measured along the pipe line. The piezometer there placed rises to C_1, which is a point in the hydraulic gradient. The equation of this line with reference to the origin A is given by the first equation of Art. 98, or

$$H_1 = (1 + m)\frac{v^2}{2g} + f\frac{l_1}{d}\frac{v^2}{2g}$$

in which H_1 is the ordinate A_1C_1, and l_1 is the abscissa AA_1, provided that the length of the pipe is sensibly equivalent to its horizontal projection. In this equation the first term of the second member is constant for a given velocity, and is represented in the figure by AB or A_1B_1; the second term varies with l_1, and is represented by B_1C_1. The gradient is therefore a straight line, subject to the provision that the pipe is laid approximately horizontal; which is usually the case in practice, since quite material vertical variations may exist in long pipes without sensibly affecting the horizontal distances.

When the variable point D_1 is taken at the outlet end of the pipe, H_1 becomes the head h, and l_1 becomes the total length l, agreeing with the formula of Art. 93, if the losses of head due to curvature and valves be omitted. When d_1 is taken very near the inlet end, l_1 becomes zero and the ordinate H_1 becomes AB, which represents the velocity-head plus the loss of head at entrance to the pipe.

When there are easy horizontal curves in a pipe line, the above conclusions are unaffected, except that the gradient BC is always vertically above the pipe, and therefore can be called straight only by courtesy, although as before the ordinate B_1C_1 is proportional to l_1. When there are sharp curves, the inclination of the hydraulic gradient becomes greater and it is depressed at each curve by an amount equal to the loss of head which there occurs. When an obstruction occurs in a pipe, or a valve is partially closed, there is a sudden depression of the gradient at the obstruction or at the valve.

If the pipe is so laid that a portion of it rises above the hydraulic gradient as at D_1 in Fig. 99*b*, an entire change of condition generally results. If the pipe is closed at C, all the piezometers

stand in the line AA, at the same level as the surface of the reservoir. When the valve at C is opened, the flow at first occurs under normal conditions, h being the head and BC the hydraulic gradient. The pressure-head at D_1 is then negative, and represented by D_1C_1. As a consequence air tends to enter the pipe, and when it does so, owing to defective joints, the continuity of the flow is broken, and then the pipe from D_1 to C is only partly filled with water. The hydraulic gradient is then shifted to BD_1, the discharge occurs at D_1 under the head A_1D_1, while the remainder of the pipe acts merely as a channel to deliver the flow. It usually happens that this change results in a great diminution of the discharge, so that it has been necessary to dig up and relay portions of a pipe line which have been inadvertently run above the hydraulic gradient. This trouble can always be avoided by preparing a profile of the proposed route, drawing the hydraulic gradient upon it, and excavating the pipe trench well below the gradient. In cases where the cost of this excavation is so great that it is resolved to lay the pipe above the gradient, all the joints of the pipe above the gradient should be made absolutely tight so that no air can enter the pipe and interrupt the flow.

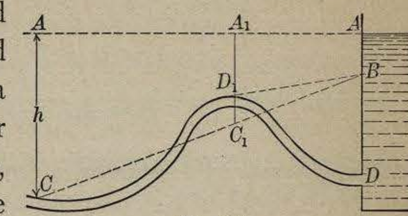

Fig. 99*b*.

When a large part of the pipe lies above the hydraulic gradient it is called a siphon. Conditions sometimes exist which require a pipe line to be laid as a siphon for a short distance. In such a case an air chamber is sometimes built at the highest elevation so that air may collect in it instead of in the pipe, and provision is made for recharging the siphon when the flow ceases by admitting water at the highest elevation, or by operating a suction-pump placed there, or by forcing water into the pipe by a pump located at a lower elevation. Probably the largest siphon ever constructed is that laid about 1885 at Kansas City, Mo., it being 42 inches in diameter, and 730 feet long, with the summit 10 feet above the general level of the pipe line. The air that collected at the summit was removed by operating a steam ejector for a few minutes each day.*

The pressure-head h_1 at any point on the pipe line distant l_1 from the reservoir may be expressed in terms of the static head on that point, the entrance-head h', and the friction-head h'' by inspection of Fig. 99*a*; thus, $$h_1 = A_1D_1 - h' - h''$$

Further, from the similar triangles in the figure,

$$h'' = (h - h')(l_1/l)$$

that is, the loss in friction in the distance l_1 is proportional to l_1. For long pipes, in which h' is small, this may be written $h'' = h(l_1\, l)$, or the friction loss at any point on the pipe line is proportional to the total head and to the distance of the point from the reservoir.

The above discussion shows that it is immaterial where the pipe enters the reservoir, provided that it enters below the hydraulic gradient point B. It is also not to be forgotten that the whole investigation rests on the assumption that the lengths l_1 and l are sensibly equal to their horizontal projections.

Prob. 99. A pipe 3 inches in diameter discharges 538 cubic feet per hour under a head of 12 feet. At a distance of 300 feet from the reservoir the depth of the pipe below the water surface in the reservoir is 4.5 feet. Compute the probable pressure-head at this point.

Art. 100. A Compound Pipe

A compound pipe is one having different sizes in different portions of its length. The change from one length to another should be made by a "reducer," which is a conical frustum several feet long, so that losses of head due to sudden enlargement or contraction are avoided (Arts. 76, 77). Let d_1, d_2, d_3, etc., be the diameters; l_1, l_2, l_3, etc., the corresponding lengths, the total length being $l_1 + l_2 +$ etc. Let v_1, v_2, etc., be the velocities in the different sections. Neglecting the loss of head at entrance and also that lost in curvature, the total head h may be placed equal to the loss of head in friction, or

$$h = f_1 \frac{l_1}{d_1}\frac{v_1^2}{2g} + f_2 \frac{l_2}{d_2}\frac{v_2^2}{2g} + \text{etc.}$$

* Engineering News, 1891, vol. 26, p. 519; 1893, vol. 29, pp. 423, 588.

Now if the discharge per second be q, and the flow be steady

$$v_1 = q/\tfrac{1}{4}\pi d_1^2 \qquad v_2 = q/\tfrac{1}{4}\pi d_2^2, \quad \text{etc.}$$

Substituting these velocities and solving for q, gives

$$q = \tfrac{1}{4}\pi \sqrt{\frac{2gh}{f_1 \dfrac{l_1}{d_1^5} + f_2 \dfrac{l_2}{d_2^5} + \text{etc.}}} \qquad (100)$$

in which the friction factors f_1, f_2, etc., corresponding to the given diameters and computed velocities are found from Table 90*a*.

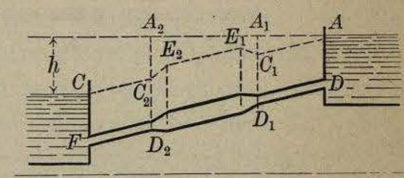

Fig. 100.

For example, consider the case of a pipe having only two sizes; let $d_1 = 2$ and $l_1 = 2800$ feet, $d_2 = 1.5$ and $l_2 = 2145$ feet, and $h = 127.5$ feet. Using for f_1 and f_2 the mean value, 0.02, and making the substitutions in the formula, there is found $$q = 26.2 \text{ cubic feet per second}$$

from which $v_1 = 8.3$ and $v_2 = 14.8$ feet per second

Now from Table 90*a* it is seen that $f_1 = 0.015$ and $f_2 = 0.015$; and repeating the computation,

$$q = 30.2 \text{ cubic feet per second}$$

whence $v_1 = 9.6$ and $v_2 = 17.1$ feet per second.

These results are probably as definite as the table of friction factors will allow, but are to be regarded as liable to an uncertainty of several percent.

To determine the diameter of a pipe which will give the same discharge as the compound one, it is only necessary to replace the denominator in the above value of q by fl/d^5, where $l = l_1 + l_2$ + etc., and d is the diameter required. Taking the values of f as equal, this gives $$\frac{l}{d^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \text{etc}$$

Applying this to the above example, it becomes

$$\frac{4945}{d^5} = \frac{2800}{2^5} + \frac{2145}{1.5^5}$$

from which $d = 1.68$ feet, or about 20 inches.

A compound pipe is sometimes used to prevent the hydraulic gradient from falling below the pipe line. Thus, it is seen in Fig. 100 that the hydraulic gradient rises at D_1 and falls at D_2, and that its slope over the larger pipe is less than over the smaller one. These slopes and the amount of rise at D_1 can be computed for a given case. Using the above numerical data, the loss of head in friction for 100 feet of the large pipe is

$$h'' = 0.015 \frac{100}{2} \frac{v_1^2}{2g} = 1.07 \text{ feet},$$

while the same for the small pipe is 4.55 feet. Hence the slope of the gradients AC_1 and C_2C is more than four times as rapid as that of the gradient E_1E_2. In the large pipe at D_1 the velocity-head is $0.01555 \times 9.6^2 = 1.43$ feet, and, supposing that no loss occurs in the reducer, the velocity-head for the small pipe is 4.55 feet. The vertical rise C_1E_1 of the hydraulic gradient at D_1 is hence the rise in pressure-head $4.55 - 1.43 = 3.12$ feet, and a fall of equal amount occurs at D_2.

When a portion of a small pipe is to be replaced by a large one, it is immaterial in what part of the length it is introduced, for it is seen that formula (100) takes no note of where the length l_1 is placed in the total distance l. The Romans knew that an increase in the diameter of a pipe after leaving the reservoir would increase the discharge, and the law passed by the Roman senate about the year 10 B.C. forbade a consumer to attach a larger pipe to the standard pipe within 50 feet of the reservoir to which the latter was connected.*

Prob. 100. At Rochester, N.Y., there is a pipe 102 277 feet long, of which 50 828 feet is 36 inches in diameter and 51 449 feet is 24 inches in diameter. Under a head of 143.8 feet this pipe is said to have discharged in 1876 about 14 cubic feet per second and in 1890 about $10\frac{1}{2}$ cubic feet per second. Compute the discharge by (100), and draw the hydraulic gradient.

Art. 101. A Pipe with a Nozzle

Water is often delivered through a nozzle in order to perform work upon a motor or for the purposes of hydraulic mining, the nozzle being attached to the end of a pipe which brings the flow from a reservoir. In such a case it is desirable that the pressure at the entrance to the nozzle should be as great as possible, and

* Herschel, Water Supply of the City of Rome (Boston, 1889), p. 77.

this will be effected when the loss of head in the pipe is as small as possible. The pressure column in a piezometer, supposed to be inserted at the end of the pipe, as shown at C_1D_1 in Fig. 101, measures the pressure-head there acting, and the height A_1C_1 measures the lost head plus the velocity-head, the latter being very small.

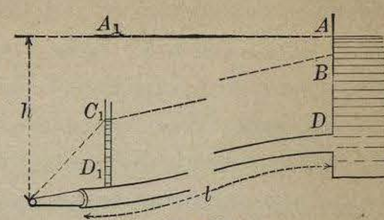

Fig. 101.

Let h be the total head on the end of the nozzle, D its diameter, and V the velocity of the issuing stream. Let d and v be the corresponding quantities for the pipe, and l its length. Then the effective velocity-head of the issuing stream is $V^2/2g$, and the lost head is $h - V^2/2g$. This lost head consists of several parts: that lost at the entrance D; that lost in friction in the pipe; that lost in curves and valves, if any; and lastly, that lost in the nozzle. Then the principle of energy gives the equation

$$h - \frac{V^2}{2g} = m\frac{v^2}{2g} + f\frac{l}{d}\frac{v^2}{2g} + m_1\frac{v^2}{2g} + m_2\frac{v^2}{2g} + m'\frac{V^2}{2g}$$

Here m is determined by Art. 89, f by Art. 90, m_1 by Art. 91, m_2 by Art. 92, while m' for the nozzle is found in the same manner as m is found for the pipe, or $m' = (1/c_1)^2 - 1$, where c_1 is the coefficient of velocity for the nozzle (Art. 83). This value of m' takes account of all losses of head in the nozzle, so that it is unnecessary to consider its length; for a perfect nozzle c_1 is unity and m' is zero.

The velocities v and V are inversely as the areas of the corresponding cross-sections (Art. 31), since the flow is steady, whence $V = v(d/D)^2$. Inserting this in the above equation and solving for v gives, if m_1 and m_2 be neglected,

$$v = \sqrt{\frac{2gh}{m + f(l/d) + (1/c_1)^2(d/D)^4}} \qquad (101)$$

for the velocity in the pipe. The velocity and discharge from the nozzle are then given by

$$V = (d/D)^2 v \qquad q = \tfrac{1}{4}\pi D^2 V$$

and the velocity head of the jet is $V^2/2g$. These equations show that the greatest value of V obtains when D is as small as possible compared to d, and that the greatest discharge occurs when D is equal to d. When the object of a nozzle is to utilize the velocity-head of a jet, a large pipe and a small nozzle should be employed. When the object is to utilize the energy of the jet in producing power by a water wheel, there is a certain relation between D and d that renders this a maximum (Art. 161).

As a numerical example, the effect of attaching a nozzle to the pipe whose discharge was computed in Art. 94 will be considered. There $l=1500$, $d=0.25$, and $h=64$ feet; $m=0.5$, $v=5.3$ feet, and $q=0.26$ cubic feet per second. Now let the nozzle be one inch in diameter at the small end, or $D=0.0833$ feet, and let its coefficient c_1 be 0.98. Here $d/D=3$, and for $f=0.025$ the velocity in the pipe is

$$v=\sqrt{\frac{2\times 32.16\times 64}{0.5+0.025\times 1500\times 4+1.041\times 81}}$$

or $v=4.2$ feet per second. The effect of the nozzle, therefore, is to reduce the velocity in the pipe. The velocity of the jet at the end of the nozzle is, however,

$$V=v(d/D)^2=37.8 \text{ feet per second,}$$

and the discharge per second from the nozzle is

$$q=\tfrac{1}{4}\pi D^2 V=0.206 \text{ cubic feet}$$

which is about 20 percent less than that of the pipe before the nozzle was attached. The nozzle, however, produces a marvelous effect in increasing the energy of the discharge; for the velocity-head corresponding to 5.3 feet per second is only 0.44 feet, while that corresponding to 37.8 feet per second is 22.2 feet, or about 50 times as great. As the total head is 64 feet, the efficiency of the pipe and nozzle is about 35 percent.

If the pressure-head h_1 at the entrance of the nozzle be observed, either by a piezometer tube or by a pressure gage, the velocity of discharge from the nozzle can be computed by the formula

$$V=\sqrt{\frac{2gh_1}{(1/c_1)^2-(D/d)^4}}$$

the demonstration of which is given in Art. 83. This can be used when a hose and nozzle is attached at any point of a pipe or at a hydrant. It can also be used to compute h_1 when V has been found. Thus, for the above example,

$$h_1=\left(\frac{1}{c_1^2}-\frac{D^4}{d^4}\right)\frac{V^2}{2g}=22.8 \text{ feet}$$

which shows that the loss of head in the nozzle is about 0.6 feet. The loss of head at entrance, for this case, is about 0.2 feet, and the loss of head in friction in the pipe is 41.0 feet.

Prob. 101. A pipe 12 inches in diameter and 4320 feet long leads from a reservoir to a gravel bank against which water is delivered from a nozzle 2 inches in diameter. The head on the end of the nozzle is 320 feet and the coefficient of velocity of the nozzle is 0.97. Compute the velocity in the pipe, the velocity-head of the jet, and the discharge.

Art. 102. House-service Pipes

A service pipe which runs from a street main to a house is connected to the former at right angles, and usually by a corporation cock or by a "ferrule." The loss of head at entrance in such cases is hence larger than in those before discussed, and m should probably be taken as at least equal to unity. The pipe, if of lead, is frequently carried around sharp corners by curves of small radius; if of iron, these curves are formed by pieces forming a quadrant of a circle into which the straight parts are screwed, the radius of the center line of the curve being but little larger than the radius of the pipe, so that each curve causes a loss of head equal nearly to double the velocity-head (Art. 91). For new iron pipes the loss of head due to friction may be estimated by the rules of Art. 90 or by Table 90*b*.

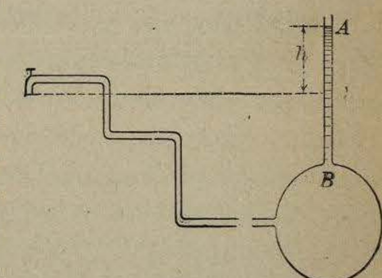

Fig. 102*a*.

A water main should be so designed that a certain minimum pressure-head h_1 exists in it at times of heaviest draft. This pressure-head may be represented by the height of the piezometer column AB, which would rise in a tube supposed to be inserted in the main, as in Fig. 102*a*. The head h which causes the flow in the pipe is then the difference in level between the top of this column and the end of the pipe, or AC. Inserting for h this value, the formulas of Arts. 94 and 95 may be applied to the investigation of service pipes in the manner there illustrated. As the sizes of common house-service pipes are regulated by the practice of the plumbers and by the market sizes obtainable, it is not often necessary to make computations regarding the flow of water through them.

The velocity of flow in the main has no direct influence upon that in the pipe, since the connection is made at right angles. But as that velocity varies, owing to the varying draft upon the main, the effective head h is subject to continual fluctuations. When there is no flow in the main, the piezometer column rises until its top is on the same level as the surface of the reservoir; in times of great draft it may sink below C, so that no water can be drawn from the service pipe.

The detection and prevention of the waste of water by consumers is a matter of importance in cities where the supply is limited and where meters are not in use. Of the many methods devised to detect this waste, one by the use of piezometers may be noticed, by which an inspector without entering a house may ascertain whether water is being drawn within, and the approximate amount per second. Let M be the street main from which a service pipe MOH runs to a house H. At the edge of the sidewalk a tube OP is connected to the service pipe, which has a three-way cock at O, which can be turned from above. The inspector, passing on his rounds in the night-time, attaches a pressure gage at P and turns the cock O so as to shut off the water from the house and allow the full pressure of the main p_1 to be registered. Then he turns the cock so that the water may flow into the house, while it also rises in OP and registers the pressure p_2. Then if p_2 is less than p_1, it is certain that waste is occurring

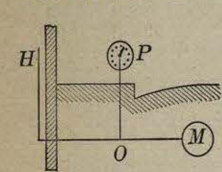

Fig. 102*b*.

within the house, and the amount of this may be approximately computed and the consumer be notified accordingly.

The pitometer, which consists of a rated Pitot tube (Art. 41), facing the current in the pipe, with a differential gage (Art. 37) to determine the pressure-head due to the current, is also used for the measurement of the flow in water mains and for the detection of water waste. A photographic record of the difference in height of the columns of liquid in the gage tube is kept, and this shows the discharge through the water main at any instant, as also all fluctuations in the flow.* (See Art. 38.)

When the pressure in the street main is very high, a pressure regulator may be placed between the main and the house in order to reduce the pressure and thus allow lighter pipes to be used in the house. Fig. 102*c* shows the principle of its action, where A represents the pipe from the main and B the pipe leading to the house. A weight W is placed upon a piston which covers the opening into the chamber C. This weight and that of the piston are sufficient to overcome a certain unit-pressure in C, and therefore the unit-pressure in B is less than that in A by that amount. For example, suppose the pressure in A to be 100 pounds per square inch, and let it be required that the pressure in B shall not rise above 60 pounds per square inch; then the piston must be so weighted that it may exert on the water in C a pressure of 40 pounds per square inch. When water is drawn out anywhere along the pipe B, the pressure in the chamber above the piston falls below 60 pounds per square inch, and hence the piston rises and water flows from A into B until the pressure is restored. Instead of a weight, a spring is generally used, or sometimes a weighted lever.

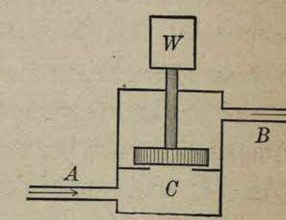

Fig. 102*c*.

Large-sized pressure regulators are also used to control and maintain a constant pressure in distributing mains in cases where

* Engineering Record, 1903, vol. 47, p. 122.

a low service level is fed from one of higher pressure, or in situations where it is desired to maintain a pressure which shall not exceed a fixed maximum.

Prob. 102. In Fig. 102*b* let the house pipe be one inch in diameter and the pressure at the gage be 34 pounds per square inch when there is no flow. The distance from the main to the gage is 16 feet and from the gage to the end of the pipe is 29 feet. At the end of the pipe, which is 5 feet higher than the gage, 2.1 gallons of water are drawn per minute. Compute the pressure at the gage.

Art. 103. Operating and Regulating Devices

In the operation of nearly every water works system certain special apparatus is employed in order to maintain nearly constant conditions within the system and under the variable draft to which it is subjected. These forms of apparatus are designed to operate automatically and so to do away with hand regulation. Many of these are designed, as described under meters in Art. 38, to trace on a chart a continuous autographic record of the pressure, of the water level, or of the discharge. Among these are pressure gages (Art. 36), water stage registers (Art. 34), and rate of flow gages (Art. 38).

Air valves are attached to water mains in situations where air is likely to accumulate within the pipe and by its presence interfere with the flow of the water or be carried along within the pipe and produce dangerous water hammer. Valves of this type permit the air within the pipe to escape, but automatically close and prevent the passage of water. They are also placed on all of the principal summits of riveted steel and other pipes so as to admit air into the pipe in case of a sudden break and thus prevent its collapse under external atmospheric pressure. In the case of cast-iron pipes, on account of the strength of their shells, this precaution is not usually necessary. The principle of the operation of the air valve is simply that of a float placed in a chamber above and connected with the pipe from which the air is to be removed. When air accumulates in the pipe, it passes up into the chamber; the float falls, and in falling, by means of a lever, operates and opens a valve. The air then escapes under the

pressure of the water until the float again rises and causes the valve to close.

Pressure regulators operating on the principle described in Art. 102 are employed for the purpose of controlling and maintaining a constant pressure in distributing systems in situations where a low service level is fed from one of higher pressure. They may also be used to regulate the flow between reservoirs situated at different elevations. In the larger sized regulators the valve which controls the flow is operated by a pair of differential pistons connecting with a chamber, the pressure in which is caused to vary with fluctuations in pressure on the two sides of the regulator. The variations in pressure within this chamber are intensified by two small-sized regulators which connect directly to the high and low pressure sides of the large regulator. That on the upstream side of the main regulator is designed to close under an increase in pressure, while that on the downstream side will tend to open as the pressure rises. The effect of any difference in pressure on the two sides of the main regulator is therefore promptly reflected in the pressure within the chamber, and the differential pistons at once move to open or close the regulating valve in the effort to maintain within the pipe the predetermined constant pressure at which the apparatus has been set. A sixteen-inch regulator of this type will control the pressure within narrow limits and pass through it, as may be necessary to accomplish this purpose, quantities up to 10 or 15 millions of gallons per day.

Relief valves for the purpose of preventing the pressure within a pipe from rising above some predetermined limit, either on account of a sudden falling off of the draft or by water hammer, are also made to operate on the principle described in Art. 102, but in the reverse direction. The regulating valve described in the preceding paragraph may also be adapted for this use by simply making the necessary adjustments of the small regulators.

In certain situations and principally in connection with the operation of filtration plants it is desirable that the flow within a pipe shall be maintained at a constant rate. This may be accomplished