The number $m$ hence rapidly increases and becomes very great when the valve is fully closed, but as the velocity is then zero there is no loss of head. The velocity $v$ here, as in other cases, refers to that in the main part of the pipe, and not to that in the contracted section formed by the valve.

Kuichling's experiments * on a gate-valve for a 24 -inch pipe give values of $m$ which are somewhat greater than those deduced by Weisbach from pipes less than 2 inches in diameter. Considering the great variation in size, the agreement is, however, a remarkable one. He found

$$
\text { for } \begin{array}{rlccccc}
d^{\prime} / d & =\frac{1}{3} & \frac{5}{12} & \frac{1}{2} & \frac{5}{8} & \frac{3}{4} & \frac{59}{72} \\
m & =0.8 & \text { 1.6. } & 3.3 & 8.6 & 22.7 . & 41.2
\end{array}
$$

and his computed value of $m$ when $d^{\prime} / d$ equals $\frac{7}{8}$ is 75.6 .
An accidental obstruction in a pipe may be regarded as causing a contraction of section, followed by a sudden expansion, and the loss of head due to it is, by Art. 76,

$$
h^{\prime \prime \prime \prime}=\left(\frac{a}{a^{\prime}}-1 \cdot\right)^{2} \frac{v^{2}}{2 g}=m \frac{v^{2}}{2 g}
$$

where $a$ is the area of the section of the pipe, and $a^{\prime}$ that of the diminished section. This formula shows that when $a^{\prime}$ is onehalf of $a$, the loss of head is equal to the velocity-head, and that $m$ rapidly increases as $a^{\prime}$ diminishes. The same formula gives the loss of head due to the sudden enlargement of a pipe from the area $a^{\prime}$ to $a$.

Air-valves are placed at high points on a pipe line in order to allow the escape of air that collects there. Mud-valves or blowoffs are placed at low points in order to clean out deposits that may be formed as well as to empty the pipe when necessary. These are arranged so as not to contract the section, and the losses of head caused by them are generally very small. When a blowoff pipe is opened and the water flows through it with the velocity $v$, the loss of head at its entrance, even when the edges are rounded, is as high as or higher than $0.56 \mathrm{v}^{2} / 2 \mathrm{~g}$, according to the experiments of Fletcher.

* Transactions American Society of Civil Engineers, 1892, vol. 26, p. 449.

In the following pages the symbol $h^{\prime \prime \prime \prime}$ will be used to denote the sum of all the losses of head due to valves and contractions of section. Then

$$
\begin{equation*}
h^{\prime \prime \prime \prime}=m_{2} \frac{v^{2}}{2 g} \tag{92}
\end{equation*}
$$

in which $m_{2}$ will denote the sum of all the values of $m$ due to these causes. In case no mention is made regarding these sources of loss they are supposed not to exist, so that both $m_{2}$ and $h^{\prime \prime \prime \prime}$ are simply zero.

Prob. 92. Which causes the greater loss of head in a 24 -inch pipe, a gate-valve one-half closed, or five $90^{\circ}$ curves of 16 feet radius?

## Art. 93. Formula for Mean Velocity

The mean velocity in a pipe can now be deduced for the condition of steady flow. The total head being $h$, and the effective velocity-head of the issuing stream being $v^{2} / 2 g$, the lost head is $h-v^{2} / 2 g$, and this must be equal to the sum of its parts, or

$$
h-\frac{v^{2}}{2 g}=h^{\prime}+h^{\prime \prime}+h^{\prime \prime \prime}+h^{\prime \prime \prime \prime}
$$

Substituting in this the values of the four lost heads, as determined in the four preceding articles, it becomes

$$
=h-\frac{v^{2}}{2 g}=m \frac{v^{2}}{2 g}+f \frac{l}{d} \frac{v^{2}}{2 g}+m_{1} \frac{v^{2}}{2 g}+m_{2} \frac{v^{2}}{2 g}
$$

and by solving for $v$ there is found

$$
\begin{equation*}
v=\sqrt{\frac{2 g h}{I+m+f(l / d)+m_{1}+m_{2}}} \tag{93}
\end{equation*}
$$

which is the general formula for the mean velocity in a pipe of constant cross-section.

The most common case is that of a pipe which has no curves, or curves of such large radius that their influence is very small, and which has no partially closed valves or other obstructions. For this case both $m_{1}$ and $m_{2}$ are zero, and, taking $m$ as 0.5 , the formula becomes

$$
\begin{equation*}
v=\sqrt{\frac{2 g h}{1.5+f(l / d)}} \tag{93}
\end{equation*}
$$

which applies to the great majority of cases in engineering practice.

In this formula the friction factor $f$ is a function of $v$ to be taken from Table $90 a$, and hence $v$ cannot be directly computed, but must be obtained by successive approximations. For example, let it be required to compute the velocity of discharge from a pipe 3000 feet long and 6 inches in diameter under a head of 9 feet. Here $l=3000, d=0.5$, and $h=9$ feet, and taking for $f$ the rough mean value 0.02 , formula $(93)_{2}$ gives

$$
v=\sqrt{\frac{2 \times 32.16 \times 9}{1.5+0.02 \times 3000 \times 2}}=2.2 \text { feet per second. }
$$

The approximate velocity is hence 2.2 feet per second and entering the table with this, the value of $f$ is found to be 0.026 . Then the formula gives

$$
v=\sqrt{\frac{2 \times 32.16 \times 9}{1.5+0.026 \times 3000 \times 2}}=1.92 \text { feet per second. }
$$

This is to be regarded as the probable value of the velocity, since the table gives $f=0.026$ for $v=1.92$. In this manner by one or two trials the value of $v$ can be computed so as to agree with the corresponding value of $f$.

To illustrate the use of the general formula $(93)_{1}$ let the pipe in the above example be supposed to have forty $90^{\circ}$ curves of 6 inches radius, and to contain two gate-valves which are half closed. Then from Arts. 91 and 92 there are found $m_{1}=$ ir. 6 for the curves and $m_{2}=4.2$ for the gates. The mean velocity then is

$$
v=\sqrt{\frac{2 \times 32.16 \times 9}{17.3+0.026 \times 6000}}=1.83 \text { feet per second, }
$$

which is but a trifle less than that found before. With a shorter pipe, however, the influence of the curves and gates in retarding the flow would be more marked.

The head required to produce a given velocity $v$ can be obtained from $(93)_{1}$ or $(93)_{2}$. Thus from the general formula the required head is

$$
h=\left(\mathrm{I}+m+f(l / d)+m_{1}+m_{2}\right) \frac{v^{2}}{2 g}
$$

in which for common computations $m=0.5$, while $m_{1}$ and $m_{2}$ are neglected.

The error in the computed velocity due to an error of one unit in the last decimal of the friction factor $f$ is always relatively less than the error in $f$ itself. For instance, where $v$ is computed for the above example with $f=0.025$, which is 4 percent less than 0.026 , its value is found to be 1.96 feet per second, or 2 percent greater than 1.92. In general the percentage of error in $v$ is less than one-half of that in $f$. It hence appears that computed velocities are liable to probable errors ranging from I to 5 percent, owing to imperfections in the tabular values of $f$ for new clean pipes. This uncertainty is as a rule still further increased by various causes, so that 5 percent is to be regarded as a common probable error in computations of velocity and discharge from pipes.

Velocities greater than 15 feet per second are very unusual in pipes, and but little is known as to the values of $f$ for such cases. For velocities less than 0.5 feet per second, the values of $f$ are also not known (Art. 110), so that only a rough reliance can be placed upon computations. The usual velocity in water mains is less than five feet per second, it being found inadvisable to allow swifter flow on account of the great loss of head in friction.

Prob. 93. Using for $f$ the mean value 0.02 compute the head required to cause a velocity of io feet per second in a pipe 15000 feet long and I8 inches in diameter.

## Art. 94. Computation of Discharge

The discharge per second from a pipe of given diameter is found by multiplying the velocity of discharge by the area of the cross-section of the pipe, or

$$
\begin{equation*}
q=\frac{1}{4} \pi d^{2} v=0.7854 d^{2} v \tag{94}
\end{equation*}
$$

in which $v$ is to be found by the method of the last article.
For example, let it be required to find the discharge in gallons per minute from a clean pipe 3 inches in diameter and 1500 feet long under a head of 64 feet. Here $d=0.25, l=1500$, and $h=$ 64 feet. Then for $f=0: 02$ the velocity is found from $(93)_{2}$ to be 5.82 feet per second; then from Table $90 a$ is found $f=0.024$ and the velocity is 5.30 feet per second. The discharge in cubic feet per second is

$$
q=0.7854 \times 0.25^{2} \times 5.30=0.260
$$

which is equal to 116.7 gallons per minute. This is the probable result, which is liable to the same uncertainty as the velocity, say about 3 percent; so that strictly the discharge should be written $116.7 \pm 3.6$ gallons per minute.

By inserting the value of $v$ from $(93)_{2}$ in the above expression for $q$ it becomes

$$
q=\frac{1}{4} \pi d^{2} \sqrt{\frac{2 g h}{1.5+f(l / d)}}
$$

and from this the head required to produce a given discharge is

$$
h=\frac{8}{\pi^{2} g}\left(\mathrm{I} \cdot 5+f(l / d) \frac{q^{2}}{d^{4}}\right.
$$

These formulas are not more convenient for precise computations than the separate expressions for $v, q$, and $h$ previously established, since $v$ must be computed in order to select $f$ from the table. For approximate computations, however, when $f$ may be taken as 0.02 , they may advantageously be used. In the English system of measures $h$ and $d$ are to be taken in feet and $q$ in cubic feet per second, and the constants in these two formulas have the values

$$
\frac{1}{4} \pi \sqrt{2 g}=6.299 \quad 8 / \pi^{2} g=0.025^{2}
$$

The last formula shows that the head required for a pipe of given diameter varies directly as the square of the proposed discharge. Thus, if a head of 50 feet delivers 8 cubic feet per second through a certain pipe, a head of about 200 feet will be necessary in order to obtain 16 cubic feet per second.

Prob. 94. What head is required to discharge 6 gallons per minute through a pipe I inch in diameter and rooo feet long ?

## Art. 95 . Computation of Diameter

It is an important practical problem to determine the diameter of a pipe to discharge a given quantity of water under*a given head and length. The last equation above serves to solve this case, if the curve and valve resistances be omitted, as all the quantities in it except $d$ are known. This equation reduces to

$$
d^{5}=\frac{8}{\pi^{2} g}(\mathrm{I} \cdot 5 d+f l) \frac{q^{2}}{h}
$$

and for the English system of measures this becomes

$$
\begin{equation*}
d=0.4780\left[(\mathrm{I} .5 d+f l) \cdot \frac{q^{2}}{h}\right]^{\frac{1}{3}} \tag{95}
\end{equation*}
$$

which is the formula for computing $d$ when $h, l$, and $d$ are in feet and $q$ is in cubic feet per second. The value of the friction factor $f$ may be taken as 0.02 in the first instance, and the $d$ in the righthand member being neglected, an approximate value of the diameter is computed. The velocity is next found by the formula

$$
v=q / a=q / 0.7854 d^{2}
$$

and from the Table $90 a$ the value of $f$ thereto corresponding is selected. The computation for $d$ is then repeated, placing in the right-hand member the approximate value of $d$. Thus by one or two trials the diameter is computed which will very closely satisfy the given conditions.

For example, let it be required to determine the diameter of a new pipe which will deliver 500 gallons per second, its length being 4500 feet and the head 24 feet. Here the discharge is

$$
q=500 / 7 \cdot 48 \mathrm{r}=66.84 \text { cubic feet per second. }
$$

The approximate value of $d$ then is

$$
d=0.479\left(\frac{0.02 \times 4500 \times 66.84^{2}}{24}\right)^{\frac{1}{5}}=3.35 \text { feet. }
$$

From this the mean velocity of flow is

$$
v=\frac{66.84}{0.7854 \times 3.35^{2}}=7.6 \text { feet per second, }
$$

and from the table the value of $f$ for this diameter and velocity is found to be o.013. Then
*

$$
d=0.479\left[(\mathrm{I} .5 \times 3.35+0.013 \times 4500) \frac{66.84^{2}}{24}\right]^{\frac{1}{5}}
$$

from which $d=3.125$ feet. With this value of $d$ the velocity is now. found to be 8.7 I feet, so that no change results in the value of $f$. Therequired diameter of the pipe is therefore 3.I feet, or abolit 37 inches; but as the regular market sizes of pipes furnish only 36 inches and 40 inches, one of these must be used, and it will be on the side of safety to select the larger.

It is very important, in determining the size of a pipe, to also consider that the interior surface may become rough by corrosion and incrustation, thus increasing the value of the friction factor and diminishing the discharge. It has been found that some waters deposit incrustations which in a few years render the values of $f$ more than double those given in Table 90a. In Art. 106 will be found values of the friction factor as determined by experiment on various pipes of different ages. The increase in $f$ from these causes is not likely to be so great in a large pipe as in a small one, but it is not improbable that for the above example they might be sufficient to make $f$ as large as 0.03 . Applying this value to the computation of the diameter from the given data there is found $d=3.6$ feet $=$ about 43 inches.

The sizes of iron pipes generally found in the market are $\frac{1}{2}, \frac{3}{4}, \mathrm{I}$, $1 \frac{1}{2}, 1 \frac{3}{4}, 2,3,4,6,8,10,12,16,18,20,24,27,30,36,40,44$, and 48 inches, while intermediate and larger sizes must be made to order. The computation of the diameter is merely a guide to enable one of these sizes to be selected, and therefore it is entirely unnecessary that the numerical work should be carried to a high degree of precision. In fact, three-figure logarithms are usually sufficient to determine reliable values of $d$ from formula (95).

Prob. 95. Compute the diameter of a pipe to deliver 50 gallons per minute under a head of 4 feet when its length is 500 feet. Also when its length is 5000 feet.

## Art. 96. Short Pipes

A pipe is said to be short when its length is less than about 500 times its diameter, and very short when the length is less than about 50 diameters. In both cases the coefficient $c_{1}$ should be estimated according to the condition of the upper end as precisely as possible, and the length $l$ should not include the first three diameters of the pipe, as that portion properly belongs to the tube which is regarded as discharging into the pipe. In attempting to compute the discharge for such pipes, it is often found that the velocity is greater than given in Table $90 a$, and hence that the friction factor $f$ cannot be ascertained. For this reason no accurate estimate can be made of the discharge from short pipes under
high heads, and fortunately it is not often necessary to use them in engineering constructions.

For example, let it be required to compute the velocity of flow from a pipe I foot in diameter and 100 feet long under a head of 100 feet, the upper end being so arranged that $c_{1}=0.80$, and hence $m=0.56$ (Art. 89). Neglecting $m_{1}$ and $m_{2}$, since the pipe has no curves or valves, formula $(93)_{1}$ for the velocity becomes

$$
v=\sqrt{\frac{2 g h}{\mathrm{I} .56+f(l / d)}}
$$

and, using for $f$ the rough mean value 0.02 and taking $l$ as 97 feet, there is found 42.9 feet per second for the mean velocity. Now there is no experimental knowledge regarding the value of the friction factor $f$ for such high velocities in iron pipes, but judging from the table it is probable that $f$ may be about 0.015 . Using this instead of 0.02 gives for $v$ the value 46 feet per second.

The general equation for the velocity of discharge deduced in Art. 93 may be applied to very short pipes by writing $l-3 d$ in place of $l$, and placing for $m$ its value in terms of the coefficient $c_{1}$. It then becomes

$$
\begin{equation*}
v=\sqrt{\frac{2 g h}{\frac{I}{c_{1}{ }^{2}}+f \frac{l-3 d}{d}}} \tag{96}
\end{equation*}
$$

If in this $l$ equals $3 d$, the velocity is $c_{1} \sqrt{2 g h}$, which is the same as for the short cylindrical tube. If $l=12 d, f=0.02$, and $c_{1}=0.82$, it gives $v=0.774 \sqrt{ }{ }_{2 g} h$, which agrees well with the value given by Art. 84 for this case. If $l=60 d$, it gives $v=0.613 \sqrt{2 g h}$, which is 2 percent greater than the value given by Art. 84 .

Prob. 96. Compute the discharge per second for a pipe $I$ inch in diameter and 40 inches long under a head of 4 feet.

## Art. 97. Long Pipes

For long pipes the loss of head at entrance becomes very small compared with that lost in friction, and the velocity-head is also small. Formula $(93)_{2}$ for the mean velocity is

$$
v=\sqrt{\frac{2 g h}{I .5+f(l / d)}}
$$

in which the first term in the denominator represents the effect of the velocity-head and the entrance-head, the mean value of the latter being 0.5 . Now it may safely be assumed that 1.5 may be neglected in comparison with the other term, when the error thus produced in $v$ is less than I percent. Taking for $f$ its mean value, this will be the case when

$$
\frac{\sqrt{1.5+0.02 l / d}}{\sqrt{0.02 l / d}}=1.01, \text { whence } \frac{l}{d}=375^{\circ}
$$

Therefore, when $l$ is greater than about 4000 d the pipe will be called long.

For long pipes under uniform flow the velocity is found from the above equation by dropping I.5, and the discharge is found by multiplying this mean velocity by the area of the cross-section. Hence the formulas for velocity and discharge are

$$
\begin{equation*}
v=\sqrt{\frac{2 g h d}{f l}} \quad q=\frac{1}{4} \pi \sqrt{\frac{2 g h d^{5}}{f l}} \tag{97}
\end{equation*}
$$

which for the English system of measures becomes

$$
\begin{equation*}
v=8.02 \sqrt{\frac{h d}{f l}} \quad q=6.30 \sqrt{\frac{h d^{5}}{f l}} \tag{97}
\end{equation*}
$$

From these expressions for $q$ the general and special formulas for computing the diameter of the pipe for a given discharge, length, and head are found to be

$$
\begin{equation*}
d^{5}=\frac{8}{\pi^{2} g} \frac{f l q^{2}}{h} \quad d=0.479\left(\frac{f l q^{2}}{h}\right)^{\frac{1}{5}} \tag{97}
\end{equation*}
$$

These equations show that for very long pipes the discharge varies directly as the $2 \frac{1}{2}$ power of the diameter, and inversely as the square root of the length.

In the above formulas, $d, h$, and $l$ are to be taken in feet, $q$ in cubic feet per second, and $f$ is to be found from Table $90 a$, an approximate value of $v$ being first obtained by taking $f$ as 0.02 . It should not be forgotten that computations of discharge or diameter from these formulas are liąble to uncertainty on account of imperfect knowledge regarding the friction factors. Especially when the velocities are lower than one or higher than fifteen feet
per second the results obtained can be regarded as rough estimates only. The value of $h$ in these formulas is really the friction-head $h^{\prime \prime}$, since in their deduction the other heads, $h^{\prime}, h^{\prime \prime \prime}$, and $h^{\prime \prime \prime \prime}$, have been neglected as insensible. Hence when the diameter $d$, the length $l$, the total head $h$, and the discharge $q$ have been measured for a long pipe the friction factor $f$ may be computed. In this manner much of the data was obtained from which Table $90 a$ has been compiled.

For circular orifices and for short tubes of equal length under the same head, the discharge varies as the square of the diameter. For pipes of equal length under a given head the discharges vary more rapidly owing to the influence of friction, for formula (97) ${ }_{2}$ shows that if $f$ be constant, $q$ varies as $d^{\frac{5}{2}}$. The relative discharging capacities of pipes hence vary approximately as the $2 \frac{1}{2}$ powers of their diameters. Thus, if two pipes of diameters $d_{1}$ and $d_{2}$ have same length and head, and if $q_{1}$ and $q_{2}$ be their discharges,

$$
q_{1} / q_{2}=d_{1}^{\frac{5}{2}} / d_{2}^{\frac{5}{2}} \quad \text { or } \quad q_{2}=\left(d_{2} / d_{1}\right)^{\frac{5}{2}} q_{1}
$$

For example, if there be two pipes of 6 and $I_{2}$ inches diameter, $d_{2} / d_{1}$ equals 2 and hence $q_{2}=5.7 q_{1}$, or the second pipe discharges nearly six times as much as the first. In a similar manner it can be shown that 32 pipes of 6 inches diameter have the same discharging capacity as I pipe 24 inches in diameter.

When the variation in the friction factor is taken into account, the formula gives

$$
q_{2}=q_{1}\left(d_{2} / d_{1}\right)^{\frac{5}{2}}\left(f_{1} / f_{2}\right)^{\frac{1}{2}}
$$

Now as the values of $f$ vary not only with the diameter but with the velocity, a solution cannot be made except in particular cases. For the above example let the velocity be about 3 feet per second; then from the table $f_{1}=0.023$ and $f_{2}=0.019$, and accordingly

$$
q_{2}=q_{1}(2)^{\frac{5}{2}}(\mathrm{I} .2)^{\frac{1}{2}}=6.2 q_{1}
$$

or the 12 -inch pipe discharges more than six times as much as the 6 -inch pipe.

Prob. 97. Compute the diameter required to deliver 15000 cubic feet per hour through a pipe 26500 .feet long under a head of 324.7 feet. If this quantity igcarried in two pipes of equal diameter, what should be their size?

## Art. 98. Piezometer Measurements

Let a piezometer tube be inserted into a pipe at any point $D_{1}$. at the distance $l_{1}$ from the reservoir measured along the pipe line. Let $A_{1} D_{1}$ be the vertical depth of this point below the water level of the reservoir ; then if the flow be stopped at the end $C$, the water rises in the tube to the point $A_{1}$. But when the flow occurs, the water level in the pie-
 zometer stands at some point $C_{1}$, and the pressurehead at $D_{1}$ is $h_{1}$, or $C_{1} D_{1}$ in the figure. The distance $A_{1} C_{1}$ then represents the velocity-head plus all the losses of head between $D_{1}$ and the reservoir. If no losses of head occur except at entrance and in friction, the value of $A_{1} C_{1}$ then is

$$
H_{1}=\frac{v^{2}}{2 g}+m \frac{v^{2}}{2 g}+f \frac{l_{1}}{d} \frac{v^{2}}{2 g}
$$

from which the piezometric height can be found when $v$ has been determined by direct measurement or by gaging.

For example, let the total length $l=3000$ feet, $d=6$ inches, $h=9$ feet, and $m=0.5$. Then, as in Art. 93, there is found $f=0.026$ and $v=1.917$ feet per second. The.position of the top of the piezometric column is then given by

$$
H_{1}=\left(\mathrm{I} .5+0.052 l_{1}\right) \times 0.05714
$$

and the height of that column above the pipe is

$$
h_{1}=\overline{A_{1} D_{1}}-H_{1}
$$

Thus if $l_{1}=1000$ feet, $H_{1}=3.06$ feet; and if $l_{1}=2000$ feet, $H_{1}=6.03$ feet. If the pipe is so laid that $A_{1} D_{1}$ is 9 feet, the cor-- responding pressure-heads are then 5.94 and 2.97 feet.

For a second piezometer inserted at $D_{2}$ at the distance $l_{2}$ from the entrance, the value $H_{2}$ is

$$
H_{2}=\frac{v^{2}}{2 g}+m \frac{v^{2}}{2 g}+f \frac{l^{2}}{\frac{v^{2}}{d}} \frac{v^{2}}{2 g}
$$

Subtracting from this the expression for $H_{1}$, there is found

$$
\begin{equation*}
H_{2}-H_{1}=f \frac{l_{2}-l_{1} v^{2}}{d 2 g} \tag{98}
\end{equation*}
$$

The second member of this formula is the head lost in friction in the length $l_{2}-l_{1}$ (Art. 90), and the first member is the difference of the piezometer elevations. Thus is again proved the principle of Art. 85 , that the difference of two piezometer elevations shows the head lost in the pipe between them; in Art. 85 the elevations $H_{1}$ and $H_{2}$ were measured upward from the datum plane, while here they have been measured downward from the water level in the reservoir.

By the help of this principle the velocity of flow in a pipe may be approximately determined. A line of levels is run between the points $D_{1}$ and $D_{2}$, which are selected so that no sharp curves occur between them, and thus the difference $H_{2}-H_{1}$ is found, while the length $l_{2}-l_{1}$ is ascertained by careful chaining. Then, from the above formula,

$$
\begin{equation*}
v=\sqrt{\frac{2 g\left(H_{2}-H_{1}\right) d}{f\left(l_{2}-l_{1}\right)}} \tag{98}
\end{equation*}
$$

from which $v$ can be computed by the help of the friction factors in Table $90 a$. For example, Stearns, in 1880 , made experiments on a conduit pipe 4 feet in diameter under different velocities of flow.* In experiment No. 2 the length $l_{2}-l_{1}$ was 1747.2 feet, and the difference of the piezometer levels was 1.243 feet. Assuming for $f$ the mean value 0.02 , and using 32.16 feet per second per second for $g$, the velocity was

$$
v^{0}=\sqrt{\frac{64.32 \times_{1.243} \times_{4}}{0.02 X_{1747}}}=3.0 \text { feet per second. }
$$

This velocity in the table of friction factors gives $f=0.015$ for a 4 -foot pipe. Hence, repeating the computation, there is found $v=3.50$ feet per second; it is accordingly uncertain whether the value of $f$ is 0.015 or 0.014 . If the latter value be used, there is found $v=3.62$ feet per second. The actual velocity, as determined by measurement of the water over a weir, was 3.738 feet

[^0]per second, which shows that the computation is in error about 4 percent.

In order that accurate results may be obtained with piezometers it is necessary, particularly under low pressure-heads, that the tubes be inserted into the pipe at right angles. If they be inclined with or against the current, the pressure-head $h_{1}$ will be greater or less than that due to the pressure at the mouth. Let $\theta$ be the angle between the direction of the flow and the inserted piezometer tube. Since the impulse in the direction of the current is proportional to the velocity-head (Art. 27), the component of this in the direction of the inserted tube tends to increase the normal pressure-height $h_{1}$ when $\theta$ is less than $90^{\circ}$ and to decrease it when $\theta$ is greater than $90^{\circ}$. Thus

$$
h_{0}=h_{1}+n \frac{v^{2}}{2 g} \cos \theta
$$

may be written as approximately applicable to the two cases in which $n$ is a coefficient


Fig. $98 b$. the value of which has not been ascertained. In this, if the tube be inserted normal to the pipe, $\theta=90^{\circ}$ and $h_{0}$ becomes $h_{1}$, the height. due to the static pressure in the pipe ; if $v=0$, the angle $\theta$ has no effect upon the piezometer readings. But if $\theta$ differs from $90^{\circ}$ by a small angle, the error in the reading may be large when the velocity in the pipe is high. Fig. $98 b$ illustrates the three cases.

The question as to the point from which the pressure-head should be measured deserves consideration. In the figures of preceding articles $h_{1}$ and $h_{2}$ have been estimated upward from the center of the pipe, and it is now to be shown that this is probably correct. Let Fig. 98 c represent a cross-section of a pipe to which are attached three piezometers as shown. If there be no velocity in the tube or pipe, the


Fig. 98 c.
water surface stands at the same level in each piezometer, and the mean pressure-head is certainly the distance of that level above the center of the cross-section. If the water in the pipe be in motion, probably the same would hold true. Referring to formula (75) ${ }_{1}$ and to Fig. $75 a$, it is also seen that if there be no velocity $h^{\prime}=h_{1}-h_{2}$, which cannot be true unless $h_{1}-h_{2}=0$, since there can be no loss of head in the transmission of static pressures; hence $h_{1}$ and $h_{\mathrm{q}}$ cannot be measured from the top of the section. In any event, since the piezometer heights represent the mean pressures, it appears that they should be reckoned upward from the center of the section. The piezometer couplings for hose devised by Freeman are arranged with connections on the top, bottom, and sides, as are also those used for the Venturi meter (Art. 38), and thus the results obtained correspond to mean pressures or pressure-heads. Even in cases where the two points of connection are so near together that the difference $H_{2}-H_{1}$, can be measured by a differential manometer (Art. 37), the method of connecting the tubes to the pipes should receive careful attention.

Prob. 98. At a point 500 feet from the reservoir, and 28 feet below its surface, a pressure gage reads i0.5 pounds per square inch; at a point 8500 feet from the reservoir and 280.5 feet below its surface, it reads 61 pounds per square inch. If the pipe is 12 inches in diameter, compute the discharge.

Art. 99. The Hydraulic Gradient
The hydraulic gradient is a line which connects the water levels in piezometers placed at intervals along the pipe ; or rather, it is the line to which the water levels would rise if piezometer tubes were inserted. In Fig. $98 a$ the line $B C$ is the hydraulic gradient, and it is now to be shown that for a pipe of uniform


Fig. 99a. size this is approximately a straight line. For a pipe discharging freely into the air, as in Fig. $98 a$, this line joins the outlet end with a point $B$ near the top of the reservoir. For a pipe with submerged discharge, as in Fig. $99 a$, it joins the lower water level with the point $B$.

Let $D_{1}$ be any point on the pipe distant $l_{1}$ from the reservoir,


[^0]:    * Transactions American Society of Civil Engineers, 1885, vol. 14, p. 4

