seems to teach that a conical frustum does not usually give as high a velocity as a standard orifice.

Under very high heads, over 300 feet, Hamilton Smith found the actual discharge to agree closely with the theoretical, or the coefficient of discharge was nearly 1.0, and in some cases slightly greater.* His tubes were about 0.9 feet long, 0.1 feet in diameter at the small end and 0.35 feet at the large end, the angle of convergence being 17°. As these figures indicate a contraction of the jet beyond the end, it cannot be supposed that the coefficient of discharge in any case was really as high as his experiments indicate. Under these high heads the cylindrical tip applied to the end of a tube produced no effect on the discharge, the jet passing through without touching its surface.

Prob. 79. When the coefficient of discharge of a tube is 0.98 and the coefficient of velocity of the jet is 0.995, compute the coefficient of contraction of the jet.

Art. 80. Inward Projecting Tubes

Inward projecting tubes, as a rule, give a less discharge than those whose ends are flush with the side of the reservoir, due to the greater convergence of the lines of direction of the filaments of water. At *A* and *B*, Fig. 80, are shown inward projecting tubes so short that the water merely touches their inner edges, and hence they may more properly be called orifices. Experiment shows that the case at *A*, where the sides of the tube are

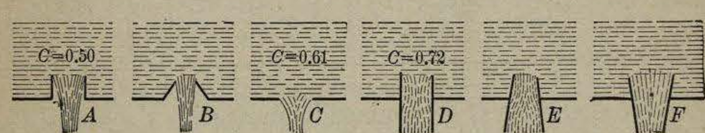

Fig. 80.

normal to the side of the reservoir, gives the minimum coefficient of discharge $c = 0.5$, while for *B* the value lies between 0.5 and that for the standard orifice at *C*. The inward projecting cylindrical tube at *D* has been found to give a discharge of about 72 percent of the theoretic discharge, while the standard tube

* Hydraulics (London and New York, 1886), p. 286.

(Art. 78) gives 82 percent. For the tubes *E* and *F* the coefficients depend upon the amount of inward projection, and they are much larger than 0.72 for both cases, when computed for the area of the smaller end.

It is usually more convenient to allow a water-main to project inward into the reservoir than to arrange it with its mouth flush to a vertical side. The case *D*, in Fig. 80, is therefore of practical importance in considering the entrance of water into the main. As the end of such a main has a flange, forming a partial bell-shaped mouth, the value of c is probably higher than 0.72. The usual value taken is 0.82, or the same as for the standard tube. Practically, as will be seen later, it makes little difference which of these is used, as the velocity in a water-main is slow and the resistance at the mouth is very small compared with the frictional resistances along its length.

Prob. 80. Find the coefficient of discharge for a tube whose diameter is one inch when the flow under a head of 9 feet is 22.1 cubic feet in 3 minutes and 30 seconds.

Art. 81. Diverging and Compound Tubes

In Fig. 81 is shown a diverging conical tube, *BC*, and two compound tubes. The compound tube *ABC* consists of two cones, the converging one, *AB*, being much shorter than the diverging one, *BC*, so that the shape roughly approximates to the form of the contracted jet which issues from an orifice in a thin plate. In the tube *AE* the curved converging part *AB* closely imitates the contracted jet, and *BB* is a short cylinder in which all the filaments of the stream are supposed to move in lines parallel to the axis of the tube, the remaining part being a frustum of a cone. The converging part of a compound tube is often called a mouthpiece and the diverging part an adjutage.

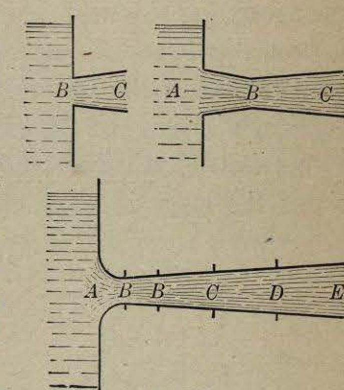

Fig. 81.

Many experiments with these tubes have shown the interesting fact that the discharge and the velocity through the smallest section, *B*, are greater than those due to the head; or, in other words, that the coefficients of discharge and velocity for this section are greater than unity. One of the first to notice this was Bernouilli in 1738, who found $c = 1.08$ for a diverging tube. Venturi in 1791 experimented on such tubes, and showed that the angle of the diverging part, as also its length, greatly influenced the discharge. He concluded that c would have a maximum value of 1.46 when the length of the diverging part was nine times its least diameter, the angle at the vertex of the cone being 5° 06′. Eytelwein found $c = 1.18$ for a diverging tube like *BC* in Fig. 81, but when this tube was used as an adjutage to a mouthpiece *AB*, thus forming a compound tube *ABC*, he found $c = 1.55$.

The experiments of Francis in 1854 on a compound tube like *ABCDE* are very interesting.* The curve of the converging part *AB* was a cycloid, *BB* was a cylinder, and the diameters at *A*, *B*, *C*, *D*, and *E* were 1.4, 0.102, 0.145, 0.234, and 0.321 feet. The piece *BB* was 0.1 feet long, and the others each 1 foot; these were made to screw together, so that experiments could be made on different lengths. A sixth piece, *EF*, not shown in the figure, was also used, which was a prolongation of the diverging cone, its largest diameter being 0.4085 feet. The tubes were cast iron, and quite smooth. The flow was measured with the tubes submerged, and the effective head varied from about 0.01 to 1.5 feet. Excluding heads less than 0.1 feet, the following shows the range in value of the coefficients of discharge:

	c for Section *BB*	c for Outer End
for tube *AB*,	0.80 to 0.94	0.80 to 0.94
for tube *AC*,	1.43 to 1.59	0.70 to 0.78
for tube *AD*,	1.98 to 2.16	0.37 to 0.41
for tube *AE*,	2.08 to 2.43	0.21 to 0.24
for tube *AF*,	2.05 to 2.42	0.13 to 0.15

* Lowell Hydraulic Experiments, 4th Edition, pp. 209–232.

The maximum discharge was thus found to occur with the tube *AE*, and to be 2.43 times the theoretic discharge that would be expected for the small section *BB*. In general the coefficients increased with the heads, the value 2.08 being for a head of 0.13 feet and 2.43 for a head of 1.36 feet; for 1.39 feet, however, c was found to be 2.26.

These coefficients of discharge are the same as the coefficients of velocity, since the tube was entirely filled. Thus, when the coefficient for the section *BB* was 2.43, the velocity was

$$v = 2.43\sqrt{2gh},$$

and the velocity-head was

$$v^2/2g = (2.43)^2h = 5.90h$$

Therefore the flow through the section *BB* was that due to a head 5.9 times greater than the actual head of 1.36 feet; or, in other words, the energy of the water flowing in *BB* was 5.9 times the theoretic energy. Here, apparently, is a striking contradiction of the fundamental law of the conservation of energy. The explanation of this apparent contradiction is the same as that given in Art. 78 for the short-tube adjutage. The increased velocity and discharge is due to the occurrence of a partial vacuum near the inner end of the adjutage *BC*. The pressure of the atmosphere on the water in the reservoir thus increases the hydrostatic pressure due to the head, and the increased flow results. The energy at the smallest section is accordingly higher than the theoretic energy, but the excess of this above that due to the head must be expended in overcoming the atmospheric pressure on the outer end of the tube, so that in no case does the available exceed the theoretic energy. No contradiction of the law of conservation therefore exists.

To render this explanation more definite, let the extreme case be considered where a complete vacuum exists near the inner end of the adjutage, if that were possible, as it perhaps might be with a tube of a certain form. Let h be the head of water in feet on the center of the smallest section. The mean atmospheric pressure on the water in the reservoir is equivalent to a head of 34 feet (Art. 4). Hence the total head which causes the discharge into the vacuum is $h + 34$

and the velocity of flow is nearly $\sqrt{2g(h+34)}$. Neglecting the resistances, which are very slight if the entrance is curved, the coefficients of velocity and discharge can now be found; thus:

$$\text{for } h = 100, \qquad v = \sqrt{2g \times 134} = 1.16\sqrt{2gh}$$
$$\text{for } h = 10, \qquad v = \sqrt{2g \times 44} = 2.10\sqrt{2gh}$$
$$\text{for } h = 1, \qquad v = \sqrt{2g \times 35} = 5.92\sqrt{2gh}$$

The coefficient hence increases as the head decreases. That this is not the case in the above experiments is undoubtedly due to the fact that the vacuum was only partial, and that the degree of rarefaction varied with the velocity. The cause of the vacuum, in fact, is to be attributed to the velocity of the stream, which by friction removes a part of the air from the inner end of the adjutage.

It follows from this explanation that the phenomena of increased discharge from a compound tube could not be produced in the absence of air. The experiment has been tried on a small scale under the receiver of an air-pump, and it was found that the actual flow through the narrow section diminished the more complete the rarefaction. It also follows that it is useless to state any value as representing, even approximately, the coefficient of discharge for such tubes.

Prob. 81. Compute the pressure per square inch in the section BB of Francis' tube when $h = 1.36$ feet and $c = 2.43$. What is the height of the column of water that can be lifted by a small pipe inserted at BB?

ART. 82. SUBMERGED TUBES

As shown in Art. 51 the effective head h which causes the flow through a submerged orifice or tube is the difference in the level of the water above and below the orifice or tube. This difference h, as in Fig. 82, also represents the loss of head occasioned by the flow through the tube. The discharge through a submerged tube is probably somewhat less than that from the same tube when discharging freely into the air. Stewart,* at the laboratory of the

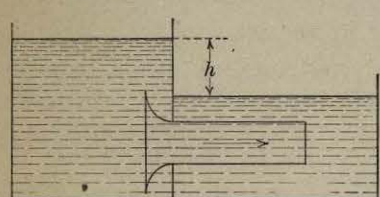

Fig. 82.

* Engineering Record, Sept. 28, 1907.

University of Wisconsin, experimented on large submerged tubes from 4 feet by 4 feet square. These tubes varied in length from 0.3 to 14.0 feet, while the heads h ranged from 0.05 to 0.30 feet. Experiments were made under various conditions of entrance by placing at the mouth of the tubes an elliptical mouthpiece as shown in Fig. 82. This mouthpiece was made in four parts, and after experiments with the straight square-edged tube had been run, others with the bottom of the mouthpiece in place, with the bottom and one side, with the bottom and two sides, and with all four of its parts in position were made.

In the following table are shown the results of these experiments; the coefficients in the first line opposite each head being those for the square-edged tube, while those in the second line are for the same tube with the full elliptical mouthpiece in position as shown.

TABLE 82. COEFFICIENTS FOR SUBMERGED TUBES

Head in Feet	Length of Tube in Feet						
	0.31	0.62	1.25	2.50	5.00	10.00	14.00
0.05	0.631	0.650	0.672	0.769	0.807	0.824	0.838
	.948			.943	.940	.927	.931
0.10	0.611	0.631	0.647	0.718	0.763	0.780	0.795
	.932			.911	.899	.892	.893
0.15	0.609	0.628	0.644	0.708	0.758	0.779	0.794
	.936			.910	.899	.893	.894
0.20	0.609	0.630	0.647	0.711	0.768	0.794	0.809
	.948			.923	.911	.906	.905
0.25	0.610	0.634	0.652	0.720	0.782	0.812	0.828
	.965			.938	.928		
0.30	0.614	0.639	0.660	0.731	0.769	0.832	0.850

From an inspection of these results it appears that the coefficients for the square-edged tubes increase both with the head and with the length of the tube, while for the tubes fitted with the mouthpiece they increase with the head but decrease with the length of the tube. This behavior is readily explained if it be remembered that the larger quantities carried with the mouthpiece in position must cause more friction and so cause a reduction in the effective head. The length of the square-edged tubes experimented on was evidently not sufficient to cause the friction in them to overcome the tendency to greater discharge due to contraction at entrance and subsequent expansion in the tube.

Prob. 82. What will be the discharge through a submerged square-edged tube 5 feet by 4 feet in section and 10 feet long, when the difference between the water levels above and below it is 0.5 feet?

Art. 83. Nozzles and Jets

For fire service two forms of nozzles are in use. The smooth nozzle is essentially a conical tube like A in Fig. 79, the larger end being attached to a hose, but it is often provided with a cylindrical tip and sometimes the larger end is curved, as shown in Fig. 83a. The ring nozzle is a similar tube, but its end is con-

Fig. 83a.

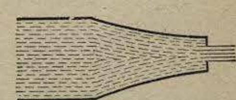

Fig. 83b.

tracted so that the water issues through an orifice smaller than the end of the tube. The experiments of Freeman show that the mean coefficient of discharge is about 0.97 for the smooth nozzle and about 0.74 for the ring nozzle.* The smooth nozzle is used much more than the ring nozzle.

Let d be the diameter of the pipe or hose and D the diameter of the outlet at the end of the nozzle, and let v and V be the corresponding velocities. Let h_1 be the pressure-head at the entrance to the nozzle; then the effective head at the entrance to the nozzle is

$$H = h_1 + \frac{v^2}{2g}$$

and the velocity at the end of the nozzle is $V = c_1\sqrt{2gH}$, where c_1 is the coefficient of velocity. The reasoning of Art. 50 applies here, if the ratio D^2/d^2 is used in place of a/A, and h_1 in place of h, and hence

$$V = c_1\sqrt{\frac{2gh_1}{1 - c^2(D/d)^4}} \qquad (83)_1$$

* Transactions American Society of Civil Engineers, 1889, vol. 21, pp. 303–482.

is the velocity of flow from the nozzle, c being the coefficient of discharge. The discharge per second is, from formula $(50)_2$,

$$q = 0.7854D^2\sqrt{\frac{2gh_1}{(1/c)^2 - (D/d)^4}} \qquad (83)_2$$

The effective head at the nozzle entrance is

$$H = \frac{1}{c_1^2}\cdot\frac{V^2}{2g} = \frac{h_1}{1 - c^2(D/d)^4}$$

and the velocity-head of the issuing jet is

$$\frac{V^2}{2g} = \frac{c_1^2h_1}{1 - c^2(D/d)^4}$$

which gives the height to which the jet would rise if there were no atmospheric resistances. In these formulas D/d is an abstract number, and to find its value D and d may be taken in any unit of measure.

When h_1 and D are in feet, g is to be taken as 32.16 feet per second per second. Then $(83)_1$ gives V in feet per second and $(83)_2$ gives q in cubic feet per second. When the gage at the nozzle entrance gives the pressure p_1 in pounds per square inch, h_1 in feet is found from $2.304p_1$. It is a common practice in figuring on fire-streams to compute the discharge in gallons per minute. For this case, if D is taken in inches,

$$q = 29.83D^2\sqrt{\frac{p_1}{(1/c)^2 - (D/d)^4}}$$

gives the discharge in gallons per minute.

For smooth nozzles the value of the coefficient of velocity c_1 is the same as that of the coefficient of discharge c, since the jet issues without contraction. The experiments of Freeman furnish the following mean values of the coefficient of discharge for smooth cone nozzles of different diameters under pressure-heads ranging from 45 to 180 feet:

Diameter in inches =	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$
Coefficient c	= 0.983	0.982	0.972	0.976	0.971	0.959

These values were determined by measuring the pressure p_1 and the discharge q, from which c can be computed by the last formula. For example, a nozzle having a diameter of 1.001 inches at the end and 2.50 inches at the base discharged 208.5 gallons per minute under a pressure of 50 pounds per square inch at the entrance. Here $D = 1.001$, $d = 2.5$, $p_1 = 50$, and $q = 208.5$, and inserting these in the formula and solving for c, there is found $c = 0.985$.

In ring nozzles the ring which contracts the entrance is usually only $\frac{1}{16}$ or $\frac{1}{8}$ inch in width. The effect of this is to diminish the discharge, but the stream is sometimes thrown to a slightly greater height. On the whole, ring nozzles seem to have no advantage over smooth ones for fire purposes. As the stream contracts after leaving the nozzle, the coefficient of velocity c_1 is greater than the coefficient of discharge c. The value of c being about 0.74, that of c_1 is probably a little larger than 0.97. In using $(83)_1$ for ring nozzles these values of c_1 and c should be inserted, but in using $(83)_2$ only the value of c is needed.

According to Freeman's experiments, the discharge of a $\frac{7}{8}$-inch ring nozzle is the same as that of a $\frac{3}{4}$-inch smooth nozzle, while the discharge of a $1\frac{1}{4}$-inch ring nozzle is about 20 percent greater than that of a 1-inch smooth nozzle. The heights of vertical jets from a $1\frac{1}{4}$-inch ring nozzle are about the same as those from a 1-inch smooth nozzle, while the jets from a $1\frac{3}{8}$-inch ring nozzle are slightly less in height than those from a $1\frac{1}{4}$-inch smooth nozzle.

The vertical height of a jet from a nozzle is very much less, on account of the resistance of the air, than the value deduced above for $V^2/2g$. For instance, let a smooth nozzle 1 inch in diameter attached to a 2.5-inch hose have $c = 0.97$ and the pressure-head $h_1 = 230$ feet; then the computation gives the velocity-head $V^2/2g$ as 221 feet, whereas the average of the highest drops in still air will be about 152 feet high and the main body of water will be several feet lower. Table 83, compiled from the results of Freeman's experiments, shows for three different smooth nozzles the height of vertical jets, column A giving the heights reached by the average of the highest drops in still air, and column B the maximum limits of height as a good effective fire-stream

TABLE 83. VERTICAL JETS FROM SMOOTH NOZZLES

Indicated Pressure at Entrance to Nozzle Pounds per Square Inch	From $\frac{3}{4}$-inch Nozzle			From 1-inch Nozzle			From $1\frac{1}{4}$-inch Nozzle		
	Height in Feet		Discharge Gallons per Minute	Height in Feet		Discharge Gallons per Minute	Height in Feet		Discharge Gallons per Minute
	A	B		A	B		A	B	
10	20	17	52	21	18	93	22	19	148
20	40	33	73	43	35	132	44	37	209
30	59	48	90	63	51	161	66	53	256
40	78	60	104	83	64	186	86	67	296
50	93	67	116	101	73	208	107	77	331
60	104	72	127	117	79	228	126	85	363
70	114	76	137	130	85	246	140	91	392
80	123	79	147	140	89	263	150	95	419
90	129	81	156	147	92	279	157	99	444
100	134	83	164	152	96	295	161	101	468

with moderate wind. The discharges given depend only on the pressure, and are the same for horizontal as for vertical jets.

The maximum horizontal distance to which a jet can be thrown is also a measure of the efficiency of a nozzle. The following, taken from Freeman's tables, gives the horizontal distances at the level of the nozzle reached by the average of the extreme drops in still air. The practical horizontal distance for an effective fire-stream is, however, only about one-half of these figures.

Pressure at nozzle entrance,	20	40	60	80	100 pounds.
From $\frac{3}{4}$-inch smooth nozzle,	72	112	136	153	167 feet.
From 1-inch smooth nozzle,	77	133	167	189	205 feet.
From $1\frac{1}{4}$-inch smooth nozzle,	83	148	186	213	236 feet.
From $1\frac{1}{8}$-inch ring nozzle,	76	131	164	186	202 feet.
From $1\frac{1}{4}$-inch ring nozzle,	78	138	172	196	215 feet.
From $1\frac{3}{8}$-inch ring nozzle,	79	144	180	206	227 feet.

The ball nozzle, often used for sprinkling, has a cup at the end of the nozzle and within the cup a ball, so that the jet issuing from the tip of the nozzle is deflected sidewise in all directions. This apparatus exhibits a striking illustration of the principle of negative pressure, for the ball is not driven away from the tip, but is held close to it by the atmospheric pressure, the negative pressure-head being caused by the high velocity of the sheet of water around the ball. The cup is usually so arranged that the ball cannot be driven out of it, for this might occur under the first impact of the jet, but when the flow has become steady, there is no tendency of this kind, and the ball is seen slowly revolving upon the cushion of water without touching any part of the cup.

Prob. 83. A nozzle $1\frac{3}{8}$ inches in diameter attached to a play-pipe $2\frac{1}{2}$ inches in diameter discharges 310.6 gallons per minute under an indicated pressure of 30 pounds per square inch. Find the velocity of the jet and the coefficient c_1.

Art. 84. Lost Head in Long Tubes

When water issues from an orifice, tube, pipe, or nozzle with the velocity v, its velocity-head is $v^2/2g$, and it is only this part of the total effective head h that can be utilized for the production of work. The lost head then is

$$h' = h - \frac{v^2}{2g}$$

Now if c_1 is the coefficient of velocity for the section where the discharge occurs, the velocity v is given by $c_1\sqrt{2gh}$, and hence

$$h' = \left(\frac{1}{{c_1}^2} - 1\right)\frac{v^2}{2g} \qquad (84)_1$$

is a general expression for the lost head in terms of the velocity-head. For the standard orifice (Art. 45), the mean value of c_1 is 0.98 and for an orifice perfectly smooth c_1 is 1.00; hence from $(84)_1$

$$h' = 0.04\frac{v^2}{2g} \quad \text{and} \quad h' = 0$$

are the losses of head for these two cases.

For the standard short cylindrical tube (Art. 78) the value of c_1 is about 0.82, and the loss of head is

$$h' = \left(\frac{1}{0.82^2} - 1\right)\frac{v^2}{2g} = 0.49\frac{v^2}{2g}$$

For the inward projecting cylindrical tube (Art. 80) the value of c_1 is about 0.72, and hence the loss of head is

$$h' = \left(\frac{1}{0.72^2} - 1\right)\frac{v^2}{2g} = 0.93\frac{v^2}{2g}$$

Accordingly the loss of head for the inward projecting tube is nearly equal to the velocity-head of the issuing stream, while that from the standard tube is about one-half the velocity-head.

When a tube is longer than three diameters, it becomes a long tube or a pipe. Here the loss of head is much greater because the water meets with frictional resistances along the interior surface, and the longer the pipe, the greater is this resistance and the slower is the velocity. The formula $(84)_1$ gives the total loss of head for this case also. For example, the experiments of Eytelwein and others have given values of c_1 for the cases below, and from these the corresponding values of the total lost head have been computed. Let l denote the length of the pipe and d its diameter, the end connected with the reservoir being arranged like the standard tube; then

for $l = 12d$ $\quad c_1 = 0.77 \quad h' = 0.69\,v^2/2g$

for $l = 36d$ $\quad c_1 = 0.67 \quad h' = 1.23\,v^2/2g$

for $l = 60d$ $\quad c_1 = 0.60 \quad h' = 1.77\,v^2/2g$

Now in each of these cases the amount $0.49\,v^2/2g$ is lost in entering the tube and in impact, as in the standard short tube. Hence the loss of head in friction in the remaining length of the pipe is $h'' = h' - 0.49v^2/2g$, or

for $l = 12d$ $\quad h'' = 0.20\,v^2/2g$

for $l = 36d$ $\quad h'' = 0.74\,v^2/2g$

for $l = 60d$ $\quad h'' = 1.28\,v^2/2g$

which shows that the frictional losses increase with the length of the pipe. The length of the pipe in which the entrance losses occur is about $3d$; hence if $3d$ be subtracted from each of the above lengths, the lengths in which the friction loss occurs are $9d$, $33d$, and $57d$, and it is seen that the above losses of head in friction are closely proportional to these lengths. By these and many other experiments it has been shown that the loss of head in friction varies directly with the length of the pipe.

The lost head has here been expressed in terms of the velocity-head, but it can also be expressed in terms of the total head h

that causes the flow. For, substituting in $(84)_1$ the value of v given by $c_1\sqrt{2gh}$, it reduces to

$$h' = (1 - c_1^2)\,h \qquad (84)_2$$

Thus, for the standard short tube $h' = 0.33\,h$; for the inward projecting tube $h' = 0.48\,h$, and for the above tube or pipe whose length is 60 diameters $h' = 0.64\,h$.

Prob 84. Find the ratio of the kinetic energy in the jet from a standard orifice to that in the jet from a standard tube, the diameters of orifice and tube being the same.

Art. 85. Inclined Tubes and Pipes

The tubes discussed in this chapter have generally been regarded as horizontal, but, if this is not the case, the formulas for velocity and discharge may be applied to them by measuring the head from the water level in the reservoir down to the center of the head of the pipe. Thus, for the nozzles of Art. 83, it is understood that the tip is at the same level as the gage which registers the pressure p_1 or the pressure-head h_1; if the tip be lower than the gage by the vertical distance d_1, the true pressure-head to be used in the formula is $h_1 + d_1$; if it be higher, the true pressure-head is $h_1 - d_1$. Then the velocity-head $v^2/2g$ is to be measured upward from the tip of the nozzle.

The theorem of Bernouilli, given in Art. 31, is true for inclined as well as for horizontal pipes under uniform flow, but it will be convenient to express it in a slightly different form. Let a_1 and a_2 be two sections of a pipe where the velocities are v_1 and v_2, and the pressure-heads are h_1 and h_2, and let the flow be steady so that the same weight of water, W, passes each section in one second. Let MN be any horizontal plane lower than the lowest section, as for instance the sea level, and let e_1 and e_2 be the elevations of a_1

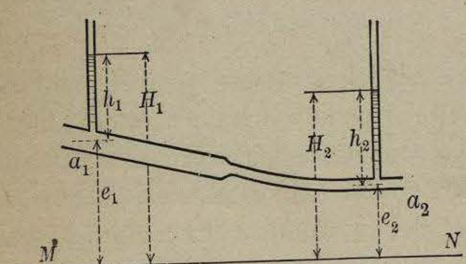

Fig. 85.

and a_2 above it. With respect to this plane the weight W at a_1 has the potential energy We_1, the pressure-energy Wh_1, and the kinetic energy $W \cdot v_1^2/2g$, or the total energy is

$$W\left(e_1 + h_1 + \frac{v_1^2}{2g}\right)$$

Similarly with respect to this plane the energy of W in a_2 is

$$W\left(e_2 + h_2 + \frac{v_2^2}{2g}\right)$$

If no losses of energy occur between the two sections, these expressions are equal, and hence

$$e_1 + h_1 + \frac{v_1^2}{2g} = e_2 + h_2 + \frac{v_2^2}{2g} \qquad (85)_1$$

and hence the theorem of Bernouilli may be stated as follows:

In any pipe, under steady flow without impact or friction, the gravity-head plus the pressure-head plus the velocity-head is a constant quantity for every section.

Now let H_1 and H_2 be the heights of the water levels in the piezometer tubes above the datum plane; then $e_1 + h_1 = H_1$ and $e_2 + h_2 = H_2$, and accordingly $(85)_1$ becomes

$$H_1 + \frac{v_1^2}{2g} = H_2 + \frac{v_2^2}{2g} \qquad (85)_2$$

or, the piezometer elevation for a_1 plus the velocity-head is equal to the sum of the corresponding quantities for any other section.

This theorem belongs to theoretical hydraulics, in which frictional resistances are not considered. Under actual conditions there is always a loss of energy or head, so that when water flows from a_1 to a_2, the first member of the above equation is larger than the second. Let Wh' be the loss in energy, then this is equal to the difference of the energies in a_1 and a_2 with respect to the datum plane, and

$$h' = (e_1 + h_1) - (e_2 + h_2) + \frac{v_1^2}{2g} - \frac{v_2^2}{2g}$$

or

$$h' = H_1 - H_2 + \frac{v_1^2}{2g} - \frac{v_2^2}{2g} \qquad (85)_3$$