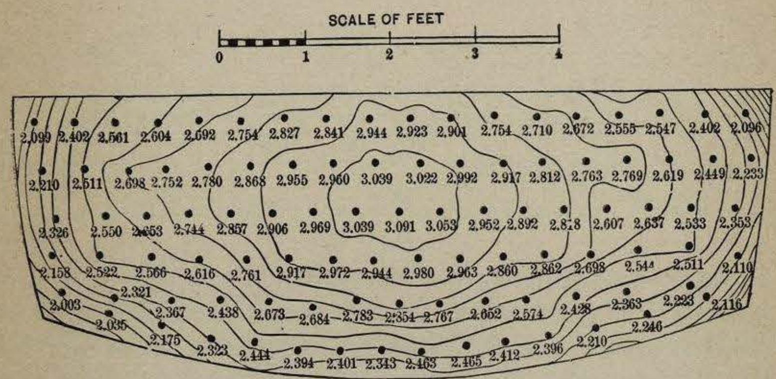

Fig. 126*a*.

Sudbury conduit when the water was about 3 feet deep, as determined by the gagings of Fteley and Stearns.* The 97 dots are the points at which the velocities were measured by a current meter (Art. 40), and the velocity for each point in feet per second is recorded below it. From these the contour curves were drawn which show clearly the manner of variation of velocity throughout this cross-section. Since the dots are distributed over the area quite uniformly, that area may be regarded as divided into 97 equal parts, in each of which the velocity is that observed, and hence the mean of the 97 observations is the mean velocity (Art. 39). Thus is found $v = 2.620$ feet per second, and this is 85 per cent of the maximum observed velocity.

Similarly Fig. 125*b* shows the results of an experiment on the New Croton Aqueduct.† In this case the average velocity de-

* Transactions American Society of Civil Engineers, 1883, vol. 12, p. 324.

† Report of The Aqueduct Commissioners, New York, 1895–1907.

termined from the 128 individual observations is 3.570, and this is 89 percent of the maximum observed velocity. A description of the methods followed in making the gagings on this aqueduct

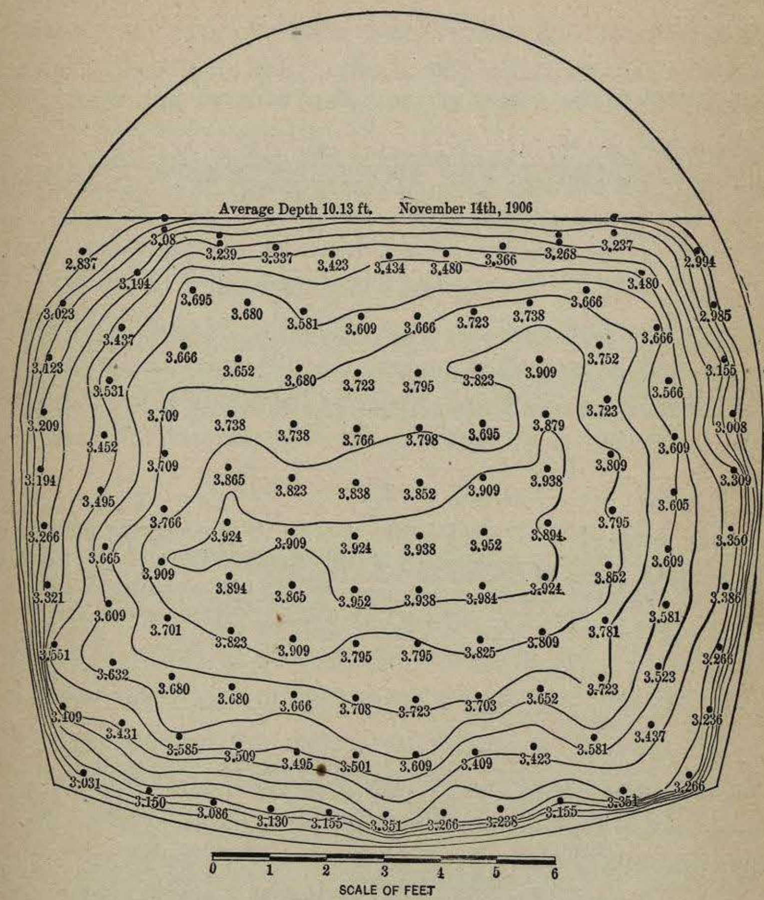

Fig. 126*b*.

is to be found at page 106 of vol. 66, Transactions American Society of Civil Engineers. See also Art. 123.

An examination of the distribution of velocities in Fig. 126*b* indicates that the maximum velocity does not occur at the center of the cross-section. This is due to the fact that the aqueduct at the point where the gaging was taken is located on a curve which tends to throw the maximum velocity away from the center and toward the outside of the curve.

If all the filaments of a stream of water in a channel have the same uniform velocity v, the kinetic energy per second of the flow is the weight of the discharge multiplied by the velocity-head; or

$$K = W\frac{v^2}{2g} = wq\frac{v^2}{2g} = wa\frac{v^3}{2g}$$

in which W is the weight of the water delivered per second, w is the weight of one cubic unit, q the discharge per second, and a the area of the cross-section. For this case, therefore, the energy of the flow is proportional to the area of the cross-section and to the cube of the velocity. Since, however, the filaments have different velocities, this expression may be applied to the actual flow by regarding v as the mean velocity. To show that this method will be essentially correct, Fig. 126a may be discussed, and for it the true energy per second of the flow is

$$K' = \frac{wa}{97}\left(\frac{v_1^3}{2g} + \frac{v_2^3}{2g} + \cdots + \frac{v_{97}^3}{2g}\right)$$

now the ratio of this true kinetic energy to the kinetic energy expressed in terms of the mean velocity is

$$\frac{K'}{K} = \frac{v_1^3 + v_2^3 + \cdots + v_{97}^3}{97v^3}$$

By cubing each individual velocity and also the mean velocity, there is found $K' = 0.9992K$, so that in this instance the two energies are practically equal, and hence it is probable that in most cases computations of energy from mean velocity give results essentially correct.

Prob. 126. Draw a vertical plane through the middle of Fig. 126b and construct a longitudinal vertical section showing the distribution of velocities. Also draw a horizontal plane through the region of maximum velocity and construct a longitudinal horizontal section. Ascertain whether the curves of velocity for these sections are best represented by parabolas or by ellipses.

Art. 127. Computations in Metric Measures

(Art. 113) The coefficient c in the Chezy formula depends upon the linear unit of measure. Let c_1 be the value when v and r are expressed in feet and c_2 the value when v and r are expressed in meters,

and let g_1 and g_2 be the corresponding values of the acceleration of gravity. Then since $c = \sqrt{8g/f}$, it is seen that

$$c_2 = c_1\sqrt{g_2/g_1} = c_1\sqrt{9.80/32.16} = 0.552\,c_1$$

Hence any value of c in the English system may be transformed into the corresponding metric value by multiplying by 0.552. The metric value of c for conduits and canals usually lies between 16 and 100.

(Art. 114) Table 127a gives values of the Chezy coefficient c for circular conduits, full or half full. In using it a tentative method must be employed, and for this purpose there may be used at first,

$$\text{mean Chezy coefficient } c = 68$$

and then, after v has been computed, a new value of c is taken from the table and a new v is found. For example, let it be required to find the velocity and discharge of a circular conduit of 1.5 meters diameter when laid on a grade of 0.8 meters in 1000 meters. First,

$$v = 68 \times \tfrac{1}{2}\sqrt{1.5 \times 0.0008} = 1.18 \text{ meters per second,}$$

and for this velocity the table gives about 77 for c. A second computation then gives $v = 1.33$ meters per second and from the table c is 78.2. With this value is found $v = 1.35$ meters per second, which may be regarded as the final result. When running full, the discharge of this conduit is $0.7854 \times 1.5^2 \times 1.35 = 2.39$ cubic meters per second.

Table 127a. Chezy Coefficients for Circular Conduits

Metric Measures

Diameter in Meters	Velocity in Meters per Second					
	0.3	0.6	0.9	1.5	3.0	4.5
0.3	53	57	60	63	67	68
0.5	57	61	64	67	71	73
0.7	61	65	68	71	76	78
0.9	64	68	70	74	79	81
1.1	66	70	72	76	81	83
1.3	68	72	74	78	83	
1.6	72	74	77	80		
2.0	74	77	79	83		
2.4	76	79	82			