TABLE 74*c*. COEFFICIENTS M FOR DAMS

Metric Measures

Up-stream Slope	Width of Crest Meters	Down-stream Slope	Head H on Crest in Meters					
			0.15	0.30	0.60	0.91	1.22	1.52
1 on 2	0.10	Vertical	1.85	2.03	2.08	2.03	2.04	2.05
1 on 2	0.20	Vertical	1.78	1.90	2.02	2.03	2.04	2.05
1 on 5	0.20	Vertical	1.83	1.84	1.85	1.86	1.87	1.87
1 on 4	0.20	Vertical		1.90	1.92	1.92	1.92	1.92
1 on 3	0.20	Vertical	2.01	2.11	2.04	1.96	1.96	1.96
1 on 2	0.00	1 on 1	2.33	2.34	2.19	2.11	2.06	2.03
1 on 2	0.10	1 on 2	1.73	1.90	1.99	2.02	2.02	2.01
1 on 2	0.20	1 on 5	1.82	1.97	1.94	1.93	1.95	1.97
Vertical	0.80	Vertical	1.43	1.47	1.57	1.66	1.77	1.87
Vertical	0.80*	Vertical	1.63	1.66	1.70	1.79	1.87	1.92
Vertical	2.00	Vertical	1.38	1.43	1.37	1.39	1.44	1.49
Vertical	2.00*	Vertical	1.50	1.56	1.57	1.58	1.60	1.63
1 on 1	Round	Vertical	1.63	1.75	1.91	1.96	1.99	2.01

* For explanation see Art. 69.

Prob. 74*c*. Compute the discharge over a submerged weir when $b = 2.35$, $H = 0.123$, and $H' = 0.027$ meters.

Prob. 74*d*. Compute the discharge over a dam, like Fig. 68*b*, when the side slopes are 1 on 2, the length of the crest 4.25 meters, and the head on the crest 1.07 meters.

CHAPTER 7

FLOW OF WATER THROUGH TUBES

ART. 75. LOSS OF ENERGY OR HEAD

A tube is a short pipe which may be attached to an orifice or be used for connecting two vessels. The most common form is a cylinder of uniform cross-section, but conical forms are also used, and in some cases a tube is made of cylinders with different diameters. The laws of flow through tubes are important as a starting-point for the theory of flow through pipes, for the discharge from nozzles, and for the discussion of many practical hydraulic problems. The theorem of Art. 31, that pressure-head plus velocity-head is a constant for a given section of a tube, is only true when there are no losses due to friction and impact. As a matter of fact such losses always exist and must be regarded in practical computations.

Energy in a tube filled with moving water exists in two forms, in potential energy of pressure and in kinetic energy of motion. Thus in the horizontal tube of Fig. 75*a* let two piezometers (Art. 37) be inserted at the sections a_1 and a_2 where the velocities are v_1 and v_2 and it is found that the water rises to the heights h_1 and h_2 above the middle of the tube. Let W be the weight of water that passes each section per second. Then in the first section the pressure energy per second is Wh_1 and the kinetic energy per second is $W \cdot v_1^2/2g$, so that the total energy of the water passing that section in one second is

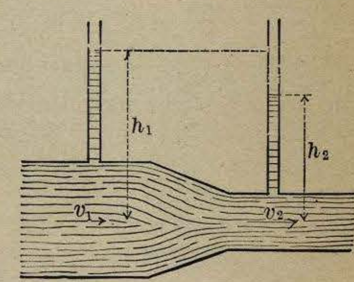

Fig. 75*a*.

$$Wh_1 + W \cdot v_1^2/2g$$

In the same manner the total energy of the water passing the second section in one second is

$$Wh_2 + W \cdot v_2^2/2g$$

but this is less than the former because some energy has been expended in friction and impact. Let Wh' be the amount of energy thus lost; then equating this to the difference of the energies in the two sections, the W cancels out and

$$h' = h_1 - h_2 + \frac{v_1^2}{2g} - \frac{v_2^2}{2g} \qquad (75)_1$$

The quantity h' is called the lost head, and the equation shows that it equals the difference of the pressure-heads plus the difference of the velocity-heads.

In hydraulics the terms "energy" and "head" are often used as equivalent, although really energy is proportional to head. In the general case, the lost head is not a loss of pressure-head only, but a loss of both pressure-head and velocity-head. When, however, the two sections are of equal area, the velocities v_1 and v_2 are equal, since the same quantity of water passes each section in one second; then the lost head h' is $h_1 - h_2$ or the loss occurs in pressure-head only. Here the loss is mainly due to the roughness of the interior surface of the tube or pipe. It should be noted that it is only necessary to measure the difference $h_1 - h_2$ and this can be done by the methods of Art. 37.

Formula $(75)_1$ is applicable to all horizontal tubes and pipes, and with a slight modification it is also applicable to inclined ones, as will be shown in Art. 85. It also applies to a flow from a standard orifice, or to the flow from an orifice to which a tube is attached.

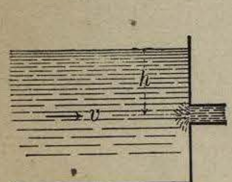

Fig. 75*b*.

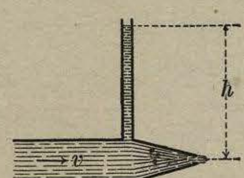

Fig. 75*c*.

Thus for the large vessel of Fig. 75*b* let the sections be taken through the vessel and through the stream as it leaves the tube. Then $h_1 = h$, and since there is no pressure outside

the tube, $h_2 = 0$; also $v_1 = 0$ and $v_2 = v$; then $h' = h - v^2/2g$. For the case in Fig. 75*c*, where the stream approaches with the velocity v_1, the formula becomes $h' = h_1 + (v_1^2 - v^2)/2g$. In both cases, if h' is made zero, these equations reduce to those established in the chapter on theoretical hydraulics, where losses of energy were not considered; thus for the second case the theoretic effective head h is equal to $h_1 + v_1^2/2g$.

In order to use $(75)_1$ for numerical computations three quantities must be known, the difference $h_1 - h_2$, and the velocities v_1 and v_2. As a direct measurement of the velocities is usually impracticable, these are generally computed from the measured discharge q and the areas a_1 and a_2 of the cross-sections; thus $v_1 = q/a_1$ and $v_2 = q/a_2$. For example, let the cross-section be circular, having diameters of 18 and 6 inches, and let the discharge be 4.7 cubic feet per second; the areas are 1.767 and 0.196 square feet, and the velocities are 2.66 and 23.94 feet per second. If the difference of the pressure-heads is 8.85 feet, the lost head is

$$h' = 8.85 + 0.01555(2.66^2 - 23.94^2) = 0.05 \text{ feet}$$

The general formula $(75)_1$ may be expressed in terms of the areas of the sections and one of the velocities. Since $a_1v_1 = a_2v_2$ it may be written

$$h' = h_1 - h_2 + \left(1 - \frac{a_1^2}{a_2^2}\right)\frac{v_1^2}{2g} \qquad (75)_2$$

or

$$h' = h_1 - h_2 + \left(\frac{a_2^2}{a_1^2} - 1\right)\frac{v_2^2}{2g} \qquad (75)_3$$

which are often convenient forms for numerical computations.

Prob. 75. In Fig. 75*a* let the areas a_1 and a_2 be 1.0 and 0.5 square feet, $h_1 - h_2 = 0.697$ feet, and $v_1 = 3.5$ feet per second. What is the value of the lost head?

Omit.

Art. 76. Loss Due to Expansion of Section

When a tube or pipe is filled with flowing water a loss of head is found to occur when the section is enlarged, so that the velocity is diminished. This case is shown in Fig. 76*a*, where v_1 and v_2 are the velocities in the smaller and larger sections and h_1 and h_2 the corresponding pressure-heads. The interior surface may be very smooth, so that friction has but little influence, and yet there will usually be more or less loss due to the fact that the velocity v_1 is changed to the smaller value v_2. Formula $(75)_1$ is here directly applicable and gives the loss of head. It is seen that $h_1 - h_2$ must be negative for this case and that its numerical value will be less than that of the difference of the velocity-heads. The general formula $(75)_1$ gives the loss of head due not only to expansion of section, but to all resistances between any two sections of a horizontal tube or pipe.

When there is a sudden enlargement of section, as in Fig. 76*b*, energy is lost in impact. In the section AB the pressure-

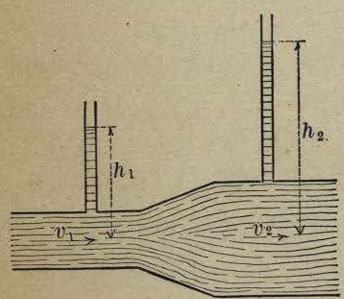

Fig. 76*a*.

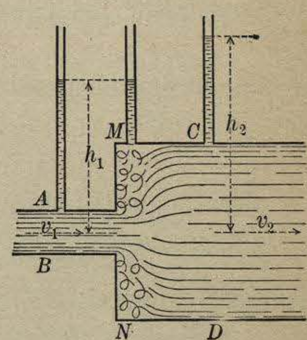

Fig. 76*b*.

head is h_1 and the velocity-head is $v_1^2/2g$, while in the section CD the pressure-head has the larger value h_2 and the velocity-head has the smaller value $v_2^2/2g$. At the section MN, near the place of sudden expansion, the pressure-head is also h_1, since the velocity v_1 is maintained for a short distance after leaving the small section; its direction, however, being changed so as to form whirls and foam. In this region the impact occurs, the velocity v_1 being finally decreased to v_2. Let a_2 be the area of the sections MN and CD, and w the weight of a cubic unit of water. Then by (15) the hydrostatic pressure normal to the section CD is wa_2h_2, and that normal to the section MN is wa_2h_1. The difference of these pressures is the force which causes the velocity v_1 to decrease to v_2, and by Art. 27, this force is equal to $W(v_1 - v_2)/g$, where W is the weight of water passing the section CD in one second. Hence

$$wa_2h_2 - wa_2h_1 = W\frac{v_1 - v_2}{g}$$

and, since W equals wa_2v_2, this equation becomes

$$h_2 - h_1 = \frac{v_2(v_1 - v_2)}{g} \qquad (76)_1$$

Inserting this value of $h_2 - h_1$ in $(75)_1$, it reduces to

$$h = \frac{(v_1 - v_2)^2}{2g}$$

which is the loss of head due to sudden expansion of section, or rather due to the sudden diminution of velocity that is caused by such expansion.

When the expansion of section is made gradually and with smooth curves, the velocity v_1 will decrease without whirl and foam, so that no loss in impact occurs. In this case the kinetic energy $w \cdot v_1^2/2g$ is changed into pressure energy, as the velocity v_1 decreases to v_2. There is, however, no distinct line of demarcation between sudden and gradual expansion, so that in many practical cases it is necessary to make measurements of the discharge and of the head $h_2 - h_1$ in order to compute the lost head h' from $(75)_1$, which is a formula applicable to all cases.

Sudden enlargement of section should always be avoided in tubes and pipes owing to the loss of head that it causes, which may often be very great. For example, let there be no pressure-head in the section a_1 and let v_1 be due to a head h so that $v_1 = \sqrt{2gh}$; let the area a_2 be four times that of a_1 so that v_2 is one-fourth of v_1. The loss of head due to sudden expansion then is

$$h' = \frac{(v_1 - v_2)^2}{2g} = \tfrac{9}{16}\,h$$

so that more than one-half of the energy of the water in a_1 is lost in impact, having been changed into heat. In the section a_2 the effective head is $\frac{7}{16}h$, of which $\frac{1}{16}h$ is velocity-head and $\frac{6}{16}h$ is pressure-head.

Formula $(76)_1$ may be expressed in terms of the areas of the sections and one of the velocities, since $a_1v_1 = a_2v_2$. The value of h' takes the two forms

$$h' = \left(1 - \frac{a_1}{a_2}\right)^2 \frac{v_1^2}{2g} = \left(\frac{a_2}{a_1} - 1\right)^2 \frac{v_2^2}{2g} \qquad (76)_2$$

and these show that no loss of head occurs when $a_1 = a_2$.

Prob. 76. In a horizontal tube like Fig. 76*a* the diameters are 6 inches and 12 inches, and the heights of the pressure-columns or piezometers are 12.16 feet and 12.96 feet above the same bench-mark. Find the loss of head between the two sections when the discharge is 1.57 cubic feet per second, and also when it is 4.71 cubic feet per second.

Art. 77. Loss Due to Contraction of Section

When a sudden contraction of section in the direction of the flow occurs, as in Fig. 77, the water suffers a contraction similar to that in the standard orifice, and hence in its expansion to fill the second section a loss of head results. Let v_1 be the velocity in the larger section and v that in the smaller, while v' is the velocity in the contracted section of the flowing stream; and let a_1, a, and a' be the corresponding areas of the cross-sections. From the formula $(76)_2$ the loss of head due to the expansion of section from a' to a is

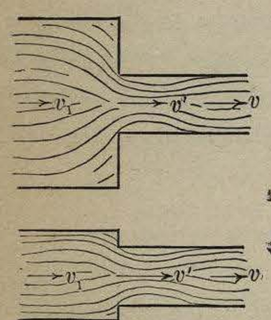

Fig. 77.

$$h' = \left(\frac{a}{a'} - 1\right)^2 \frac{v^2}{2g} = \left(\frac{1}{c'} - 1\right)^2 \frac{v^2}{2g} \qquad (77)_1$$

in which c' is the coefficient of contraction of the stream or the ratio of a' to a (Art. 44).

The value of c' depends upon the ratio between the areas a and a_1. When a is small compared with a_1, the value of c' may be taken at 0.62 as for orifices (Art. 44). When a is equal to a_1, there is no contraction or expansion of the stream and c' is unity. Let d and d_1 be the diameters corresponding to the areas a and a_1, and let r be the ratio of d to d_1. Then experiments seem to indicate that an expression of the form

$$c' = m + \frac{n}{1.1 - r}$$

gives the law of variation of c' with r. Placing $c' = 0.62$ and $r = 0$ gives one equation between m and n; placing $c' = 1.00$ and $r = 1$ gives another equation; and the solution of these furnishes the values of m and n. Thus is found

$$c' = 0.582 + \frac{0.0418}{1.1 - r} \qquad (77)_2$$

from which approximate values of c' can be computed:

for	$r = 0.0$	0.4	0.6	0.7	0.8	0.9	0.95	1.0
	$c' = 0.62$	0.64	0.67	0.69	0.72	0.79	0.86	1.00

from which intermediate values may often be taken without the necessity of using the formula.

For a case of gradual contraction of section, such as shown in Fig. 75*a*, the loss of head is less than that given by formula $(77)_1$, and it can only be determined from three measured quantities by the help of the general formulas of Art. 75. If the change of section is made so that the stream has no subsequent enlargement, loss of head is avoided; for, as the above discussions show, it is the loss in velocity due to sudden expansion which causes the loss of head.

The loss due to sudden contraction of a tube or pipe is often much smaller than that due to sudden expansion. For instance, let the diameter of the large section be three times that of the smaller, and the velocity in the large section be 2 feet per second, then the loss of head which occurs when the flow passes from the small to the large section is, by Art. 76,

$$h' = 0.01555(18 - 2)^2 = 4.0 \text{ feet}$$

But if the flow occurs in the opposite direction, the ratio r is $\frac{1}{3}$, the coefficient c' is about 0.64, and the loss of head is

$$h' = 0.01555\left(\frac{1}{0.64} - 1\right)^2 18^2 = 1.6 \text{ feet}$$

When, however, the ratio r is higher than 0.77, the loss due to sudden contraction is greater than that due to sudden expansion. Thus, if the diameter of the small section be nine-tenths that of the large one and the velocity in the large section be 2 feet per second, the loss of head when the flow passes from the small to the large section is

$$h' = 0.01555\left(\frac{1}{0.81} - 1\right)^2 2^2 = 0.0034 \text{ feet}$$

But if the flow occurs in the opposite direction, the ratio r is 0.9, the coefficient c' is 0.79, and the loss of head is

$$h' = 0.01555\left(\frac{1}{0.79} - 1\right)^2 2.47^2 = 0.0066 \text{ feet}$$

As formula $(77)_2$ is an empirical one the results derived from it are tc be regarded as approximate.

Prob. 77. Compute the loss of head when a pipe which discharges 1.57 cubic feet per second suddenly diminishes in section from 12 to 6 inches in diameter.

Art. 78. The Standard Short Tube

An adjutage is a tube inserted into an orifice, and the short-tube adjutage, consisting of a cylinder whose length is about three times its diameter, is the most common form. For convenience it will be called the standard short tube, because its theory and coefficients form a starting-point with which all other adjutages may be compared. This short tube is of little value for the measurement of water, since the coefficients for standard orifices are much more definitely known. The discussion here given is for the case where the inner edge is a sharp, definite corner like that of the standard orifice (Art. 43). When the tube is only two diameters in length, the stream passes through without touching it, as in Fig. 78a, and the discharge is the same as from the orifice. When it is lengthened sufficiently, the stream expands and fills the tube, as in Fig. 78b, and the discharge is much increased. By observations on glass tubes it is seen that the stream usually contracts after leaving the inner end of the tube and then expands. This contraction

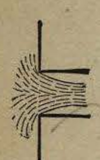

Fig. 78a.

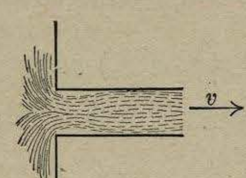

Fig. 78b.

may be apparently destroyed by agitating the water or by striking the tube, and the entire tube is then filled, yet if a hole is bored in the tube near its inner end, water does not flow out, but air enters, showing that a negative pressure exists.

An estimate of the velocity and discharge from this short-tube adjutage may be made as follows: Let h be the head on the inner end of the tube and v the velocity of the outflowing water. The head h equals the velocity-head $v^2/2g$ plus all the losses of head. At the inner edge a loss of $0.11\, v^2/2g$ occurs in entering the tube, as in the standard orifice (Art. 56), and then there is a loss of $(v' - v)^2/2g$ when the contracted stream suddenly expands so that its velocity v' is reduced to v (Art. 76). If a' and a are the areas of these two sections, their ratio a'/a is the coefficient of contraction c'. Then

$$h = 0.11\frac{v^2}{2g} + \left(\frac{1}{c'} - 1\right)^2 \frac{v^2}{2g} + \frac{v^2}{2g}$$

Now, taking for c' its mean value 0.62, this equation reduces to $v = 0.82\sqrt{2gh}$, or the coefficient of velocity of the issuing jet is 0.82. Since the cross-section of the stream at the outer end of the tube is the same as that of the tube, the coefficient of contraction for that end is unity, and hence (Art. 46) the mean value of the coefficient of discharge is also 0.82.

While this theoretic discussion does not take account of losses due to the small frictional resistances along the sides of the tube after the stream has expanded, the mean results of the experiments of Venturi and Bossut give closely the same coefficient. Hence both theory and practice agree in establishing as an average value for the short tube,

$$\text{Coefficient of discharge } c = 0.82$$

This coefficient, however, ranges from 0.83 for low heads to 0.79 for high heads. It is greater for large tubes than for small ones, its law of variation being probably the same as for orifices (Art. 47), but sufficient experiments have not been made to state definite values in the form of a table.

A standard orifice gives on the average about 61 percent of the theoretic discharge, but by the addition of a tube this may be increased to 82 percent. The velocity-head of the jet from the tube is, however, much less than that from the orifice. For, let v be the velocity and h the head, then (Art. 45) for the standard orifice

$$v = 0.98\sqrt{2gh} \quad \text{or} \quad v^2/2g = 0.96\,h$$

and similarly for the standard tube

$$v = 0.82\sqrt{2gh} \quad \text{or} \quad v^2/2g = 0.67\,h$$

Accordingly the velocity-head of the stream from the standard orifice is 96 percent of the theoretic velocity-head, and that of the stream from the standard tube is only 67 percent. Or if jets are directed vertically upward from a standard orifice and tube, as in Fig. 78*c*, that from the former rises to the height 0.96 h, while that from the latter rises to the height 0.67 h, where h is the head measured downward from the surface of water in the reservoir to the point of exit from the orifice.

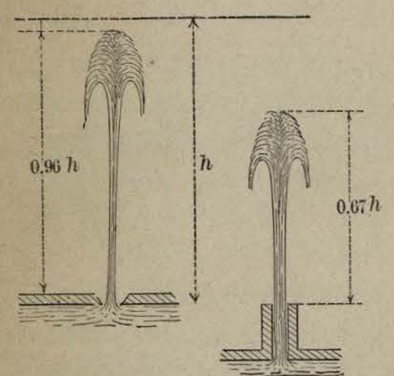

Fig. 78*c*.

The energy lost in the stream from the standard orifice is hence 4 percent of the theoretic energy, but 33 percent is lost in the stream from the standard tube. In reality energy is never lost, but is merely transformed into other forms of energy. In the tube the one-third of the total energy which has been called lost is only lost because it cannot be utilized as work; it is, in fact, transformed into heat, which raises the temperature of the water. The above explanation shows that most of this loss is due to impact resulting from sudden expansion of the stream.

The loss of head in the flow from the short tube is large, but not so large as might be expected from theoretical considerations based on the known coefficients for orifices. When the tube has a length of only two diameters, the water does not touch its

inner surface, and the flow occurs as from a standard orifice. The velocity in the plane of the inner end is then 61 percent of the theoretic velocity, since the mean coefficient of discharge is 0.61. Now when the tube is sufficiently increased in length, its outer end will be filled, and if the contraction still exists, it might be inferred that the coefficient for that end would be also 0.61; this would give a velocity-head of $(0.61)^2\,h$ or 0.37 h, so that the loss of head would be 0.63 h. Actually, however, the coefficient is found to be 0.82 and the loss of head only 0.33 h. It hence appears that further explanation is needed to account for the increased discharge and energy.

In the first place, a loss of about 0.04 h occurs at the inner end of the tube in the same manner as in the standard orifice, and only the head 0.96 h is then available for the subsequent phenomena. If the coefficient c' for the contracted section has the value 0.62, the velocity in that section is

$$v' = \frac{0.82}{0.62}\sqrt{2gh} = 1.32\sqrt{2gh}$$

and the velocity-head for that section is

$$v'^2/2g = 1.75\,h$$

and consequently the pressure-head in that section is

$$0.96\,h - 1.75\,h = -0.79\,h$$

There exists therefore a negative pressure or partial vacuum near the inner end of the tube which is sufficient to lift a column of water to a height of about three-fourths the head. This conclusion has been confirmed by experiment for low heads, and was in fact first discovered experimentally by Venturi. For high heads it is not valid, since in no event can atmospheric pressure raise a column of water higher than about 34 feet (Art. 4); probably under high heads the coefficient of contraction of the stream in the tube becomes much greater than 0.62.

The cause of the increased discharge of the tube over the orifice is hence a partial vacuum, which causes a portion of the atmospheric head of 34 feet to be added to the head h, so that the flow at the contracted section occurs as if under the head $h + h_1$. The occurrence of this partial vacuum is attributed to the friction of the water on the air. When the flow begins, the stream is surrounded by air of the normal atmospheric pressure which is imprisoned as the stream fills the tube. The friction of the moving water carries some of this air out with it, thus rarefying the remaining air. This rarefaction, or negative pressure, is followed by an increased velocity of flow, and the process continues until the air around the contracted section is so rarefied that no more is removed, and the flow then remains permanent, giving the results ascertained by experiment. The partial vacuum causes neither a gain nor loss of head, for although it increases the velocity-head at the contracted section to $1.75\,h$, there must be expended $0.79\,h$ in order to overcome the atmospheric pressure at the outer end of the tube. The experiments of Buff have proved that in an almost complete vacuum the discharge of the tube is but little greater than that of the orifice.*

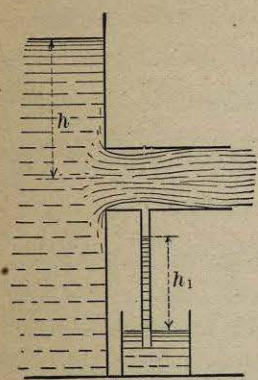

Fig. 78*d*.

Prob. 78. When the coefficient of contraction for the contracted section is 0.70, find the probable coefficient of discharge and also the negative pressure-head.

Art. 79. Conical Converging Tubes

Conical converging tubes are used when it is desired to obtain a high efficiency in the energy of the stream of water. At A, Fig. 79, is shown a simple converging tube, consisting of a frustum of a cone, and at B is a similar frustum provided with a cylindrical tip. The proportions of these converging tubes, or mouthpieces, vary somewhat in practice, but the cylindrical tip when employed is of a length equal to about $2\frac{1}{2}$ times its inner diameter, while the conical part is eight or ten times the length of that

* Annalen der Physik und Chemik, 1839, vol. 46, p. 242.

diameter, the angle at the vertex of the cone being between 10 and 20 degrees.

The stream from a conical converging tube like A suffers a contraction at some distance beyond the end. The coefficient of discharge is higher than that of the standard tube, being generally between 0.85 and 0.95, while the coefficient of velocity is higher still. Experiments made by d'Aubuisson and Castel on conical converging tubes 0.04 meters long and 0.0155 meters in diameter at the small end, under a head of 3 meters, furnish the coefficients of discharge and velocity given in Table 79.

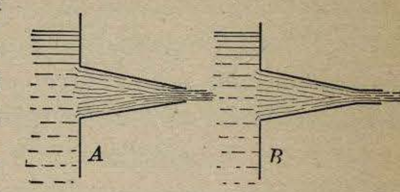

Fig. 79.

Table 79. Coefficients for Conical Tubes

Angle of Cone	Discharge c	Velocity c_1	Contraction c'
0° 00′	0.829	0.829	1.00
1 36	.866	.867	
4 10	.912	.910	
7 52	.930	.932	0.998
10 20	.938	.951	.986
13 24	.946	.963	.983
16 36	.938	.971	.966
21 00	.919	.972	.945
29 58	.895	.975	.918
48 50	.847	.984	.861

The former of these was determined by measuring the actual discharge (Art. 46), and the latter by the range of the jet (Art. 45). The coefficient of contraction as computed from these is given in the last column, and this applies to the jet at the smallest section, some distance beyond the end of the tube. While these values show that the greatest discharge occurred for an angle of about $13\frac{1}{2}°$, they also indicate that the coefficient of velocity increases with the convergence of the cone, becoming about equal to that of a standard orifice for the last value. Hence the table