and at this same age the values for its lower end were:

Velocity in feet per second, $v =$	1	2	3	4	5	6
Cylinder joint 4 ft. diam.,	$f =$ 0.046	0.034	0.032	0.031		

Similarly the $3\frac{1}{2}$-foot-diameter taper joint pipe above referred to, when 11 years old, gave the following values for the friction factor:

Velocity in feet per second, $v =$	1	2	3	4	5	6
Taper joint $3\frac{1}{2}$ ft. diam.,	$f =$ 0.050	0.036	0.034	0.032		

Experiments on the 6-foot Jersey City Water Supply Company* taper joint pipe gave the following values for the friction factor at ages of 2 months to $5\frac{1}{2}$ years:

Velocity in feet per second, $v =$	1	2	3	4
at $\frac{1}{6}$ year,	$f =$ 0.021	0.022	0.022	0.022
at $1\frac{1}{3}$ years,	$f =$ 0.029	0.026	0.026	0.025
at $2\frac{1}{3}$ years,	$f =$ 0.034	0.029	0.027	0.027
at $5\frac{1}{2}$ years,	$f =$ 0.036	0.034	0.035	

Gagings by Marx, Wing, and Hoskins† of the flow through a steel riveted pipe 6 feet in diameter with butt joints when new, and again after two years' use furnish the following values of the friction factor f:

Velocity in feet per second, $v =$	1	2	3	4	5	6
1897,	$f =$ 0.021	0.021	0.022	0.021		
1899,	$f =$ 0.038	0.027	0.025	0.024	0.023	0.023

These results indicate a marked diminution with age in carrying capacity. This reduction is in part due to the formation of blisters in the asphaltum coating, which is generally used, in part, to the formation of tubercules or rust spots and in part to vegetable growths and incrustations formed by deposits from the water.

The so-called lock-bar pipe (Fig. 107b) was first used on the Coolgardie line in Australia and since 1900 has been introduced to a considerable extent in the United States. In this style of pipe the transverse joints are made up with rivets, as in the ordinary riveted pipe, but the

* Here published by courtesy of Jersey City Water Supply Company.

† Transactions American Society of Civil Engineers, 1898, vol. 40, p. 471; and 1900, vol. 44, p. 34.

longitudinal joints are made by clamping the edges of the plates under heavy pressure into a grooved bar which thus holds them together and makes a joint of exceptional strength. No longitudinal rivets therefore interfere with the flow, and as the plates of which the pipe is made can be used with their longer edges parallel to the axis of the pipe, the number of transverse joints can be reduced from 50 to 60 per cent. The carrying capacity of this style of pipe is probably materially in excess of that of riveted pipe, but no recorded experiments are available from which values of the friction factor can be stated.

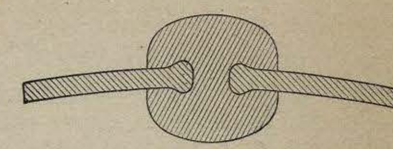

Fig. 107b.

Prob. 107. Construct curves showing the progressive increase with age in the value of the friction factor f for riveted steel pipes of 42, 48, and 60 inches in diameter.

Art. 108. Wood Pipes

Wood pipes were used in several American cities during the years 1750–1850, these being made of logs laid end to end, a 3 or 4 inch hole having been first bored through each log. Pipes formed of redwood staves were first used in California about 1880, these staves being held in place by bands of wrought-iron arranged so that they could be tightened by a nut and screw. Several long lines of these large conduit pipes have been built in the Rocky mountains and Pacific states. They have also been used there for city mains to a limited extent and recently have been introduced in the East on main distributing lines.

Gagings of a wood pipe 6 feet in diameter were made by Marx, Wing, and Hoskins, in connection with those of the steel pipe cited in Art. 107. The values of the friction factor f deduced from their results for velocities ranging from 1 to 5 feet per second are

Velocity in feet per second, $v =$	1	2	3	4	5
1897,	$f =$ 0.026	0.019	0.017	0.016	
1899,	$f =$ 0.019	0.018	0.017	0.017	0.017

These show that this wood pipe became smoother after two years' use, while the steel pipe became rougher.

T. A. Noble's gagings of wood pipes 3.67 and 4.51 feet in diameter furnish similar values of f.* For the smaller pipe f ranges from 0.021 to 0.019, with velocities ranging from 3.5 to 4.8 feet per second. For the larger pipe f ranges from 0.019 to 0.016, with velocities ranging from 2.3 to 4.7 feet per second. From Adams' measurements on a pipe 1.17 feet in diameter the values of f range from 0.027 to 0.020, with velocities ranging from 0.7 to 1.5 feet per second. Noble's discussion of all the recorded gagings on wood pipes show certain unexplained discrepancies, and he proposes special empirical formulas to be used for precise computations. Wooden stave pipes after being in service some time may undergo considerable alterations in form, as the circle is apt to be deformed into an ellipse.

By the help of the formulas of the preceding pages, computations for the velocity and discharge of steel and wood pipes under given heads may be readily made. As such pipes are generally long, the formulas of Art. 97 will usually apply. In designing a pipe line a liberal factor of safety should be introduced by taking a value of f sufficiently large so that the discharge may not be found deficient after a few years' use has deteriorated its surface.

Prob. 108. What is the discharge, in gallons per day, of a wood stave pipe 5 feet in diameter when the slope of the hydraulic gradient is 47.5 feet per mile?

Art. 109. Fire Hose

Fire hose is generally $2\frac{1}{2}$ inches in diameter, and lined with rubber to reduce the frictional losses. The following values of the friction factor f have been deduced from the experiments of Freeman.†

Velocity in feet per second,	$v =$ 4	6	10	15	20
Unlined linen hose,	$f =$ 0.038	0.038	0.037	0.035	0.034
Rough rubber-lined cotton,	$f =$ 0.030	0.031	0.031	0.030	0.029
Smooth rubber-lined cotton,	$f =$ 0.024	0.023	0.022	0.019	0.018
Discharge, gallons per minute =	61	92	153	230	306

By the help of this table computations may be made on flow of water through fire hose in the same manner as for pipes. It is

* Transactions American Society of Civil Engineers, 1902, vol. 49, pp. 112, 143.

† Transactions American Society of Civil Engineers, 1889, vol. 21, p. 303; 346.

seen that the friction factors for the best hose are slightly less than those given for $2\frac{1}{2}$-inch pipes in Table 90*a*.

When the hose line runs from a steamer to the nozzle, instead of from a reservoir, the head h is that due to the pressure p at the steamer pump (Art. 11). If this hose line is of uniform diameter the velocity in the hose and nozzle may be computed by Art. 101 and the discharge is then readily found. For example, let the hose be $2\frac{1}{2}$ inches in diameter and 400 feet long, the pressure at the steamer be 100 pounds per square inch, which corresponds to a head of 230.4 feet, and the nozzle be $1\frac{1}{8}$ inches in diameter with a coefficient of velocity of 0.98. Then, neglecting the loss of head at entrance, and using for f the value 0.03, the velocity from the nozzle is found to be 66.0 feet per second, which gives a velocity-head of 67.7 feet and a discharge of 180 gallons per minute. The head lost in friction is $230.4-67.7=162.7$ feet, of which 2.8 feet are lost in the nozzle and the remainder in the hose.

Sometimes the hose near the steamer is larger in diameter than the remaining length. Let l_1 be the length and d_1 the diameter of the larger hose, and l_2 and d_2 the same quantities for the smaller hose. Let c_1 be the coefficient of velocity for a smooth nozzle, D its diameter, and V the velocity of the stream issuing from the nozzle. By reasoning as in Arts. 93 and 101, and neglecting losses of head at entrance and in curvature, there is found for the velocity at the end of the nozzle

$$V=\sqrt{\frac{2gh}{f_1\frac{l_1}{d_1}\left(\frac{D}{d_1}\right)^4+f_2\frac{l_2}{d_2}\left(\frac{D}{d_2}\right)^4+\frac{1}{c_1^2}}} \qquad (109)$$

and the discharge is given by $q=\frac{1}{4}\pi D^2V$. For example, let $h=230.4$, $l_1=100$, $l_2=300$ feet; $d_1=3$, $d_2=2.5$, $D=1.125$ inches; $c_1=0.98$, and $f_1=f_2=0.03$. Then, by the formula $V=69.7$ feet per second, which gives a velocity-head of 75.5 feet and a discharge of 190 gallons per minute. This example is the same as that of the preceding paragraph, except that a larger hose is used for one-fourth of the length, and it is seen that its effect is to increase the velocity-head nearly 12 per cent and the discharge nearly 6 per cent. For this case the head lost in friction is 154.9 feet, of which 3.1 feet are lost in the nozzle and the remainder in the 400 feet of hose.

In using the above formula the tip of the nozzle is supposed to be on the same level with the pressure gage at the steamer pump and the head h is given in feet by $2.304\,p$, where p is the gage reading in pounds per square inch. When the tip of the nozzle is a vertical distance z above this gage, h is to be replaced by $h-z$ in the formula; when it is the same vertical distance below the gage, h is to be replaced by $h+z$. In the former case gravity decreases and in the latter case it increases the velocity and discharge. The above formula applies also to the case of a hose connected to a hydrant, if h is the effective-head at the entrance, that is, the pressure-head plus the velocity-head in the hydrant. In Art. 201 will be found further discussions regarding pumping through fire hose.

At a hydrant of diameter d_1 the pressure-head is h_1. To this is attached a hose of length l and diameter d_1 and to the end of the hose a nozzle of diameter D and velocity coefficient c_1. Neglecting losses at entrance and in curvature the formula for computing the velocity of the jet issuing from the nozzle, when its tip is held at the same level as the gage that indicates the pressure-head, is

$$V = \sqrt{\frac{2gh_1}{f\dfrac{l}{d}\left(\dfrac{D}{d}\right)^4 + \dfrac{1}{c_1^2}}}$$

Prob. 109. When the pressure-gage at the steamer indicates 83 pounds per square inch, a gage on the leather hose 800 feet distant reads 25 pounds. Compute the value of the friction factor f, the discharge per minute being 121 gallons. If the second gage be at the entrance to a $1\frac{1}{4}$-inch nozzle, compute its coefficient of velocity.

Art. 110. Other Formulas for Flow in Pipes

The formulas thus far presented in this chapter are based upon the assumption that all losses of head vary with the square of the velocity. This is closely the case for the velocities common in engineering practice, but for velocities smaller than 0.5 feet per second the losses of head due to friction have been found to vary at a less rapid rate, and in fact nearly as the first power of the velocity. Probably at usual velocities the loss of head in friction is composed of two parts, a small part varying directly with the velocity which is due to cohesive resistance along the surface, and a large part varying as the square of the velocity which is due to impact as illustrated in Fig. 90. This was recognized by the early hydraulicians who, after defining the friction head and friction factor as in (90), by the formula

$$h'' = f\frac{l}{d}\frac{v^2}{2g}$$

endeavored to express f in terms of the velocity v. Thus, D'Aubisson deduced

$$f = 0.0269 + \frac{0.00484}{v}$$

and Weisbach advocated the form

$$f = 0.0144 + \frac{0.00172}{\sqrt{v}}$$

Darcy, on the other hand, expressed f in terms of d, namely,

$$f = 0.0199 + \frac{0.00167}{d}$$

All these expressions are for English measures, v being in feet per second and d in feet. Later investigations show, however, that f varies with both v and d, and the best that can now be done is to tabulate its values as in Table 90a. In fact it may be said that the theory of the flow of water in pipes at common velocities is not yet well understood.

Many attempts have been made to express the velocity of flow in a long pipe by an equation of the form

$$v = \alpha \cdot d^\beta (h/l)^\gamma$$

in which α, β, and γ are to be determined from experiments in which v, d, h, and l have been measured. The exponential formula deduced by Lampe for clean cast-iron pipes varying in diameter from one to two feet is

$$v = 77.7\, d^{0.694}(h/l)^{0.555} \qquad (110)$$

in which d, h, and l are to be taken in feet, and v will be found in feet per second. From this are derived

$$q = 61.0\, d^{2.694}(h/l)^{0.555} \qquad d = 0.217\, q^{0.371}(l/h)^{0.206}$$

by which discharge and diameter may be computed. Other investigators find different values of β and γ, the values $\beta = \frac{2}{3}$ and $\gamma = \frac{1}{2}$ being frequently advocated.

The formula of Chezy (Art. 113), that of Kutter (Art. 118), that of Bazin (Art. 122), and that of Williams and Hazen (Art. 124), are often used for long pipes, care being taken to select the proper value of c for the first, of n for the second, of m for the third, and of c for the fourth. The formulas of Kutter and Bazin are sometimes more advantageous than the others since in using them the roughness of the surface of the pipe can better be taken into account.

The formulas of this chapter do not apply to very small pipes and very low velocities, and it is well known that for such conditions the loss of head in friction varies as the first power of the velocity. This was shown in 1843 by Poiseuille, who made experiments in order to study the phenomena of the flow of blood in veins and arteries. For pipes of less than 0.03 inches diameter he found the head h to be given by $h = C_1lv/d_2$ where C_1 is a constant factor for a given temperature, v is the velocity, d the diameter, and l the length of the pipe. Later researches indicate that the laws expressed by this equation also hold for large pipes provided the velocity be very small, and that there is a certain critical velocity at which the law changes and beyond which $h = C_2lv^2/d$, as for the common cases in engineering practice. This critical point appears to be that where the filaments cease to move in parallel lines and where the impact disturbances illustrated in Fig. 90 begin. For a very small pipe the velocity may be high before this critical point is reached; for a large pipe it happens at very low velocities. Experiments devised by Reynolds enable the impact disturbance to be actually seen as the critical velocity is passed, so that its existence is beyond question. It may also be noted that the velocity of flow through a submerged sand filter bed varies directly as the first power of the effective head.

Prob. 110. Solve Problems 94 and 95 by the use of the above formulas of Lampe.

Art. 111. Computations in Metric Measures

Nearly all the formulas of this chapter are rational in form, the coefficient of velocity c_1, the factors f and f_1, and the factors m, m_1, m_2, and m' are abstract numbers which have the same values in all systems of measures.

(Art. 90) The mean value of the friction factor f is 0.02, and Table 111a gives closer values corresponding to metric arguments. For

Table 111a. Friction Factors for Clean Iron Pipes

Arguments in Metric Measures

Diameter in Centimeters	Velocity in Meters per Second					
	0.3	0.6	1.0	1.5	2.5	4.5
1.5	0.047	0.041	0.036	0.033	0.030	0.028
3.	.038	.032	.030	.027	.025	.023
8.	.031	.028	.026	.024	.023	.021
16.	.027	.026	.025	.023	.021	.019
30.	.025	.024	.023	.021	.019	.017
40.	.024	.023	.022	.019	.018	.016
60.	.022	.020	.019	.017	.015	.013
90.	.019	.018	.016	.015	.013	.012
120.	.017	.016	.015	.013	.012	
180.	.015	.014	.013	.012		

example, let l = 3000 meters, d = 30 centimeters = 0.3 meters, and v = 1.75 meters per second. Then from the table f is 0.022, and

$$h'' = 0.022 \times \frac{3000}{0.3} \times \frac{1.75^2}{19.6} = 34.3 \text{ meters,}$$

which is the probable loss of head in friction. By the use of Table 111b approximate computations may be made more rapidly, thus for this case the loss of head for 100 meters of pipe is found to be 1.10 meters, hence for 3000 meters the loss of head is 33 meters.

(Art. 94) The metric value of $\frac{1}{4}\pi\sqrt{2g}$ is 3.477 and that of $8/\pi^2 g$ is 0.2653.

(Art. 95) When (95) is used in the metric system, the constant 0.4789 is to be replaced by 0.6075; here q is to be in cubic meters per second, and l and d in meters.

TABLE 111*b*. FRICTION HEAD FOR 100 METERS OF CLEAN IRON PIPE

Metric Measures

Diameter in Centimeters	Velocity in Meters per Second					
	0.3	0.6	1.0	1.5	2.5	4.5
	Meters	Meters	Meters	Meters	Meters	Meters
1.5	1.44	5.02	12.2			
3.	0.58	1.96	5.10	10.3	26.6	
8.	.18	0.64	1.66	3.45	9.23	27.1
16.	.08	.30	.80	1.65	4.09	12.3
30.	.04	.15	.39	0.80	2.02	5.85
40.	.03	.10	.28	.54	1.43	4.13
60.	.02	.06	.16	.33	0.80	2.24
90.	.01	.04	.09	.19	.46	1.38
120.		.02	.06	.12	.32	
180.		.01	.04	.08		

(Art. 97) In $(97)_2$ the two constants are 4.43 and 3.48 instead of 8.02 and 6.30. In $(97)_3$ the constant is 0.607 instead of 0.479.

(Arts. 106, 107, and 108) The friction factors f for cast iron, steel and wood pipes may be taken for metric arguments by using the velocities in meters per second, namely, by writing 0.3, 0.6, 0.9, 1.2, 1.5, 1.8 meters per second, instead of 1, 2, 3, 4, 5, 6 feet per second.

(Art. 109) For fire hose the values of the friction factor f for metric data are as follows, for hose 6.35 centimeters in diameter:

Velocity, meters per second,	$v = 1.22$	1.83	3.05	4.57	6.10
Unlined linen hose,	$f = 0.038$	0.038	0.037	0.035	0.034
Rough rubber-lined cotton,	$f = 0.030$	0.031	0.031	0.030	0.029
Smooth rubber-lined cotton,	$f = 0.024$	0.023	0.022	0.019	0.018
Discharge, liters per minute,	= 231	348	579	871	1158

(Art. 110) In the metric system the formulas for the friction factor f are the same as those in the text, except that the numerator of the last term is to be divided by 3.28 in the formulas of D'Aubisson and Darcy and by 1.81 in that of Weisbach. Lampe's formula is

$$v = 54.1\, d^{0.694}(h/l)^{0.555}$$

and his formulas for discharge and diameter are

$$q = 42.5\, d^{2.694}(h/l)^{0.555} \qquad d = 0.249\, q^{0.371}(h/l)^{0.206}$$

in which d, h, and l are in meters, v in meters per second, and q in cubic meters per second.

Prob. 110*a*. Compute the diameter, in centimeters, for a pipe to deliver 500 liters per minute under a head of 2 meters, when its length is 100 meters. Also when the length is 1000 meters.

Prob. 110*b*. Compute the velocity-head and discharge for a pipe 1 meter in diameter and 856 meters long under a head of 64 meters. Compute the same quantities when a smooth nozzle 5 centimeters in diameter is attached to the end of the pipe.

Prob. 110*c*. A compound pipe has the three diameters 15, 20, and 30 centimeters, the lengths of which are 150, 600, and 430 meters. Compute the discharge under a head of 16 meters.

Prob. 110*d*. A steel-riveted pipe 1.5 meters in diameter is 7500 meters long. Compute the velocity and discharge under a head of 30.5 meters.

Prob. 110*e*. The value of C_1 in Poiseuille's formula for small pipes is 0.0000177 for English measures at 10° centigrade. Show that its value is 0.0000690 for metric measures.

Prob. 110*f*. In Fig. 105*b* let the pipe AB be 3000 meters long and 30 centimeters in diameter, BCD be 800 meters long and 20 centimeters in diameter, BCE be 1000 feet long and 20 centimeters in diameter, and EF be 300 meters long and 30 centimeters in diameter. Compute the velocity and discharge for each pipe when the total lost head H is 12.5 meters.