that is, the lost head is equal to the difference in level of the water surfaces in the piezometer tubes plus the differences of the velocity-heads. When the pipe is of the same size at the two sections, the velocities v_1 and v_2 are equal when the flow is uniform, and the lost head is simply

$$h' = H_1 - H_2 \qquad (85)_4$$

Piezometers or pressure gages hence furnish a very convenient method of determining the head lost in friction in a pipe of uniform size. For a pipe of varying section the velocities v_1 and v_2 must also be known, in order to use $(85)_3$ for finding the lost head.

Prob. 85. A large Venturi water meter placed in a pipe of 57.823 square feet cross-section had an area of 7.047 square feet at the throat. When the discharge was 54.02 cubic feet per second, the elevations of the water levels in the piezometers at a_1 and a_2 in Fig. 38*a* were 99.858 and 98.951 feet. Compute the loss of head between the two sections.

ART. 86. VELOCITIES IN A CROSS-SECTION

Thus far the velocity has been regarded as uniform over the cross-section of the tube or pipe. On account of the roughness of the surface, however, the velocity along the surface is always smaller than that near the middle of the cross-section. There appears to be no theoretical method of finding the law which connects the velocity of a filament with its distance from the center of the pipe, and yet it is probable that such a law exists. The mean velocity is evidently greater than the velocity at the surface and less than the velocity at the middle, and if the position of a filament were known whose velocity is the same as the mean velocity, a Pitot tube (Art. 41) with its tip at that position would directly measure the mean velocity.

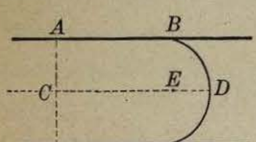

Fig. 86*a*.

Let Fig. 86*a* be a longitudinal section of a pipe, and let AB be laid off to represent the surface velocity v_s and CD to represent the central velocity v_c. Then the velocity v at any distance y from the axis will be an abscissa parallel to the axis and limited by the line AC and the curve BD. Suppose this curve to be a parabola whose equation is $y^2 = mx$, the origin being at D and x measured toward the left. When y is equal to the radius of the pipe r, the value of x is $v_c - v_s$ and hence $m = r^2/(v_c - v_s)$. The velocity v_y at the distance y above the axis is $v_c - x$, and accordingly

$$v_y = v_c - (v_c - v_s)\, y^2/r^2 \qquad (86)_1$$

It thus is seen that the velocity at any distance from the axis cannot be found unless the surface and central velocities are known. The position of the filament having the same velocity as the mean velocity v can, however, be determined, since the mean velocity is the mean length of the solid of revolution whose section is shown by the broken lines. This solid consists of a cylinder having the volume $\pi r^2 v_s$ and a paraboloid having the volume $\frac{1}{2}\pi r^2(v_c - v_s)$, and the sum of these is $\frac{1}{2}\pi r^2(v_c + v_s)$. Dividing this by the area of the cross-section gives $\frac{1}{2}(v_c + v_s)$ as the value of the mean velocity, and inserting this for v_y in the above equation there is found $y = 0.71r$ for the ordinate of a filament whose velocity is the same as mean velocity v. If the parabolic curve gives the true law of variation of velocity, a Pitot tube with its tip placed $0.29r$ below the top of the pipe would measure the mean velocity directly.

The first measurements of velocities of filaments were made by Freeman in 1888 with the Pitot tube.* They were on jets issuing from fire nozzles and also from a $1\frac{1}{8}$-inch tube under high velocities. For smooth nozzles the velocities were practically constant for a distance of $0.6r$ from the center, and then rapidly decreased, and the ratio of the surface velocity to the central velocity was about 0.77. For the pipe the velocities decreased quickly near the center, but more rapidly toward the surface. The velocity curve for the nozzle lies outside and that for the pipe lies within the parabolic curve represented by the equation $(86)_1$.

Bazin made experiments in 1893 on jets from standard orifices, using also the Pitot tube.† He found the velocities near the center to be smaller than others within $0.2r$ of the surface. Thus

* Transactions American Society of Civil Engineers, 1889, vol. 21, p. 412.

† Experiments on the Contraction of the Liquid Vein. Trautwine's translation, New York, 1896.

if $v_y = c\sqrt{2gh}$, the following are some of his values of c for a vertical circular and a vertical square orifice, h being always the head on the center.

$r =$	+0.8	+0.6	+0.2	0.0	−0.2	−0.6	−0.8
$c =$	0.68	0.64	0.62	0.63	0.64	0.72	0.86
$c =$	0.71	0.67	0.64	0.64	0.65	0.71	0.82

These are for velocities in the plane of the orifice, and he found similar variations for a section of the jet at a distance from the orifice of about one-half its diameter.

Judd and King,* in their experiments on orifices (Art. 45), traversed the jets with a Pitot tube and found that at the contracted section the velocity in all parts of the jet was uniform.

Cole, in 1897, made measurements of velocities in pipes,† using the Pitot tube with a differential gage (Art. 37). For pipes 4, 6, and 12 inches in diameter he found the ratio of the mean velocity to the center velocity to range from 0.91 to 1.01, while for a 16-inch pipe he found it to range from 0.83 to 0.86. His velocity curves show that the surface velocity was 60 percent or more of the center velocity.

Williams, Hubbell, and Fenkell, in 1899, made numerous measurements of velocities in water mains with the Pitot tube, and arrived at the conclusions that the ratio of the mean velocity to the central velocity was about 0.84, and that the surface velocity was about one-half the central velocity.‡ These ratios agree with an ellipse better than with a parabola. Let the curve BD in Fig. 86*a* be an ellipse having the semi-axes ED and BE, the ellipse being tangent to the pipe surface at B. As before, let AB represent the surface velocity v_s and CD the central velocity v_c; then ED is $v_c - v_s$ and BE is the radius r. The equation of the ellipse with respect to E as an origin is

$$(v_c - v_s)^2 y^2 + r^2 x^2 = (v_c - v_s)^2 r^2$$

* Engineering News, Sept. 27, 1906.

† Transactions American Society of Civil Engineers, 1902, vol. 47, p. 276.

‡ Transactions American Society of Civil Engineers, 1902, vol. 47, p. 63.

in which x is measured toward the right and y upward. The velocity v_y at any distance y from the axis CD is $v_s + x$, and accordingly

$$v_y = v_s + (v_c - v_s)\sqrt{1 - y^2/r^2} \qquad (86)_2$$

Now the mean velocity is the mean length of the solid of revolution formed by the cylinder whose volume is $\pi r^2 v_s$ and the semi-ellipsoid whose volume is $\frac{2}{3}\pi r^2(v_c - v_s)$. The volume of the solid is hence $\pi r^2(\frac{2}{3}v_c + \frac{1}{3}v_s)$ and the mean velocity is $\frac{2}{3}v_c + \frac{1}{3}v_s$. Inserting this for v_y in $(86)_2$, there is found $y = 0.75r$ for the position of the filament having the same velocity as the mean velocity, while the parabola gave $y = 0.71r$. If v_s is one-half of v_c, the mean velocity under the elliptic law is $\frac{2}{3}v_c + \frac{1}{3}v_s = 0.83v_c$, while under the parabolic law it is $\frac{1}{2}v_c + \frac{1}{2}v_s = 0.75v_c$.

Much irregularity is observed in velocity curves plotted from actual measurements, this being due to pulsations in the water and to errors of observations. The above experiments were on pipes having diameters of 12, 16, 30, and 42 inches and under velocities ranging from 0.5 to 7.5 feet per second; and they are a very valuable addition to the knowledge of this subject. The conclusion that v_s is one-half of v_c is, however, one that appears to be liable to some doubt. The conclusion that the mean velocity v is about $0.84v_c$ appears well established, and a Pitot tube with its tip at the center of the pipe will hence determine a fair value of the mean velocity, several readings being taken in order to eliminate errors of observation.

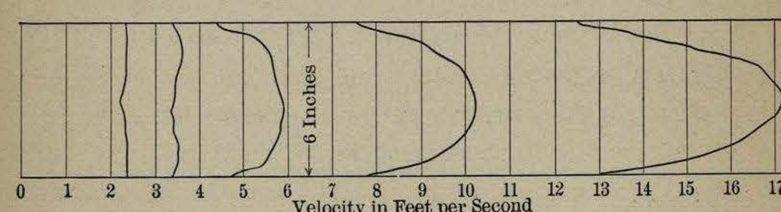

Fig. 86*b*.

In the case of fountain flow (Art. 87), Lawrence and Braunworth * found that the velocities in the cross-section depend on whether or not the flow out of the top of the pipe occurs as in a

* Transactions American Society of Civil Engineers, vol. 57.

jet or as over a weir. Thus, in Fig. 86*b* the velocity curves for a vertical 6-inch cast-iron pipe are shown for velocities ranging from 2 to 17 feet per second. These velocities were obtained from the expression $v = \sqrt{2gh}$, where h was measured by a Pitot tube.

Prob. 86. Let $v_s = 3$ and $v_c = 6$ feet per second. Plot the parabola from formula $(86)_1$ and the ellipse from formula $(86)_2$.

Art. 87. Fountain Flow

When a stream of water rises and flows out of the top of a vertical pipe of diameter D, the flow, if the head H to which it rises above the top of the pipe is small, is practically the same as that over a thin-edged circular weir. As H increases there comes a transition period during which the character of the flow resembles neither that over a circular weir nor that of a jet. Lawrence and Braunworth * experimented on the fountain flow of water from pipes 2, 4, 6, 9, and 12 inches in diameter. They measured the heads H both by means of a Pitot tube and by sighting on two rods and across the top of the pipe. The water discharged during the experiments was measured volumetrically. From the discussion of these experiments the following formulas were deduced:

$$q = 8.80\,D^{1.25}H^{1.35} \quad \text{and} \quad q = 5.57\,D^{1.99}H^{0.53}$$

the first being for weir and the second for jet flow. Here D and H are in feet and q in cubic feet per second, H being measured by means of sighting across the top of the flow as above described.

For cases in which the head H is measured with a Pitot tube the formulas deduced were

$$q = 8.80\,D^{1.29}H^{1.29} \quad \text{and} \quad q = 5.84\,D^{2.025}H^{0.53}$$

the first of these, as before, being applicable to weir and the second to jet flow.

In general the average results given by these formulas are correct within 3 percent for the jet condition, while for the condition of the weir flow using the Pitot tube for the measurement of the head the average accuracy is within 4 percent. Single

* Transactions American Society of Civil Engineers, vol. 57, p. 209.

measurements cannot be depended upon closer than to within about twice the above limits of accuracy.

In the following table are shown the computed discharges in cubic feet per second for various sizes of pipes under various heads, the heads being observed by means of a Pitot tube.

Table 87. Discharges in Cubic Feet per Second for Fountain Flow from Vertical Pipes

Head in Feet	Diameter of Pipe in Inches							
	1	2	4	6	8	12	18	24
0.02			0.014	0.023	0.033	0.055	0.092	0.134
0.04		0.014	0.035	0.055	0.080	0.133	0.223	0.324
0.06		0.023	0.059	0.093	0.136	0.227	0.380	0.549
0.08	0.010	0.032	0.085	0.136	0.197	0.330	0.550	0.802
0.10	0.011	0.039	0.114	0.180	0.262	0.439	0.731	1.08
0.15	0.014	0.054	0.184	0.307	0.442	0.742	1.28	1.84
0.20	0.016	0.065	0.243	0.438	0.645	1.08	1.87	2.66
0.30	0.020	0.082	0.325	0.662	1.03	1.81	3.12	4.45
0.40	0.023	0.096	0.385	0.832	1.36	2.66	4.50	6.50
0.50	0.026	0.108	0.435	0.975	1.65	3.35	5.98	8.62
0.75	0.033	0.133	0.539	1.23	2.18	4.73	9.50	14.80
1.00	0.038	0.155	0.627	1.43	2.57	5.73	12.27	20.20
1.50	0.047	0.192	0.778	1.77	3.18	7.22	16.25	28.08
2.00	0.055	0.224	0.906	2.06	3.71	8.41	19.15	33.75
3.00	0.068	0.278	1.16	2.56	4.60	10.42	23.80	42.55
4.00	0.079	0.324	1.32	2.98	5.36	12.15	27.70	49.60
5.00	0.089	0.365	1.47	3.36	6.03	13.67	31.20	55.80
6.00	0.098	0.401	1.62	3.70	6.64	15.05	34.40	61.40
7.00	0.107	0.435	1.76	4.02	7.20	16.34	37.30	66.70
8.00	0.115	0.467	1.89	4.31	7.73	17.55	40.05	71.60
9.00	0.122	0.498	2.01	4.59	8.23	18.66	42.65	76.20
10.00	0.129	0.527	2.13	4.86	8.70	19.79	45.1[illegible]	80.55

In the above table the condition of weir flow obtains for all figures above the upper horizontal lines, the condition intermediate between weir and jet flow holds for all figures between the two sets of horizontal lines, while that of jet flow obtains for all figures below the second set of horizontal lines.

At the point where the condition of weir flow changes to that of jet flow both of the above equations should theoretically hold true.

By equating the second members of these equations the critical head at which the nature of the flow changes is found to be about 0.6 D for all values of H between 0.1 and 3.0 feet. Practically, however, the exact point at which the change occurs cannot be exactly determined.

Prob. 87. Compute the flow from a vertical pipe 14 inches in diameter when the head above the top of the pipe, as measured by a Pitot tube, is 0.04 feet. Also compute the discharge when the head is 7.6 feet.

Art. 88. Computations in Metric Measures

Nearly all the formulas of this chapter are rational and may be used in all systems of measures. In the metric system lengths are to be taken in meters, areas in square meters, velocities in meters per second, discharges in cubic meters per second, and using for the acceleration constants the values given in Table 9*c*.

(Art. 83) The coefficients of discharge and velocity for smooth fire nozzles 2.0, 2.5, 3.0, and 3.5 centimeters in diameter are 0.983, 0.972, 0.973, and 0.959, respectively. In using the formula $(83)_2$ the values of d and h_1 should be taken in meters, but in finding the ratio D/d the values of D and d may be in centimeters or any other convenient unit. The constant g being 9.80 meters per second, the discharge q will be in cubic meters per second. When it is desired to use the gage reading p_1 in kilograms per square centimeter and to take D in centimeters, the formula

$$q = 65.96\, c_1 D^2 \sqrt{\frac{p_1}{1 - c_1^2 (D/d)^4}}$$

may be used for finding the discharge in liters per minute.

Prob. 88*a*. Compute the loss of head which occurs when a pipe, discharging 18.5 cubic meters per second, suddenly enlarges in diameter from 1.25 to 1.50 meters.

Prob. 88*b*. Find the coefficient of discharge for a tube 8 centimeters in diameter when the flow under a head of 4 meters is 18.37 cubic meters in 5 minutes and 15 seconds.

Prob. 88*c*. Compute the discharge from a smooth nozzle 2.5 centimeters in diameter, attached to a hose 7.5 centimeters in diameter, when the pressure at the entrance is 5.2 kilograms per square centimeter.

CHAPTER 8

FLOW OF WATER THROUGH PIPES

Art. 89. Fundamental Ideas

Pipes made of clay were used in very early times for conveying water. Pliny says that they were two digits (0.73 inches) in thickness, that the joints were filled with lime macerated in oil, and that a slope of at least one-fourth of an inch in a hundred feet was necessary in order to insure the free flow of water.* The Romans also used lead pipes for conveying water from their aqueducts to small reservoirs and from the latter to their houses. Frontinus gives a list of twenty-five standard sizes of pipes,† varying in diameter from 0.9 to 9 inches, which were made by curving a sheet of lead about ten feet long and soldering the longitudinal joint. The Romans had confused ideas of the laws of flow in pipes, their method of water measurement being by the area of cross-section, with little attention to the head or pressure. They knew that the areas of circles varied as the squares of the diameters, and their unit of water measurement was the quinaria, this being a pipe $1\frac{1}{4}$ digits in diameter; then the denaria pipe, which had a diameter of $2\frac{1}{2}$ digits, was supposed to deliver 4 quinarias of water.

In modern times lead pipes have also been used for house service, but these are now largely superseded by either iron pipes or iron pipes lined with lead or tin. For the mains of city water supplies cast-iron pipes are most common, and since 1890 steel-riveted pipes have come into use for large sizes. Lap-welded wrought-iron or steel pipes are used in some cases where the pressure is very high, and large wooden stave pipes are in use in the western part of the United States.

* Natural History, book 31, chapter 31, line 5.

† Herschel, Water Supply of the City of Rome (Boston, 1899), p. 36.