that the drop is independent of the length of the weir. All of these laws except the last have been previously deduced by the discussion of experiments.

The path of the stream after leaving the weir is closely that of a parabola. In the plane of the crest the mean velocity is

$$V = q/b(H-d)$$

and the direction of this may be taken as approximately horizontal. The range of a stream on a horizontal plane at the distance y below the middle of the weir notch is then readily found. For, if x be this range which is reached in the time t, then $x = Vt$, and also $y = \frac{1}{2}gt^2$; whence, by the elimination of t, there results $gx^2 = 2V^2y$, and accordingly the horizontal range at the depth y is

$$x = M\frac{H^{\frac{3}{2}} - d^{\frac{3}{2}}}{H-d}\sqrt{\frac{2y}{g}}$$

in which d is given by (70). For example, take a case where $H = 3$ feet, $G = 23$ feet, and $v = 0.5$ feet per second. From (70) the value of d is found to be 1.17 feet. Now, when $y = 50$ feet, the last formula gives $x = 12.5$ feet, which is the horizontal distance of the middle of the stream from the vertical plane through the crest.

Prob. 70. In the above example what velocity of approach is necessary in order that there may be no drop in the plane of the crest? What is the range for this case?

Art. 71. Triangular Weirs

Triangular weirs are sometimes used for the measurement of water, the arrangement being shown in Fig. 71. Let b be the width of the orifice at the water level, and H the head of water on the vertex. Let an elementary strip of the depth δy be drawn at a distance y below the water level. From similar triangles the length of this strip is $(H-y)b/H$ and the elementary discharge through it then is

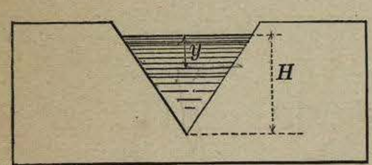

Fig. 71.

$$\delta Q = \frac{b}{H}(H-y)\,\delta y\sqrt{2gy} = \frac{b}{H}\sqrt{2g}\,(Hy^{\frac{1}{2}} - y^{\frac{3}{2}})\,\delta y$$

The integration of this between the limits H and 0 gives the theoretic discharge through the triangular weir, namely,

$$Q = \tfrac{4}{15}\,b\sqrt{2g}\cdot H^{\frac{3}{2}} \qquad (71)_1$$

If the sides of the triangle are equally inclined to the vertical, as should be the case in practice, and if this angle be α, the surface width b may be expressed in terms of α and H, so that the last formula becomes

$$Q = \tfrac{8}{15}\tan\alpha\cdot\sqrt{2g}\cdot H^{\frac{5}{2}} \qquad (71)_2$$

The discharge is thus equal to a constant multiplied by the $2\frac{1}{2}$ power of the measured depth.

Triangular weirs are used but little, as in general they are only convenient when the quantity of water to be measured is small. Such a weir must have sharp inner corners, so that the stream may be fully contracted, and the sides should have equal slopes. The angle at the lower vertex should be a right angle, as this is the only case for which coefficients are known with precision. The depth of water above this lower vertex is to be measured by a hook gage in the usual manner at a point several feet upstream from the notch. Making the angle at the vertex a right angle, and applying a coefficient, the actual discharge per second is given by the expression

$$q = c\cdot\tfrac{8}{15}\sqrt{2g}\,H^{\frac{5}{2}}$$

in which H is the head of water above the vertex. Experiments made by Thomson* indicate that the coefficient c varies less with the head than for ordinary weirs; this, in fact, was anticipated, since the sections of the stream are similar in a triangular notch for all values of H, and hence the influence of the contractions in diminishing the discharge should be approximately the same. As the result of his experiments the mean value of c for heads between 0.2 and 0.8 feet may be taken as 0.592, and hence the mean discharge in cubic feet per second through a right-angled triangular weir may be written

$$q = 2.53\,H^{\frac{5}{2}} \qquad (71)_3$$

* British Association Report, 1858, p. 133.

in which, as usual, H must be expressed in feet. About 4 feet is probably the greatest practicable value for H, and this gives a discharge of only 81 cubic feet per second. When velocity of approach exists, H in this formula should be replaced by $H + 1.4\,h$, as for rectangular weirs with end contractions.

Prob. 71. A triangular orifice in the side of a vessel has a horizontal base b and an altitude d, the head of water on the base being h and that on the vertex being $h + d$. Show that the theoretic discharge through the orifice is $\frac{1}{15}\sqrt{2g(b/d)} \cdot [4(h+d)^{\frac{5}{2}} - (4\,h + 10\,d)h^{\frac{3}{2}}]$.

Art. 72. Trapezoidal Weirs

Trapezoidal weirs are sometimes used instead of rectangular ones, as the coefficients vary less in value. The theoretic discharge through a trapezoidal weir which has the length b on the crest, the head H, and the length $b + 2z$ on the water surface, as seen in Fig. 72, is the sum of the discharges through a rectangle of area bH and a triangle of area zH. Taking the former from $(61)_1$ and the latter from $(71)_2$, and replacing $\tan\alpha$ by z/H

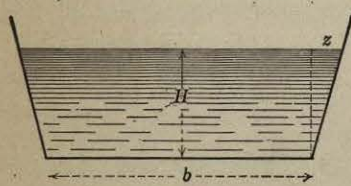

Fig. 72.

$$Q = \tfrac{2}{15}\sqrt{2g}(5b + 4z)H^{\frac{3}{2}}$$

is the theoretic discharge. Here z/H, which is the slope of the ends, may be any convenient number, and it is usually taken as $\frac{1}{4}$, as first recommended by Cippoletti.*

The reasoning from which this conclusion was derived is based upon Francis' rule that the two end contractions in a standard rectangular weir diminished the discharge by a mean amount $3.33 \times 0.2\,H^{\frac{3}{2}}$ (Art. 65), or in general by the amount $c \cdot \frac{2}{3}\sqrt{2\,g} \times 0.2\,H^{\frac{3}{2}}$. If the sides are sloped, however, the discharge through the two end triangles is $c \cdot \sqrt{2g} \times zH^{\frac{3}{2}}$. If, now, the slope is just sufficient so that the extra discharge balances the effect of the end contractions, these two quantities are equal. Equating them, and supposing that c has the same value in each,

* Cippoletti, Canal Villoresi, 1887; see Engineering Record, Aug. 13, 1892.

there results $z = \frac{1}{4}H$. Hence for such a trapezoidal weir the discharge should be the same as that from a suppressed rectangular weir of length b, or, according to Francis, $q = 3.33bH^{\frac{3}{2}}$. Cippoletti, however, concluded from his experiments that the coefficient should be increased about one percent, and he recommended

$$q = 3.367\,bH^{\frac{3}{2}} \tag{72}$$

as the formula for discharge over such a trapezoidal weir when no velocity of approach exists.

Experiments by Flinn and Dyer* indicate that the coefficient 3.367 is probably a little too large. In 32 tests with trapezoidal weirs of from 3 to 9 feet length on the crest and under heads ranging from 0.2 to 1.4 feet, they found 28 to give discharges less than the formula, the percentage of error being over 3 percent in eight cases. The four cases in which the discharge was greater than that given by the formula show a mean excess of about 3.5 percent. The mean deficiency in all the 32 cases was nearly 2 percent. These experiments are not very precise, since the actual discharge was computed by measurements on a rectangular weir, so that the results are necessarily affected by the errors of two sets of measurements. Cippoletti's formula, given above, may hence be allowed to stand as a fair one for general use with trapezoidal weirs in which the slope of the ends is $\frac{1}{4}$. It can, of course, be written in the form

$$q = c \cdot \tfrac{2}{3}\sqrt{2g} \cdot bH^{\frac{3}{2}}$$

where the coefficient c has the mean value 0.629.

When velocity of approach exists, H in this formula is to be replaced by $H + 1.4\,h$, where h is the head due to that velocity. In order to do good work, however, h should not exceed 0.004 feet. Other precautions to be observed are that the cross-section of the canal should be at least seven times that of the water in the plane of the crest, and that the error in the measured head should not be greater than one-third of one percent. On the whole, however, the coefficients for the standard rectangular weir with end contractions are so definitely established, and those for trapezoidal weirs so imperfectly known,

* Transactions American Society of Civil Engineers, 1894, vol. 32, pp. 9–33.

that the use of the latter cannot be recommended in any case where the greatest degree of precision is required.

The above formula for the theoretic discharge may be applied to the Cippoletti trapezoidal weir by putting $z = \frac{1}{4}H$, and introducing a coefficient; thus,

$$q = c\,\tfrac{2}{3}\sqrt{2g}\cdot bH^{\frac{3}{2}}(1 + 0.2\,H/b)$$

is a formula for the actual discharge, in which the values of c are probably not far from those given in Table 63 for rectangular contracted weirs. Here the term $0.2\,H/b$ shows the effect of the two end triangles in increasing the discharge.

Prob. 72. For a head of 0.7862 feet on a Cippoletti weir of 4 feet length the actual discharge in 420 seconds was 3912.3 cubic feet. Compute the discharge by the above formula, and find the percentage of error.

Art. 73. Oblique Weirs

In certain cases weirs or dams are built obliquely across streams and in others there may be either a curve or one or more angles in the line of the crest. When the volume of the flow in the stream is small, so that the water may at all points approach the crest in a direction sensibly at right angles to it, the discharge will be proportional to the crest length and may be computed by the formulas already given. When, however, the flow of the stream becomes so great that the water approaches the crest in an oblique direction, the discharge tends to approximate that over a weir placed at right angles to the axis of the stream. This, however, is not strictly true in case the obliquity be material. In such a case the discharge for the same head is increased above that over a weir built normal to the axis of the stream. This condition is sometimes taken advantage of where it is desired to keep down the effect of backwater during times of flood, but such an arrangement causes a loss of available head during times of medium and low water. The problem of the regulation of river heights is, under certain conditions, an important one and is well exemplified by the conditions at the Chaudiere Dam, Ottawa.*

Achiel † experimented on weirs inclined to the axis of the channel at angles varying from 15 to 90°. These weirs were placed in channels 1.64 and 3.28 feet in width, the end contractions were suppressed, and the nappe was thoroughly aerated; their height was 0.82 feet and the heads ranged from 0.04 to 0.60 feet. From these experiments the formula $F_c = 1 - H/Gr$ was deduced. Here H is the measured head on the weir, G the height of the weir crest above the channel of approach, and r a number taken from the table below. F_c then is a correction factor by which the values of the coefficient for a vertical thin-edged weir are to be multiplied in order to obtain the coefficients for each unit of length of the oblique weir. This formula does not hold when the ratio H/G is greater than 0.62, and this ratio should be smaller as the obliquity of the weir increases. In general it can be said that outside the range of the few experiments which have been made but little is known on this subject.

* Engineering News, June 30, 1910.

† Zeitschrift Verein Deutschen Ingenieure; see abstract in Engineering Record, July 3, 1909.

Angle of weir	= 15°	30°	45°	60°	75°	90°
r for broad channels	= 1.4	2.8	5.0	9.1	26.3	∞
r for narrow channels	= 1.2	2.1	3.6	7.7	26.3	∞

Prob. 73. What is the coefficient for an oblique sharp-edged weir with contractions suppressed, 19 feet long and two feet in height when the head is 0.6 feet and the obliquity of the weir 45 degrees?

Art. 74. Computations in the Metric System

The formulas for discharge in Arts. 61–64 are rational and may be used in all systems, the coefficients c being abstract numbers. In the metric system b and H are often expressed in centimeters, but they should be reduced to meters for use in the formulas, and then q will be in cubic meters per second. The mean value of $\sqrt{2g}$ is 4.427 and that of $1/2g$ is 0.05102.

(Art. 62) The head h in meters corresponding to the mean velocity of approach is to be computed from the formula

$$h = 0.05102\,(q/A)^2 \qquad (74)_1$$

in which A is in square meters. For example, take a weir where $B = 200$, $G = 90$, $b = 45.1$, $H = 26.28$ centimeters, and $c = 0.620$.

Then by $(63)_1$ the discharge q' is 0.1112 cubic meters per second, and from $(74)_1$ the head h is 0.0002 meters.

(Art. 63) Table 74a gives values of the coefficient c for weirs with end contractions, with arguments in the metric system. Thus, if H = 5.45 centimeters and b = 0.45 meters, there is found, by interpolation, c = 0.626, which is liable to a probable error of about two units in the third decimal place.

TABLE 74a. COEFFICIENTS c FOR CONTRACTED WEIRS

Arguments in Metric Measures

Effective Head in Centimeters	Length of Weir in Meters						
	0.2	0.3	0.6	0.9	1.5	3.0	5.8
3.	0.633	0.640	0.647	0.653	0.654	0.656	0.657
5.	.618	.624	.634	.638	.640	.641	.642
7.	.606	.613	.622	.625	.627	.629	.630
9.	.601	.608	.616	.619	.621	.624	.625
12.	.596	.602	.609	.613	.615	.618	.620
15.	.591	.597	.605	.608	.611	.615	.617
18.	.588	.593	.601	.605	.608	.613	.615
22.		.589	.597	.603	.606	.612	.614
26.			.594	.599	.604	.610	.613
30.			.590	.595	.601	.608	.611
35.			.586	.592	.597	.605	.610
45.				.585	.593	.601	.608

(Art. 64) Coefficients c for weirs without end contractions, with metric arguments, are given in Table 74b, which has been prepared by the help of Table 64.

(Art. 65) When b and H are in meters and q in cubic meters per second, Francis' formula for suppressed weirs takes the form

$$q = 1.84\, bH^{\frac{3}{2}} \qquad (74)_2$$

and for weirs with end contractions,

$$q = 1.84\,(b - 0.2\,H)\,H^{\frac{3}{2}} \qquad (74)_3$$

the number 1.84 being a mean value of $c \cdot \frac{2}{3}\sqrt{2g}$.

(Art. 67) Table 67 applies to any system of measures, and the formula $q = 1.84\, b(nH)^{\frac{3}{2}}$ then gives the discharge in cubic meters per

TABLE 74b. COEFFICIENTS c FOR SUPPRESSED WEIRS

Arguments in Metric Measures

Effective Head in Centimeters	Length of Weir in Meters						
	5.8	3.0	2.0	1.5	1.2	0.9	0.6
3.	0.658	0.659	0.659	0.660			
5.	.642	.643	.644	.645	0.647	0.649	0.652
7.	.632	.633	.634	.635	.637	.640	.643
9.	.626	.628	.629	.631	.633	.636	.639
12.	.621	.623	.625	.628	.630	.633	.636
15.	.619	.621	.624	.627	.630	.633	.637
18.	.618	.620	.623	.627	.630	.634	.638
22.	.618	.620	.624	.628	.632	.636	.640
26.	.619	.622	.627	.631	.635	.639	.645
30.	.619	.624	.628	.633	.637	.641	
35.	.620	.626	.631	.635	.640	.645	
45.	.622	.630	.635	.641	.645		

second, if b and H be in meters. The metric values of M for use in $(67)_2$ are found by multiplying those in the text by 0.5522.

(Art. 69) The formulas of the first paragraph are transformed into metric measures by replacing 3.33 by 1.84 and 3.01 by 1.72. For formula $(69)_1$ the value of M for dams may range from about 1.4 to 2.3. Table 74c gives metric values of M as deduced from the experiments made by Bazin in 1897, and by Rafter in 1898. The explanation of this table is in all respects like that of Table 69a. All values of M given in Art. 69 may be reduced to metric measures by multiplying by 0.5522, this being the ratio of the value of $\sqrt{2g}$ expressed in meters to that expressed in feet.

(Art. 71) The metric formula for discharge over the triangular weir is $q = 1.40\, H^{\frac{5}{2}}$.

(Art. 72) The metric formula for Cippoletti's trapezoidal weir takes the form $q = 1.86\, bH^{\frac{3}{2}}$.

Prob. 74a. Compute the head that produces a velocity of approach of 50.5 centimeters per second.

Prob. 74b. What are the discharges, in liters per minute, over a suppressed weir 2.35 meters long when the heads on the crest are 12.3, 12.4, and 12.5 centimeters?

TABLE 74c. COEFFICIENTS M FOR DAMS

Metric Measures

Up-stream Slope	Width of Crest Meters	Down-stream Slope	Head H on Crest in Meters					
			0.15	0.30	0.60	0.91	1.22	1.52
1 on 2	0.10	Vertical	1.85	2.03	2.08	2.03	2.04	2.05
1 on 2	0.20	Vertical	1.78	1.90	2.02	2.03	2.04	2.05
1 on 5	0.20	Vertical	1.83	1.84	1.85	1.86	1.87	1.87
1 on 4	0.20	Vertical		1.90	1.92	1.92	1.92	1.92
1 on 3	0.20	Vertical	2.01	2.11	2.04	1.96	1.96	1.96
1 on 2	0.00	1 on 1	2.33	2.34	2.19	2.11	2.06	2.03
1 on 2	0.10	1 on 2	1.73	1.90	1.99	2.02	2.02	2.01
1 on 2	0.20	1 on 5	1.82	1.97	1.94	1.93	1.95	1.97
Vertical	0.80	Vertical	1.43	1.47	1.57	1.66	1.77	1.87
Vertical	0.80*	Vertical	1.63	1.66	1.70	1.79	1.87	1.92
Vertical	2.00	Vertical	1.38	1.43	1.37	1.39	1.44	1.49
Vertical	2.00*	Vertical	1.50	1.56	1.57	1.58	1.60	1.63
1 on 1	Round	Vertical	1.63	1.75	1.91	1.96	1.99	2.01

* For explanation see Art. 69.

Prob. 74c. Compute the discharge over a submerged weir when $b = 2.35$, $H = 0.123$, and $H' = 0.027$ meters.

Prob. 74d. Compute the discharge over a dam, like Fig. 68b, when the side slopes are 1 on 2, the length of the crest 4.25 meters, and the head on the crest 1.07 meters.

CHAPTER 7

FLOW OF WATER THROUGH TUBES

ART. 75. LOSS OF ENERGY OR HEAD

A tube is a short pipe which may be attached to an orifice or be used for connecting two vessels. The most common form is a cylinder of uniform cross-section, but conical forms are also used, and in some cases a tube is made of cylinders with different diameters. The laws of flow through tubes are important as a starting-point for the theory of flow through pipes, for the discharge from nozzles, and for the discussion of many practical hydraulic problems. The theorem of Art. 31, that pressure-head plus velocity-head is a constant for a given section of a tube, is only true when there are no losses due to friction and impact. As a matter of fact such losses always exist and must be regarded in practical computations.

Energy in a tube filled with moving water exists in two forms, in potential energy of pressure and in kinetic energy of motion. Thus in the horizontal tube of Fig. 75a let two piezometers (Art. 37) be inserted at the sections a_1 and a_2 where the velocities are v_1 and v_2 and it is found that the water rises to the heights h_1 and h_2 above the middle of the tube. Let W be the weight of water that passes each section per second. Then in the first section the pressure energy per second is Wh_1 and the kinetic energy per second is $W \cdot v_1^2/2g$, so that the total energy of the water passing that section in one second is

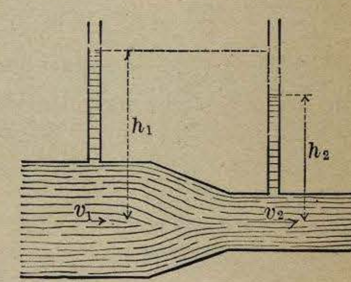

Fig. 75a.

$$Wh_1 + W \cdot v_1^2/2g$$