By equating the second members of these equations the critical head at which the nature of the flow changes is found to be about 0.6 D for all values of H between 0.1 and 3.0 feet. Practically, however, the exact point at which the change occurs cannot be exactly determined.

Prob. 87. Compute the flow from a vertical pipe 14 inches in diameter when the head above the top of the pipe, as measured by a Pitot tube, is 0.04 feet. Also compute the discharge when the head is 7.6 feet.

Art. 88. Computations in Metric Measures

Nearly all the formulas of this chapter are rational and may be used in all systems of measures. In the metric system lengths are to be taken in meters, areas in square meters, velocities in meters per second, discharges in cubic meters per second, and using for the acceleration constants the values given in Table 9*c*.

(Art. 83) The coefficients of discharge and velocity for smooth fire nozzles 2.0, 2.5, 3.0, and 3.5 centimeters in diameter are 0.983, 0.972, 0.973, and 0.959, respectively. In using the formula $(83)_2$ the values of d and h_1 should be taken in meters, but in finding the ratio D/d the values of D and d may be in centimeters or any other convenient unit. The constant g being 9.80 meters per second, the discharge q will be in cubic meters per second. When it is desired to use the gage reading p_1 in kilograms per square centimeter and to take D in centimeters, the formula

$$q = 65.96\, c_1 D^2 \sqrt{\frac{p_1}{1 - c_1^2(D/d)^4}}$$

may be used for finding the discharge in liters per minute.

Prob. 88*a*. Compute the loss of head which occurs when a pipe, discharging 18.5 cubic meters per second, suddenly enlarges in diameter from 1.25 to 1.50 meters.

Prob. 88*b*. Find the coefficient of discharge for a tube 8 centimeters in diameter when the flow under a head of 4 meters is 18.37 cubic meters in 5 minutes and 15 seconds.

Prob. 88*c*. Compute the discharge from a smooth nozzle 2.5 centimeters in diameter, attached to a hose 7.5 centimeters in diameter, when the pressure at the entrance is 5.2 kilograms per square centimeter.

CHAPTER 8

FLOW OF WATER THROUGH PIPES

Art. 89. Fundamental Ideas

Pipes made of clay were used in very early times for conveying water. Pliny says that they were two digits (0.73 inches) in thickness, that the joints were filled with lime macerated in oil, and that a slope of at least one-fourth of an inch in a hundred feet was necessary in order to insure the free flow of water.* The Romans also used lead pipes for conveying water from their aqueducts to small reservoirs and from the latter to their houses. Frontinus gives a list of twenty-five standard sizes of pipes,† varying in diameter from 0.9 to 9 inches, which were made by curving a sheet of lead about ten feet long and soldering the longitudinal joint. The Romans had confused ideas of the laws of flow in pipes, their method of water measurement being by the area of cross-section, with little attention to the head or pressure. They knew that the areas of circles varied as the squares of the diameters, and their unit of water measurement was the quinaria, this being a pipe $1\frac{1}{4}$ digits in diameter; then the denaria pipe, which had a diameter of $2\frac{1}{2}$ digits, was supposed to deliver 4 quinarias of water.

In modern times lead pipes have also been used for house service, but these are now largely superseded by either iron pipes or iron pipes lined with lead or tin. For the mains of city water supplies cast-iron pipes are most common, and since 1890 steel-riveted pipes have come into use for large sizes. Lap-welded wrought-iron or steel pipes are used in some cases where the pressure is very high, and large wooden stave pipes are in use in the western part of the United States.

* Natural History, book 31, chapter 31, line 5.

† Herschel, Water Supply of the City of Rome (Boston, 1899), p. 36.

The simplest case of the flow of water through a pipe is that where the diameter of the pipe is constant and the discharge occurs entirely at the open end. This case will be discussed in Arts. 90–99, and afterwards will be considered the cases of pipes of varying diameter, a pipe with a nozzle at the end, and pipes with branches. Most of the principles governing the simple case apply with slight modification to the more complex ones. Pipes used in engineering practice range in diameter from ½ inch up to 10 feet or more.

The phenomena of flow for this common case are apparently simple. The water from the reservoir, as it enters the pipe, meets with more or less resistance, depending upon the manner of connecting, as in tubes (Art. 80). Resistances of friction and cohe-

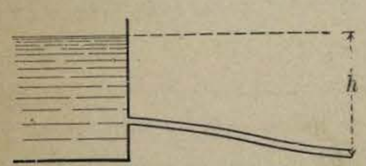

Fig. 89a.

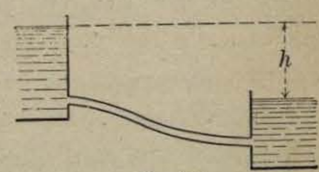

Fig. 89b.

sion must then be overcome along the interior surface, so that the discharge at the end is much smaller than in the tube (Art. 84). When the flow becomes steady, the pipe is entirely filled throughout its length; and hence the mean velocity at any section is the same as that at the end, since the size is uniform. This velocity is found to decrease as the length of the pipe increases, other things being equal, and becomes very small for great lengths, which shows that nearly all the head has been lost in overcoming the resistances. The length of the pipe is measured along its axis, following all the curves, if there be any. The velocity considered is the mean velocity, which is equal to the discharge divided by the area of the cross-section of the pipe. The actual velocities in the cross-section are greater than this mean near the center and less than it near the interior surface of the pipe, the law of distribution being that explained in Art. 86.

The object of the discussion of flow in pipes is to enable the discharge which will occur under given conditions to be deter-

mined, or to ascertain the proper size which a pipe should have in order to deliver a given discharge. The subject cannot, however, be developed with the definiteness which characterizes the flow from orifices and weirs, partly because the condition of the interior surface of the pipe greatly modifies the discharge, partly because of the lack of experimental data, and partly on account of defective theoretical knowledge regarding the laws of flow. In orifices and weirs errors of two or three percent may be regarded as large with careful work; in pipes such errors are common, and are generally exceeded in most practical investigations. It fortunately happens, however, that in most cases of the design of systems of pipes errors of five and ten percent are not important, although they are of course to be avoided if possible, or, if not avoided, they should occur on the side of safety.

The head which causes the flow is the difference in level from the surface of the water in the reservoir to the center of the end, when the discharge occurs freely into the air as in Fig. 89*a*. If h be this head, and W the weight of water discharged per second, the theoretic potential energy per second is Wh; and if v be the actual mean velocity of discharge, the kinetic energy of the discharge is $W \cdot v^2/2g$. The difference between these is the energy which has been transformed into heat in overcoming the resistances. Thus the total head is h, the velocity-head of the outflowing stream is $v^2/2g$, and the lost head is $h - v^2/2g$. If the lower end of the pipe is submerged, as in Fig. 89*b*, the head h is the difference in elevation between the two water levels.

The total loss of head in a straight pipe of uniform size consists of two parts, as in a long tube (Art. 84). First, there is a loss of head h' due to entrance, which is the same as in a short cylindrical tube, and secondly there is a loss of head h'' due to the frictional resistance of the interior surface. The loss of head at entrance is always less than the velocity-head and in this chapter it will be expressed by the formula

$$h' = m\frac{v^2}{2g} \qquad (89)_1$$

in which m is 0.93 for the inward projecting pipe, 0.49 for the standard end, and 0 for a perfect mouthpiece, as shown in Art. 84. When the condition of the end is not specified, the value used for m will be 0.5, which supposes that the arrangement is like the standard tube, or nearly so. For short pipes, however, it may be necessary to consider the particular condition of the end, and then m is to be computed from

$$m = (1/c_1)^2 - 1 \qquad (89)_2$$

in which the coefficient c_1 is to be selected from the evidence presented in the last chapter.

It should be noted that the loss of head at entrance is very small for long pipes. For example, it is proved by actual gagings that a clean cast-iron pipe 10 000 feet long and 1 foot in diameter discharges about $4\frac{1}{4}$ cubic feet per second under a head of 100 feet. The mean velocity then is, if q be the discharge and a the area of the cross-section,

$$v = \frac{q}{a} = \frac{4.25}{0.7854} = 5.41 \text{ feet per second,}$$

and the probable loss of head at entrance hence is

$$h' = 0.5 \times 0.01555 \times 5.41^2 = 0.23 \text{ feet,}$$

or only one-fourth of one per cent of the total head. In this case the effective velocity-head of the issuing stream is only 0.45 feet, which shows that the total loss of head is 99.55 feet, of which 99.32 feet are lost in friction.

Prob. 89. Under a head of 20 feet a pipe 1 inch in diameter and 100 feet long discharges 15 gallons per minute. Compute the loss of head at entrance.

Art. 90. Loss of Head in Friction

The loss of head due to the resisting friction of the interior surface of a pipe is usually large, and in long pipes it becomes very great, so that the discharge is only a small percentage of that due to the head. Let h be the total head on the end of the pipe where the discharge occurs, $v^2/2g$ the velocity-head of the issuing stream, h' the head lost at entrance, and h'' the head lost in friction. Then if the pipe is straight, so that no other losses of head occur,

$$h = h' + h'' + \frac{v^2}{2g}$$

Inserting for the entrance-head h' its value from Art. 89, this equation becomes

$$h = m\frac{v^2}{2g} + h'' + \frac{v^2}{2g}$$

which is a fundamental formula for the discussion of flow in straight pipes of uniform size.

The head lost in friction may be determined for a particular case by measuring the head h, the area a of the cross-section of the pipe, and the discharge per second q. Then q divided by a gives the mean velocity v, and from the above equation, inserting for m its value from $(89)_2$, there is found

$$h'' = h - \frac{1}{c_1^2} \cdot \frac{v^2}{2g}$$

which serves to compute h'', the value of c_1 being first selected according to the condition of the end. This method is not a good one for short pipes because of the uncertainty regarding the coefficient c_1 (Art. 84), but for long pipes it gives precise results.

Another method, and the one most generally employed, is by the use of piezometers (Art. 85). A portion of the pipe being selected which is free from sharp curves, two piezometer tubes are inserted into which the water rises, or the pressure-heads are measured by gages (Art. 36). The difference of level of the water surfaces in the piezometer tubes is then the head lost in the pipe between them (Art. 85), and this loss is caused by friction alone if the pipe be straight and of uniform size.

By these methods many observations have been made upon pipes of different sizes and lengths under different velocities of flow, and the discussion of these has enabled the approximate laws to be deduced which govern the loss of head in friction, and tables to be prepared for practical use. These laws are:

1. The loss of head in friction is directly proportional to the length of the pipe.
2. It is inversely proportional to the diameter of the pipe.
3. It increases nearly as the square of the velocity.
4. It is independent of the pressure of the water.
5. It increases with the roughness of the interior surface.

These five laws may be expressed by the formula

$$h'' = f\frac{l}{d}\frac{v^2}{2g} \qquad (90)$$

in which l is the length of the pipe, d its diameter, f is an abstract number which depends upon the degree of roughness of the surface, and $v^2/2g$ is the velocity-head due to the mean velocity.

This formula may be justified by reasonings based on the assumption that what has been called the loss in friction is really caused by impact of the particles of water against each other. Fig. 90 represents a pipe with the roughness of its surface enormously exaggerated and imperfectly shows the disturbances thereby caused. As any particle of water strikes a protuberance on the surface, it is deflected and its velocity diminished, and then other particles of water in striking against it also undergo a diminution of velocity.

Fig. 90.

Now in this case of impact the resisting force F acting over each square unit of the surface is to be regarded as varying with the square of the velocity (Arts. 27 and 76). The total resisting friction for a pipe of length l and diameter d is then πdlF, and the work lost in one second is $dl\pi Fv$. Let W be the weight of water discharged in one second, then Wh'' is also the energy lost in one second. But $W = wq$, if w be the weight of a cubic unit of water and q the discharge per second, and the value of q is $\frac{1}{4}\pi d^2v$. Then, equating the two expressions for the lost energy, and replacing F by Cv^2 where C is a constant, there results

$$h'' = \frac{4}{w}\frac{l}{d}F = \frac{4C}{w}\frac{l}{d}v^2.$$

Now C must increase with the roughness of the surface and hence this expression is the same in form as (90), and it agrees with the five laws of experience.

Values of h'' having been found by experiments, in the manner described above, values of the quantity f can be computed. In this way it has been found that f varies not only with the roughness of the interior surface of the pipe, but also with its diameter, and with the velocity of flow. From the discussions of Fanning, Smith, and others, the mean values of f given in Table 90*a* have been compiled, which are applicable to clean cast-iron and wrought-iron pipes, either smooth or coated with coal-tar, and laid with close joints.

Table 90*a*. Friction Factors for Clean Iron Pipes

Diameter in Feet	Velocity in Feet per Second						
	1	2	3	4	6	10	15
0.05	0.047	0.041	0.037	0.034	0.031	0.029	0.028
0.1	.038	.032	.030	.028	.026	.024	.023
0.25	.032	.028	.026	.025	.024	.022	.021
0.5	.028	.026	.025	.023	.022	.020	.019
0.75	.026	.025	.024	.022	.021	.019	.018
1.	.025	.024	.023	.022	.020	.018	.017
1.25	.024	.023	.022	.021	.019	.017	.016
1.5	.023	.022	.021	.020	.018	.016	.015
1.75	.022	.021	.020	.018	.017	.015	.014
2.	.021	.020	.019	.017	.016	.014	.013
2.5	.020	.019	.018	.016	.015	.013	.012
3.	.019	.018	.016	.015	.014	.013	.012
3.5	.018	.017	.016	.014	.013	.012	
4.	.017	.016	.015	.013	.012	.011	
5.	.016	.015	.014	.013	.012		
6.	.015	.014	.013	.012	.011		

The quantity f may be called the friction factor, and the table shows that its value ranges from 0.05 to 0.01 for new clean iron pipes. A rough mean value, often used, is

$$\text{Friction factor } f = 0.02$$

It is seen that the tabular values of f decrease both when the diameter and when the velocity increases, and that they vary most rapidly for small pipes and low velocities. The probable error of a tabular value of f is about one unit in the third decimal place, which is equivalent to an uncertainty of 10 percent when $f = 0.011$, and to 5 percent when $f = 0.021$. The effect of this is to render computed values of h'' liable to the same uncertainties; but the effect upon computed velocities and discharges is much less, as will be seen in Art. 93.

To determine, therefore, the probable loss of head in friction, the velocity v must be known, and f is taken from Table 90a for the given diameter of pipes. The formula (90) then gives the probable loss of head in friction. For example, let l = 10 000 feet, d = 1 foot, v = 5.41 feet per second. Then from Table 90a the factor f is 0.021, and

$$h'' = 0.021 \times \frac{10\,000}{1} \times 0.455 = 95.5 \text{ feet,}$$

which is to be regarded as an approximate value, liable to an uncertainty of 5 percent.

TABLE 90b. FRICTION HEAD FOR 100 FEET OF CLEAN IRON PIPE

Diameter in Feet	Velocity in Feet per Second						
	1	2	3	4	6	10	15
	Feet	Feet	Feet	Feet	Feet	Feet	Feet
0.05	1.46	5.10	10.3	16.9	34.7		
0.1	0.59	1.99	4.20	6.97	14.5	37.3	
0.25	.20	0.70	1.46	2.40	5.37	13.7	29.4
0.5	.09	.32	0.70	1.14	2.46	6.22	13.3
0.75	.05	.21	.45	0.73	1.57	3.94	8.40
1.	.04	.15	.32	.55	1.12	2.80	5.95
1.25	.03	.11	.25	.42	0.85	2.11	4.48
1.5	.02	.09	.20	.33	.67	1.66	3.50
1.75	.02	.07	.16	.26	.54	1.33	2.80
2.	.02	.06	.13	.21	.45	1.09	2.27
2.5	.01	.05	.10	.16	.34	0.81	1.68
3.	.01	.04	.07	.12	.26	.67	1.40
3.5	.01	.03	.06	.10	.21	.53	
4.		.02	.05	.08	.17	.42	
5.		.02	.04	.06	.13		
6.		.01	.03	.05	.10		

From Table 90a and formula (90) the losses of head in friction for 100 feet of clean cast-iron pipe have been computed for different values of d and f and are given in Table 90b, from which approximate computations may be rapidly made. Thus, for the above data, by interpolation in Table 90b, there is found 0.952 feet for the loss in 100 feet of pipe, and then for 10 000 feet the loss of head is 95.2 feet.

Prob. 90. Determine the actual loss of head in friction from the following experiment: l = 60 feet, h = 8.33 feet, d = 0.0878 feet, q = 0.03224 cubic feet per second, and c = 0.8. Compute the probable loss for the same data from formula (90) and also from Table 90b.

ART. 91. LOSS OF HEAD IN CURVATURE

Thus far the pipe has been regarded as straight, so that no losses of head occur except at entrance and in friction. But when the pipe is laid on a curve, the water suffers a change in direction whereby an increase of pressure is produced in the direction of the radius of the curve and away from its center (Art. 156). This increase in pressure causes eddying motions of the water, from which impact results and energy is transformed into heat. The total loss of head h''' due to any curve evidently increases with its length, and should be greater for a small pipe than for a large one. Hence the loss of head due to the curvature of a pipe may be written

$$h''' = f_1 \frac{l}{d} \frac{v^2}{2g} \qquad (91)_1$$

in which l is the length of the curve, d the diameter of the pipe, v the mean velocity of flow, and f_1 is an abstract number called the curve factor, that depends upon the ratio of the radius of the curve to the diameter of the pipe. Let R be the radius of the circle in which the center line of the pipe is laid. Then, if R is infinity, the pipe is straight and $f_1 = 0$; but as the ratio R/d decreases, the value of f_1 increases.

There are few experiments from which to determine the values of f_1. Weisbach, about 1850, from a discussion of his own experiments and those of Castel, deduced a formula for the value of $f_1 l/d$ for curves of one-fourth of a circle,* and from this the following values of the curve factor f_1 have been computed:

for R/d =	20	10	5	3	2	1.5	1.0
f_1 =	0.004	0.008	0.016	0.030	0.047	0.072	0.184

These values of f_1 are applicable only to small smooth iron pipes where the entire curve is without joints, since most of the pipes

* Die Experimentale Hydraulik (Freiberg, 1855), p. 159. Mechanics of Engineering (New York, 1870), vol. 1, p. 898.

on which the above experiments were made were probably of this kind.

Freeman, in 1889, made measurements of the loss of head in fire hose 2.49 and 2.64 inches in diameter, and the curves were complete circles of 2, 3, and 4 feet radius.* From the results given for the smaller hose the following values of the curve factor f_1 have been found:

for $R/d =$	19.2	14.4	9.6
$f_1 =$	0.0033	0.0034	0.0048

while for the larger hose the values are

for $R/d =$	16.2	13.6	8.1
$f_1 =$	0.0036	0.0046	0.0045

These values are in fair agreement with those given above for the small iron pipes.

Williams, Hubbell, and Fenkell, in 1898 and 1899, made measurements in Detroit on cast-iron water mains having curves of 90°. From their results for a 30-inch pipe the values of the curve factor f_1 have been computed and are found to be as follows:

for $R/d =$	24	16	10	6	4	2.4
$f_1 =$	0.036	0.037	0.047	0.060	0.062	0.072

while from their work on a 12-inch pipe the values are

for $R/d =$	4	3	2	1
$f_1 =$	0.05	0.06	0.06	0.20

Of these values, those derived from the larger pipe are the most reliable, and it is seen that they are much greater than the values deduced from Weisbach's investigations on small pipes. Probably some of this increase is due to the circumstance that the curves had rougher surfaces and that the joints were nearer together than on the straight portions. These experiments † were made with the Pitot tube in the manner explained in Arts. 41 and 86. They show that the law of distribution of the velocities in the cross-section is quite different from that for a straight pipe,

* Transactions American Society of Civil Engineers, 1889, vol. 21, p. 363.
† Transactions American Society of Civil Engineers, 1902, vol. 47.

the maximum velocity being not at the center, but between the center and the outside of the curve.

From the experiments of Schoder,* on 6-inch pipe and bends of 90°, the following values of f have been computed for velocities of 5 and 16 feet per second:

	for $R/d =$	20	15	10	6	5	2
$v = 5$,	$f_1 =$	0.008	0.004	0.010	0.020	0.018	0.049
$v = 16$,	$f_1 =$	0.008	0.009	0.011	0.021	0.022	0.059

The data given by Davis,* from his experiments on pipe about $2\frac{1}{16}$ inches in diameter for bends of 90°, enable the following values of f_1 to be computed for velocities of 5 and 15 feet per second:

	for $R/d =$	10	6	5	4	2	1
$v = 5$,	$f_1 =$	0.023	0.024	0.027	0.032	0.081	0.323
$v = 15$,	$f_1 =$	0.027	0.051	0.052	0.058	0.144	0.394

From the experiments of Brightmore,† on pipes 4 inches in diameter and for bends of 90°, the values of f_1 given below have been computed for velocities of 5 and 10 feet per second:

	for $R/d =$	10	6	5	4	2	1
$v = 5$,	$f_1 =$	0.013	0.033	0.034	0.036	0.105	0.406
$v = 10$,	$f_1 =$	0.013	0.034	0.040	0.046	0.127	0.365

While the above values of f_1 are few in number, and not wholly in accord, yet they may serve as a basis for roughly estimating the loss of head due to curvature. For example, let there be two curves of 24 and 16 feet radius in a pipe 2 feet in diameter, each curve being a quadrant of a circle. The ratios R/d are 12 and 8, and the values of f_1, taken from those deduced above from the large Detroit pipe, are 0.044 and 0.053. The lengths of the curves are 37.7 and 25.1 feet, and then from $(91)_1$

$$h''' = 0.044 \frac{37.7}{2} \frac{v^2}{2g} = 0.83 \frac{v^2}{2g}$$

$$h''' = 0.053 \frac{25.1}{2} \frac{v^2}{2g} = 0.66 \frac{v^2}{2g}$$

* Transactions American Society of Civil Engineers, vol. 52.
† Proceedings Institution of Civil Engineers, vol. 169.

are the losses of head for the two cases. Here it is seen that the easier curve gives the greater loss of head. By the use of the values of f_1 deduced from Weisbach's investigation, the loss of head is much smaller and the sharper curve gives the greater loss of head, since the coefficients of the velocity-head are found to be 0.13 and 0.14 instead of 0.83 and 0.66. The subject of losses in curves is, indeed, in an uncertain state, since sufficient experiments have not been made either to definitely establish the validity of $(91)_1$ or to determine authoritative values of the curve factor f_1. Probably it will be found that f_1 varies with the diameter d as well as with the ratio R/d.

When there are several curves in a pipe line, the value of $f_1(l/d)$ for each curve is to be found and then these are to be added in order to find the total loss of head. Thus, in general,

$$h''' = m_1 \frac{v^2}{2g} \qquad (91)_2$$

is the total loss of head, in which m_1 represents the sum of the values of $f_1(l/d)$ for all the curves. It must be remembered, however, that this loss of head is occasioned by the fact that the pipe is curved and that it is to be added to the loss caused by friction along the entire length of the pipe. In other words the curve factor f_1 does not include the friction factor f.

The lost head due to curvature in a pipe line is usually low compared with that lost in friction, since the number of curves is usually made as small as possible. For example, take a pipe 1000 feet long and 3 inches in diameter, which has ten curves, five being of 90° and 6 inches radius and five being of 57°.3 and 5 feet radius. From (90), using 0.02 for the mean friction factor, the loss of head in friction is $80\, v^2/2g$. From $(91)_1$, using the curve factors deduced from Weisbach, the loss of head for the five sharp curves is $0.74\, v^2/2g$, and that for the five easy curves is $0.4\, v^2/2g$.

Prob. 91. If the central angle of a curve of 18 inches radius is 57°.3, what is the length of the curve? If a hose, $2\frac{1}{2}$ inches in diameter, is laid on this curve, compute the loss in head due to curvature when the velocity in the hose is 30 feet per second and also when it is 15 feet per second.

Art. 92. Other Losses of Head

Thus far the cross-section of the pipe has been supposed to be constant, so that no losses of head occur except at entrance (Art. 89), in friction (Art. 90), and in curvature (Art. 91). But if the pipe contains valves, or has obstructions in its cross-section, or is of different diameters, other losses occur which are now to be considered.

The figures show three kinds of valves for regulating the flow in pipes: A being a valve consisting of a vertical sliding-gate, B a cock-valve formed by two rotating segments, and C a throttle-valve or circular disk which moves like a damper in a stovepipe.

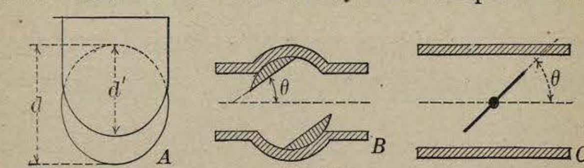

Fig. 92.

The loss of head due to these may be very large when they are sufficiently closed so as to cause a sudden change in velocity. It may be expressed by

$$h'''' = m \frac{v^2}{2g}$$

in which m has the following values, as determined by Weisbach from his experiments on pipes of small diameter.* For the gate-valve let d' be the vertical distance that the gate is lowered below the top of the pipe; then

for $d'/d =$	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$
$m =$	0.0	0.07	0.26	0.81	2.1	5.5	17	98

For the cock-valve let θ be the angle through which it is turned, as shown at B in Fig. 92; then

for $\theta =$	0°	10°	20°	30°	40°	50°	55°	60°	65°
$m =$	0°	0.29	1.6	5.5	17	53	106	206	486

In like manner, for the throttle-valve the coefficients are:

for $\theta =$	5°	10°	20°	30°	40°	50°	60°	65°	70°
$m =$	0.24	0.52	1.5	3.9	11	33	118	256	750

* Mechanics of Engineering, vol. 1, Coxe's translation, p. 902.