in which h/H can be computed from the equivalent expression $(2\,cH/3\,(H+G))$.[2] For example, from the above data the value of h/H is 0.0095, whence the quantity in the parenthesis is 1.019 and $q = 4.16 \times 1.019 = 4.24$ cubic feet per second.

Prob. 64. Compute the discharge per second over a weir without end contractions when b = 0.995 feet, H = 0.7955 feet, G = 4.6 feet.

Art. 65. Francis' Formulas

The formulas most extensively used for computing the flow through weirs are those established by Francis in 1854* from the discussion of his numerous and carefully conducted experiments, but as they are stated without tabular coefficients they are to be regarded as giving only mean approximate results. The experiments were made on large weirs, most of them 10 feet long, and with heads ranging from 0.4 to 1.6 feet, so that the formulas apply particularly to such, rather than to short weirs and low heads. The shape and details of the crest of the weirs are shown in Fig. 65 and the head was measured as described in Art. 60. The length b and the head H being expressed in feet, the discharge per second, when there is no velocity of approach, is, for weirs without end contractions, or suppressed weirs,

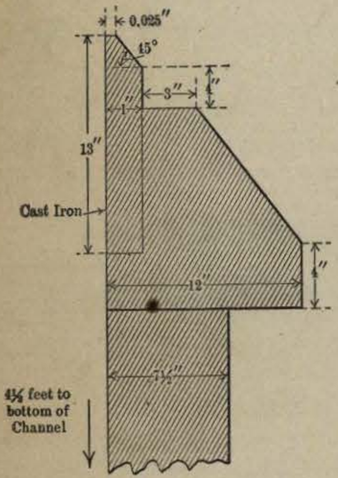

Fig. 65.

$$q = 3.33\, bH^{\frac{3}{2}} \qquad (65)_1$$

and for weirs with two end contractions,

$$q = 3.33\,(b - 0.2\,H)H^{\frac{3}{2}} \qquad (65)_2$$

Here it was considered by Francis that the effect of each end contraction is to diminish the effective length of the weir by

* Lowell Hydraulic Experiments (4th edition, New York, 1883), p. 133.

0.1 H. In these formulas b and H must be taken in feet and q will be found in cubic feet per second.

It is seen that the number 3.33 is $c \cdot \frac{2}{3}\sqrt{2g}$, where c is the true coefficient of discharge. The 88 experiments from which this mean value was deduced show that the coefficient 3.33 actually ranged from 3.30 to 3.36, so that by the use of the mean value an error of one per cent in the computed discharge may occur. When such an error is of no importance, the formula may be safely used for weirs longer than 4 feet and heads greater than 0.4 feet.

Francis' method of correcting for velocity of approach differs from that of Hamilton Smith, and is the same as that explained in Art. 50. The head h causing the velocity of approach is computed in the usual way, and then the formulas are written, for weirs without end contractions,

$$q = 3.33\, b[(H+h)^{\frac{3}{2}} - h^{\frac{3}{2}}]$$

and for weirs with end contractions,

$$q = 3.33\,(b - 0.2\,H)[(H+h)^{\frac{3}{2}} - h^{\frac{3}{2}}]$$

It is necessary that this method of introducing the velocity of approach should be strictly observed, since the mean number 3.33 was deduced for this form of expression.

Prob. 65. What modification would you introduce in $(65)_2$, if the weir has one end with and the other end without contraction?

Art. 66. Other Weir Formulas

Fteley and Stearns* in the discussion of their experiments on standard weirs proposed the formula

$$Q = 3.33\, bH^{\frac{3}{2}} + 0.007\, b \qquad (66)_1$$

in which correction for end contraction is made as in the Francis formula (Art. 65). They also proposed the following corrections for velocity of approach for use in the above formula $(66)_1$.

$$H + h = H + 1.50\frac{v^2}{2g} \qquad H + h = H + 2.05\frac{v^2}{2g}$$

* Transactions American Society of Civil Engineers, vol. 12.

the former of which is applicable to suppressed weirs and the latter to weirs having end contractions, v being the mean velocity of approach.

Among the most recent formulas for the flow over weirs are those of Bazin* who experimented on sharp crests varying in height from 0.79 to 3.72 feet and in length from 1.64 to 6.56 feet. From his discussion of his own results as well as those of Fteley and Stearns, he deduced the following formulas for weirs without end contractions

$$Q = \mu \sqrt{2g} \cdot bH^{\frac{3}{2}} \quad \text{and} \quad Q = m \sqrt{2g} \cdot bH^{\frac{3}{2}} \qquad (66)_2$$

The first of these formulas is applicable to cases where there is no velocity of approach, while the second, by means of the coefficient m, corrects for any approach velocity which may exist. The relations between m, μ, and H are

$$m = \mu\left[1 + 0.55\left(\frac{H}{G+H}\right)^2\right] \qquad \mu = 0.405 + \frac{0.00984}{H}.$$

where G is the height of the weir crest above the bottom of the channel of approach. It is thus seen that m varies with the head and also with the height of the weir above the bottom of the channel, both of which factors influence the velocity of approach. On the other hand μ varies only with the head.

Table 66. Bazin's Coefficients m for Suppressed Weirs

Head in Feet	Height G of Weir Crest, in Feet				
	0.79	1.15	1.64	2.46	3.72
0.20	0.447	0.445	0.444	0.444	0.443
0.39	.447	.440	.435	.433	.431
0.59	.458	.446	.438	.432	.427
0.79	.470	.455	.443	.434	.426
0.98	.482	.464	.418	.437	.427
1.18	.495	.473	.454	.441	.428
1.38			.460	.444	.429

* Annales des ponts et chaussées, 1898; translated by Maichal and Trautwine in Proceedings Engineers' Club of Philadelphia, vols. 5, 7, and 9.

In the above table are given some of the values of the coefficient m determined by Bazin's experiments for varying heads and heights G of standard sharp-crested weirs. These coefficients are applicable only to weirs having suppressed end contractions. While these formulas give results agreeing well with many weir gagings under ordinary heads, the expression for μ cannot be regarded as a rational one since it becomes infinite when H is zero.

Prob. 66. What will be the value of m in the case of a weir 2.50 feet high when H is 1.25 feet?

Art. 67. Submerged Weirs

When the water on the downstream side of the weir is allowed to rise higher than the level of the crest, the weir is said to be submerged. In such cases an entire change of condition results, and the preceding formulas are inapplicable. Let H be the head above the crest measured upstream from the weir by the hook gage in the usual manner, and let H' be the head above the crest of the water downstream from the weir measured by a second hook gage. If H be constant, the discharge is uninfluenced until the lower water rises to the level of the crest, provided that free access of air is allowed beneath the descending sheet of water. But as soon as it rises slightly above the crest so that H' has small values, the contraction is suppressed and the discharge hence increased. As H' increases, however, the discharge diminishes until it becomes zero when H' equals H. Submerged weirs cannot be relied upon to give precise measurements of discharge on account of the lack of experimental knowledge regarding them, and should hence always be avoided if possible.

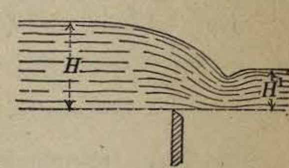

Fig. 67.

The following method for estimating the discharge over submerged weirs without end contractions is taken from the discussion given by Herschel* of the experiments made by Francis and by Fteley and Stearns. The observed head H is first multiplied

* Transactions American Society of Civil Engineers, 1885, vol. 14, p. 194.

by a number n, which depends upon the ratio of H' to H, and then the discharge is to be computed by using the modified Francis' formula

$$q = 3.33\, b\, (nH)^{\frac{3}{2}} \qquad (67)_1$$

The values of n deduced by Herschel* are given in Table 67. They are liable to a probable error of about one unit in the second decimal place when H' is less than 0.2 H, and to greater errors in the remainder of the table, values of n less than 0.70 being in particular uncertain. It is seen that H' may be nearly one-fifth of H without affecting the discharge more than two percent.

TABLE 67. FACTORS FOR SUBMERGED WEIRS

$\frac{H'}{H}$	n	$\frac{H'}{H}$	n	$\frac{H'}{H}$	n	$\frac{H'}{H}$	n
0.00	1.000	0.18	0.989	0.38	0.935	0.58	0.856
.01	1.004	.20	0.985	.40	0.929	.60	0.846
.02	1.006	.22	0.980	.42	0.922	.62	0.836
.04	1.007	.24	0.975	.44	0.915	.64	0.824
.06	1.007	.26	0.970	.46	0.908	.66	0.813
.08	1.006	.28	0.964	.48	0.900	.70	0.787
.10	1.005	.30	0.959	.50	0.892	.75	0.750
.12	1.002	.32	0.953	.52	0.884	.80	0.703
.14	0.998	.34	0.947	.54	0.875	.90	0.574
.16	0.994	.36	0.941	.56	0.866	1.00	0.000

A rational formula for the discharge over submerged weirs may be deduced in the following manner. The theoretic discharge may be regarded as composed of two portions, one through the upper part $H - H'$, and the other through the lower part H'. The portion through the upper part is given by the usual weir formula, $H - H'$ being the head, or

$$Q_1 = \tfrac{2}{3}\sqrt{2g}\cdot b(H - H')^{\frac{3}{2}}$$

and that through the lower part is given by the formula for a submerged orifice (Art. 51), in which b is the breadth, H' the height, and $H - H'$ the effective head, or

$$Q_2 = bH'\sqrt{2g(H - H')}$$

* Transactions American Society of Civil Engineers, 1885, vol. 14, p. 194.

The addition of these gives the total theoretic discharge,

$$Q = \tfrac{2}{3}\sqrt{2g}\cdot b(H - H')^{\frac{3}{2}} + \sqrt{2g}\cdot bH'(H - H')^{\frac{1}{2}}$$

which may be put into the more convenient form,

$$Q = \tfrac{2}{3}\sqrt{2g}\cdot b(H + \tfrac{1}{2}H')(H - H')^{\frac{1}{2}}$$

The actual discharge per second may now be written,

$$q = c\cdot \tfrac{2}{3}\sqrt{2g}\cdot b(H + \tfrac{1}{2}H')(H - H')^{\frac{1}{2}}$$

in which c is the coefficient of discharge.

Fteley and Stearns adopted the above formula for the discharge, or placing M for $c \cdot \frac{2}{3}\sqrt{2g}$, they wrote,*

$$q = \mathrm{M}b(H + \tfrac{1}{2}H')(H - H')^{\frac{1}{2}} \qquad (67)_2$$

and from their experiments deduced the following values of the coefficient M:

for	$H'/H =$	0.00	0.04	0.08	0.12	0.16	0.2	0.3
	M =	3.33	3.35	3.37	3.35	3.32	3.28	3.21
for	$H'/H =$	0.4	0.5	0.6	0.7	0.8	0.9	1.0
	M =	3.15	3.11	3.09	3.09	3.12	3.19	3.33

These are for suppressed weirs; for contracted weirs few or no experiments are on record.

Thus far in this article velocity of approach has not been considered. This may be taken into account in the usual way by determining the velocity-head h, and thus correcting H. But it is unnecessary, on account of the limited use of submerged weirs, and the consequent lack of experimental data, to develop this branch of the subject. What has been given above will enable an approximate probable estimate to be made of the discharge in cases where the water accidentally rises above the crest, and further than this the use of submerged weirs cannot be recommended.

Prob. 67. Compute by the two methods the discharge over a submerged weir when $b = 8$, $H = 0.46$, and $H' = 0.22$ feet.

* Transactions American Society of Civil Engineers, 1883, vol. 12, p. 103.

Art. 68. Rounded and Wide Crests

When the inner edge of the crest of a weir is rounded as at *A* in Fig. 68, the discharge is materially increased as in the case of orifices (Art. 53), or rather the coefficients of discharge become much larger than those given for the standard sharp crests. The degree of rounding influences so much the amount of increase that no definite values can be stated, and the subject is here merely mentioned in order to emphasize the fact that a rounded inner edge is always a source of error. If the radius of the rounded edge is small, the sheet of escaping water is at a point below the top (*a* in the figure), which has the practical effect of increasing the measured head by a constant quantity. The experiments of Fteley and Stearns show that when the radius is less than one-half an inch, the discharge can be computed from the usual weir formula, seven-tenths of the radius being first added to the measured head *H*.

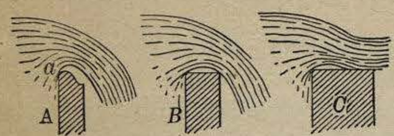

Fig. 68.

Two wide-crested weirs with square inner corners are shown in Fig. 68, the one at *B* being of sufficient width so that the descending sheet may just touch the outer edge, causing the flow to be more or less disturbed, while that at *C* has the sheet adhering to the crest for some distance. In both cases the crest contraction occurs, although water instead of air may fill the space above the inner corner. For *B* the discharge may be equal to or greater than that of the standard weir having the same head *H*, depending upon whether the air has or has not free access beneath the sheet in the space above the crest. For *C* the discharge is always less than that of the standard weir.

Table 68 is an abstract from the results obtained by Fteley and Stearns,* and gives the corrections in feet to be subtracted from the depths on a wide crest, like *C* in Fig. 68, in order to obtain the depths on a standard sharp-crested suppressed weir giving the same discharge.

* Transactions American Society of Civil Engineers, 1883, vol. 12, p. 96.

Table 68. Corrections for Wide Crests

Head on Wide Crest Feet	Width of Crest in Inches						
	2	4	6	8	10	12	24
0.05	0.010	0.009	0.009	0.009	0.009	0.009	0.009
.10	.016	.018	.017	.017	.017	.017	.017
.20	.012	.029	.031	.032	.033	.033	.034
.30		.030	.041	.045	.047	.048	.050
.40		.022	.045	.055	.060	.062	.066
.50		.006	.041	.060	.069	.074	.082
.60			.031	.059	.075	.083	.097
.70			.017	.052	.075	.089	.112
.80			.000	.040	.071	.091	.125
.90				.027	.062	.089	.137
1.00				.011	.050	.082	.149
1.20					.021	.061	.168
1.40						.032	.180

The U. S. Geological Survey* during 1903 caused to be made at the laboratory of Cornell University a series of experiments on broad-crested weirs. These experiments covered crest widths of from 0.479 to 16.302 feet and heads from 0.2 to 5.0 feet. Without here going into detail, it was concluded from the results obtained that a coefficient of 2.64 may be used in the formula $q = cbH^{\frac{3}{2}}$ for all cases of broad-crested weirs exceeding 3.0 feet in breadth and under heads in excess of 2.0 feet. For heads of less than 2.0 feet the coefficients are variable and dependent on both the head and the width of the crest as well as on whether or not the nappe or water sheet remains attached to or becomes detached from the downstream face of the weir. For heads of less than 0.5 feet the sheet is very unstable and the coefficients fluctuate correspondingly. From 0.5 to 2.0 feet the coefficients are still somewhat variable and uncertain but become quite steady for higher heads and on crests exceeding 3.0 feet in width. In general when the sheet becomes detached, the coefficient becomes equal to that for a sharp-crested weir; when the sheet is adherent, the coefficient may drop to 2.60. The possible range

* Water Supply and Irrigation Paper, No. 200, U. S. Geological Survey.

in coefficients for such cases is hence seen to be from 2.60 to 3.33.

Prob. 68. Compute the discharge for a weir like C in Fig. 68 when the width of crest is 1.5 feet, the head 0.85 feet, and the length of weir 10 feet.

Art. 69. Waste Weirs and Dams

Waste weirs are constructed at the sides of reservoirs in order to allow the surplus water to escape. They are usually arranged so that the end contractions are suppressed. When the crest is narrow and the front vertical, so that the descending sheet of water has air upon its lower side, the discharge is approximately given by Francis' weir formula (Art. 65),

$$q = 3.33\, bH^{\frac{3}{2}}$$

in which b is the length of the crest, and H the head measured some distance back from the crest. When the crest is wide and the approach to it is inclined, as is often the case, the discharge is somewhat smaller. For a crest about three feet wide and level, with an inclined approach back of it, Francis deduced

$$q = 3.01\, bH^{1.53}$$

which, for a head of one foot, gives a discharge ten percent less than that of the first formula.

In constructing a waste weir the discharge q is generally known or assumed, and it is required to determine b and H. The latter being taken at 1, 2, or 3 feet, as may be judged safe and proper, b is found by one of these formulas. For example, let the crest be wide, q be 87 cubic feet per second, and H be 2.0 feet, then

$$\log b = \log 87 - \log 3.01 - 1.53 \log 2$$

from which $\log\ b = 1.0004$, whence $b = 10.0$ feet. When, however, the crest is narrow, the first formula gives $b = 9.2$ feet. Evidently no great precision is needed in computing the length of a waste weir, since it is difficult to determine the exact discharge which is to pass over it, and an ample factor of safety should be introduced to cover unusual floods.

The above formulas may be used for obtaining the approximate flow of a stream in which a dam with level crest has been built. The water, however, is often received upon an apron of timber or masonry, and the inclination of this, as well as the inclination of the approach to the crest, materially modifies the discharge. The formula,

$$q = c \cdot \tfrac{2}{3}\sqrt{2g} \cdot bH^{\frac{3}{2}} = \mathrm{M}bH^{\frac{3}{2}} \qquad (69)_1$$

is usually employed for dams, and it is found that the value of M, for English measures, may range under different circumstances from 2.5 to 4.2. This formula is modified below for the influence of velocity of approach (Art. 62).

Experiments were made by Bazin in 1897* on dams from 1.6 to 2.5 feet high with heads of water on the crests ranging from 0.2

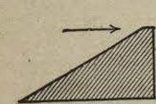

Fig. 69*a*.

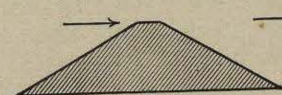

Fig. 69*b*.

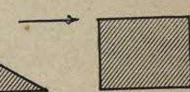

Fig. 69*c*.

to 1.4 feet. For the case of Fig. 69*a* the approach had an inclination of 1 on 2 and the front was vertical; when the width of the crest was 0.33 feet, the coefficient M varied from 3.24 to 4.12 as the head increased from 0.27 to 1.41 feet; when the width of the crest was 0.66 feet, M varied from 3.10 to 3.89 for similar heads. For the case of Fig. 69*b* both approach and apron had slopes of 1 on 2 and the crest was 0.66 feet wide; here M increased from 2.83 to 3.75 as the head ranged from 0.22 to 1.42 feet. For Fig. 69*c*, with a crest 2.62 feet wide, M ranged from 2.47 to 2.76, but when the upstream corner was rounded to a radius of 4 inches, it ranged from 2.71 to 3.12. Here it is seen that widening the crest decreases the discharge, as already noted in Art. 68, and that the apron produces a similar influence.

Experiments on a larger scale were made by Rafter in 1898, for the U. S. Deep Waterways Commission at the canal of the Cornell hydraulic laboratory, in which the flow over dams

* Annales des ponts et chaussées, 1898; translated by Rafter in Transactions American Society of Civil Engineers, 1900, vol. 44, p. 254.

was measured by a standard weir. The results of these experiments are given in Table 69*a*, the first five being for dams of the form shown in Fig. 69*a*, the next three for dams like Fig. 69*b*, and the next four for dams like Fig. 69*c*, those marked with an asterisk having the upstream corner rounded

TABLE 69*a*. COEFFICIENTS M FOR DAMS

Upstream Slope	Width of Crest Feet	Down-stream Slope	Head H on Crest in Feet						
			0.5	1.0	1.5	2.0	3.0	4.0	5.0
1 on 2	0.33	Vertical	3.35	3.68	3.82	3.77	3.68	3.70	3.71
1 on 2	0.66	Vertical	3.22	3.44	3.59	3.66	3.68	3.70	3.71
1 on 5	0.66	Vertical	3.31	3.33	3.34	3.35	3.38	3.39	3.39
1 on 4	0.66	Vertical		3.44	3.46	3.48	3.48	3.48	3.48
1 on 3	0.66	Vertical	3.64	3.82	3.83	3.69	3.55	3.55	3.55
1 on 2	0.00	1 on 1	4.21	4.24	4.09	3.97	3.83	3.74	3.68
1 on 2	0.66	1 on 2	3.14	3.42	3.45	3.61	3.66	3.66	3.64
1 on 2	0.33	1 on 5	3.30	3.57	3.60	3.51	3.47	3.54	3.57
Vertical	2.62	Vertical	2.60	2.67	2.75	2.84	3.01	3.21	3.39
Vertical	2.62 *	Vertical	2.96	3.01	3.03	3.08	3.25	3.38	3.47
Vertical	6.56	Vertical	2.50	2.60	2.54	2.48	2.51	2.61	2.70
Vertical	6.56 *	Vertical	2.71	2.83	2.84	2.84	2.86	2.90	2.94
1 on 1	Round	Vertical	2.95	3.17	3.31	3.45	3.56	3.61	3.65

to a radius of 4 inches. The last line of the table refers to a section whose top was 5 feet wide and rounded to a radius of 3.37 feet, the rounding beginning on the upstream side 1.00 foot below the crest. The height of these dams varied from 4.56 to 4.91 feet, and the length of the crest was in all cases 6.58 feet.*

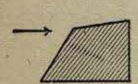
Fig. 69*d*.

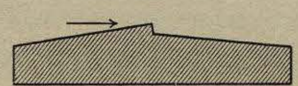
Fig. 69*e*.

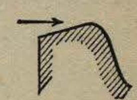
Fig. 69*f*.

Rafter also made experiments on some other forms of dams. The one shown in Fig. 69*d* had a vertical front 4.57 feet deep, and the two back slopes were 1 on 6 and 1 on $\frac{3}{4}$, the width of the former being 4.5 feet; the values of M for this case ranged from

* Transactions American Society of Civil Engineers, 1900, vol. 44, p. 266.

3.33 to 3.46 for heads ranging from 1.0 to 6.0 feet. The one shown in Fig. 69*e* had a total width of about 23 feet and a height of 4.53 feet, the slopes of the approach and apron being 1 on 6, and that just below the crest about 1 on $\frac{1}{4}$, the vertical depth of this being 0.75 feet; for this the mean values of M ranged from 3.07 to 3.27 for heads ranging from 1.0 to 6.0 feet, the smaller coefficients being due to the contact of the water with the apron.

For ogee dams similar in cross-section to Fig. 69*f*, experiments were made in 1903 * by the U. S. Geological Survey. The widths a of the various crests ranged from 3.0 to 6.0 feet, the radii r from 1.0 to 3.0 feet, and the rises c from 0.75 feet to 2.88 feet. From a discussion of these results it was concluded that the coefficient M has a value of $(3.78 - 0.16\,s)\,H^{\frac{1}{20}}$, where s is the ratio of a to c in Fig. 69*g*. For example, when $s = 3.0/1.5$ and $H = 4.0$ feet, then $M = 3.70$.

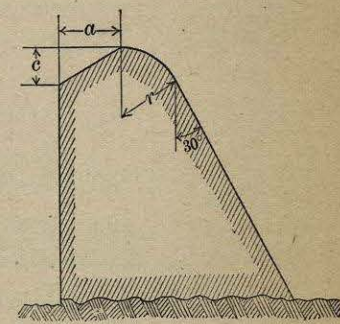

Fig. 69*g*.

In the table on the next page are shown the principal results of the above experiments on models of ogee dams:

The height of the crests above the bottom of the channel of approach of all the models was 11.25 feet and the heads were measured at two points, one 10.3 feet and the other 16.059 feet upstream from the weir crest. It was found that in general the reading of the gage nearest the weir was not affected by the surface curve for heads of less than three feet on the crest. The water which was used in these experiments was measured over a sharp-crested standard weir 6.65 feet high and having a crest 15.93 feet in length.

By the use of these coefficients the discharge of a stream over a dam may be computed with a good degree of precision. For-

* Water Supply and Irrigation Paper No. 200, p. 131.

TABLE 69*b*. COEFFICIENTS M FOR OGEE DAMS

	1	2	3	4	5	6	7	8
a, feet	3.00*	3.00†	3.00*	3.00†	3.00*	4.50*	‡4.83*	6.00*
c, feet	0.75	0.75	1.50	1.50	2.88	1.00	1.00	1.00
r, feet	3.00	3.00	3.00	3.00	3.00	2.00	2.00	1.00
Head in Feet	Value of Coefficient M							
0.50	3.31		3.21	3.27	3.15	3.18	3.23	3.28
1.00	3.44	3.29	3.48	3.37	3.45	3.30	3.34	3.49
2.00	3.42	3.36	3.67	3.51	3.75	3.42	3.52	3.42
3.00	3.46	3.43	3.72	3.57	3.87	3.49	3.64	3.31
4.00	3.52	3.53	3.74	3.67	3.88	3.53	3.70	3.30
5.00		3.72		3.82				

* Length of crest 15.969 feet, contractions suppressed.
† Length of crest 7.938 feet, with one end contraction.
‡ This model had upstream corner rounded to radius of 4 inches.

mula $(62)_1$ may be used to find the head corresponding to the velocity of approach, and then

$$q = \text{M}b(H+h)^{\frac{3}{2}} \qquad (69)_2$$

gives the discharge in cubic feet per second. For example, when $\text{M} = 3.45$, $b = 1.50$ feet, $H = 1.25$ feet, $h = 0.02$ feet, then $q = 1110$ cubic feet per second. A fair estimate of the probable error of a coefficient M is from 3 to 4 percent.

The following formula has been found to give good results in automatically applying a correction for the velocity of approach for heads above 0.5 feet.

$$q = \frac{\text{M}bH^{\frac{3}{2}}}{1 - H/3\,(G+H)}$$

where G is the height of the weir crest above the bottom of the approach channel. It will be noted that in form the term $H/3\,(G+H)$ is similar to the correction for velocity of approach used by Bazin (Art. 66).

Prob. 69. Find the length of a waste weir which will be ample to discharge a rainfall of one inch per hour on a drainage area of 3.65 square miles,

the head on the crest of the weir being 2.12 feet. Also when the head is 4.24 feet.

ART. 70. THE SURFACE CURVE

The surface of the water above a weir or dam assumes a curve whose equation is a complex one, but some of the laws that govern the drop in the plane of the crest may be deduced. Let H be the head on the level of the crest measured in perfectly level water at some distance back of the weir, and let d be the depression or drop of the curve below this level in the plane of the weir (Fig. 70). Then the discharge per second q can be expressed in terms of H and d by formula $(50)_4$, placing H for h_2 and d for h_1, and making $h_0 = 0$. This formula becomes, after replacing $\frac{2}{3}\sqrt{2g}$ by M, and Q by q,

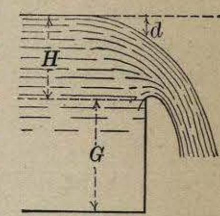

Fig. 70.

$$q = \text{M} \cdot b\,(H^{\frac{3}{2}} - d^{\frac{3}{2}})$$

This expression, it may be remarked, is the true weir formula, and only the practical difficulties of measuring H and d prevent its use. This may be written

$$d^{\frac{3}{2}} = H^{\frac{3}{2}} - q/\text{M}b$$

from which the drop d in the plane of crest of the weir can be found. Let B be the breadth of the feeding canal, G its depth below the crest, and v the mean velocity of approach; then also

$$q = B\,(G+H)v$$

and inserting this in the expression for $d^{\frac{3}{2}}$ it becomes

$$d^{\frac{3}{2}} = H^{\frac{3}{2}} - \frac{B}{\text{M}b}(G+H)\,v \qquad (70)$$

which is an expression for the drop of the curve in terms of the dimensions of the weir, the total head, and the velocity of approach.

The approximate value of the coefficient M is about 3.3 for English measures, but precise values of d cannot be computed unless M and H are known with accuracy. The formula, however, serves to exemplify the laws which govern the drop of the curve in the plane of the weir. It shows that the drop increases with the head on the crest and with the length of a contracted weir, that it decreases with the breadth and depth of the feeding canal, and that it decreases with the velocity of approach. It also shows for suppressed weirs, where $B = b$,