a horizontal plane 1.41 meters below the vertical orifice, which was under a head of 7.19 meters. Compute the coefficient of velocity.

Prob. 59*b*. An orifice 3 centimeters square was under a constant head of 4 meters, and during 230 seconds the jet flowed into a tank which was found to contain 1122 liters. Show that the coefficient of discharge was 0.612.

Prob. 59*c*. Find from the table the coefficient of discharge for a standard circular orifice 2.5 centimeters in diameter under a head of 2.5 meters.

Prob. 59*d*. Compute the discharge through a standard orifice 7.5 centimeters square under a head of 8 meters.

Prob. 59*e*. Compute the time required to empty a canal lock 7 meters wide and 32 meters long through an orifice of 0.9 square meters area, the head on the center of the orifice being 5.1 meters when the lock is filled.

CHAPTER 6

FLOW OF WATER OVER WEIRS

Art. 60. Standard Weirs

A weir is a notch in the top of the vertical side of a vessel or reservoir through which water flows. The notch is generally rectangular, and the word "weir" will be used to designate a rectangular notch unless otherwise specified, the lower edge of the rectangle being truly horizontal, and its sides vertical. The lower edge of the rectangle is called the "crest" of the weir. In

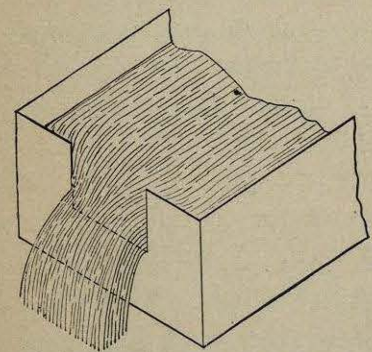

Fig. 60*a*.

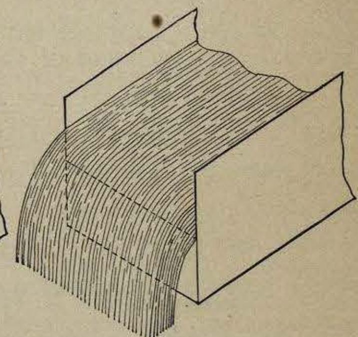

Fig. 60*b*.

Fig. 60*a* is shown the outline of the most usual form, where the vertical edges of the notch are sufficiently removed from the sides of the reservoir or feeding canal, so that the sides of the stream may be fully contracted; this is called a weir with end contractions. In the form of Fig. 60*b* the edges of the notch are coincident with the sides of the feeding canal, so that the filaments of water along the sides pass over without being deflected from the vertical planes in which they move; this is called a weir without end contractions, or with end contractions suppressed. Both kinds of weirs are extensively used for the measurement of water in engineering operations.

It is necessary in order to make accurate measurements of discharge by a weir that the same precaution should be taken as for orifices (Art. 54), namely, that the inner edge of the notch shall be a definite angular corner so that the water in flowing out may touch the crest only in a line, thus insuring complete contraction, as in Fig. 61. In precise observations a thin metal plate will be used for a crest, while in common work it may be sufficient to have the crest formed by a plank of smooth hard wood with its inner corner cut to a sharp right angle and its outer edge beveled. The vertical edges of the weir should be made in the same manner for weirs with end contractions, while for those without end contractions the sides of the feeding canal should be smooth and be prolonged a slight distance beyond the crest. It is also necessary to observe the same precautions as for orifices to prevent the suppression of the contraction (Art. 52), namely, that the distance from the crest of the weir to the bottom of the feeding canal, or reservoir, should be greater than three times the head of water on the crest. For a weir with end contractions a similar distance should exist between the vertical edges of the weir and the sides of the feeding canal. A standard weir is one in which these arrangements have been carefully carried out.

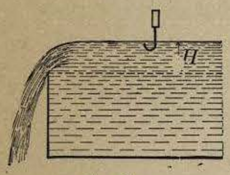

Fig. 60*c*.

The head of water *H* upon the crest of a weir is usually much less than the breadth of the crest *b*. The value of *H* should not be less than 0.1 feet, and it should not exceed 4.5 feet in order to keep within the range of experiments on the standard weir. The least value of *b* in practice is about 0.5 feet, and it does not often exceed 20 feet. Weirs are extensively used for measuring the discharge of small streams, and for determining the quantity of water supplied to hydraulic motors; the practical importance of the subject is so great that numerous experiments have been made to ascertain the laws of flow, and the coefficients of discharge.

Since the head on the crest of a weir is small, it must be determined with precision in order to avoid error in the computed discharge. The hook gage illustrated in Art. 35 is generally used for accurate work in connection with hydraulic motors, and the simpler form, consisting of a hook set into a leveling rod, is usually of sufficient precision for many cases. For rough gagings of streams the heads may be determined by setting a post a few feet upstream from the weir and on the same level as the crest, and measuring the depth of the water over the top of the post by a scale graduated to tenths and hundredths of a foot, the thousandths being either estimated or omitted entirely.

The head *H* on the crest of the weir is in all cases to be measured several feet upstream from the crest, as indicated in Fig. 60*c*. This is necessary because of the curve taken by the surface of the water in approaching the weir. The distance to which this curve extends back from the crest of the weir depends upon many circumstances (Art. 70), but it is generally considered that perfectly level water will be found at 2 or 3 feet back of the crest for small weirs, and at 6 or 8 feet for very large weirs. It is desirable that the hook should be placed at least one foot from the sides of the feeding canal, if possible. As this is apt to render the position of the observer uncomfortable, some experimenters have placed the hook in a pail a few feet away from the canal, the water being led to the pail by a pipe which joins the feeding canal several feet back from the crest, and the water should enter this pipe, not at its end, but through a number of holes drilled at intervals along its circumference. Piezometers (Art. 36) consisting of a glass tube and scale are also sometimes used for large heads, the water being led to the tube by such a pipe. A rough method of measuring the head is to hold a common foot rule on a post set with its top on the same level as the crest and upstream from it.

In a case where it is desired to obtain the highest degree of accuracy care should be taken to reproduce as nearly as possible the conditions which obtained under the experiments from which the coefficients to be used were obtained. This is particularly true of the manner in which the head is to be measured. Thus Poncelet and Lesbros, whose experimental results have been recomputed by Hamilton Smith, measured the head in a reservoir 11.48 feet upstream from the weir. Francis * in some of his experiments measured the head with a hook gage in a wooden stilling box, having a hole one inch in diameter in its bottom which was placed at a level of about four inches below the crest of the weir and about 6 feet upstream from it. Fteley and Stearns † measured the head with a hook gage in a pail placed below the weir, the pail being connected to the channel above the weir at a point 6 feet upstream from the crest. Bazin ‡ in his work on standard thin-edged weirs measured the head in pits 16.40 feet upstream from the weir. One pit was placed on each side of the channel of approach and connected with it through an opening 4 inches in diameter, the opening being exactly flush and at right angles to the channel.

A valuable discussion by Horton,§ in which he tabulates the results of many experiments made on weirs up to 1907, is strongly recommended for reference.

In cases where the flow of water to be measured is constant it is best that a number of observations of the head on the measuring weir should be taken and their mean used in computing the quantity. In most practical cases, however, the flow is constantly fluctuating, and, in order that the total quantity may be accurately determined, observations at frequent intervals must be taken. It may be best in some cases, for convenience or where a high degree of refinement is required, to install an instrument such as that described in Art. 34 for automatically and continuously recording the head. Where such a record has been obtained, it will not do to simply average the heads and use the resulting figure in the formula for the discharge. Since the discharges vary with the three-halves power of the head, it is necessary to compute them for various instants which are so selected that the computed discharges can be fairly averaged before multiplying by the total time between the beginning and end of the tests in order to obtain the total quantity which has passed over the weir. No definite rules can be laid down for this procedure, but every case

* Lowell Hydraulic Experiments (4th edition, New York, 1883).

† Transactions American Society of Civil Engineers, vol. 12.

‡ Translated in Proceedings of Engineers Club, Philadelphia, vols. 7, 9, 10.

§ Water Supply and Irrigation Paper No. 200, U. S. Geological Survey.

should be studied and a plan be adopted which will give the results desired with the required degree of accuracy.

Prob. 60. The trough of a weir, several feet back from the crest, is 6 feet wide, and the depth of water in it is 1.96 feet. What is the mean velocity in this trough when the flow over the weir is 4.24 cubic feet per second?

Art. 61. Formulas for Discharge

Referring to the demonstration of Art. 48 it is seen that a rectangular orifice becomes a weir when the head on its top is zero. Let b be the breadth of the notch, commonly called the length of the crest, and H the head of water on the crest. Then replacing h_1 by 0 and h_2 by H, the theoretic discharge per second is

$$Q = \tfrac{2}{3}\sqrt{2g}\cdot bH^{\frac{3}{2}} \qquad (61)_1$$

The head H is not the depth measured in the vertical plane of the crest, for since the deduction of the formula assumes nothing regarding the fall due to the surface curve, and regards the velocity at any point vertically over the crest as due to the head upon that point below the free water surface, it seems that H should be measured with reference to that surface, as is actually done by the hook gage. The above formula then gives the theoretic discharge per second, provided that there be no velocity at the point where H is measured, which can only be the case when the area of the weir opening is very small compared to that of the cross-section of the feeding canal. This condition would be fulfilled for a rectangular notch at the side of a large pond.

When there is an appreciable velocity of approach of the water at the point where H is measured by the hook gage, the above formula must be modified. Let v be the mean velocity in the feeding canal at this section; this velocity may be regarded as due to a fall, h, from the surface of still water at some distance upstream from the hook, as shown in Fig. 61. Now the true head on the crest of the weir is $H + h$, since this would have been the reading of the hook gage had it been placed where the water had no velocity. Hence the theo-

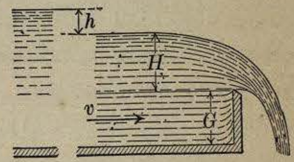

Fig. 61.

retic discharge per second over the weir is

$$Q = \tfrac{2}{3}\sqrt{2g}\cdot b\,(H+h)^{\frac{3}{2}}$$

in which H is read by the hook and h is to be determined from the mean velocity v.

The actual discharge is always less than the theoretic discharge, due to the contraction of the stream and the resistances of the edges of the weir. To take account of these a coefficient is applied to the theoretic formulas in the same manner as for orifices; these coefficients being determined by experiment, the formulas may then be used for computing the actual discharge. It was also proposed by Hamilton Smith to modify the head h, owing to the fact that the velocity of approach is not constant throughout the section, but greater near the surface than near the bottom, as in conduits and streams (Art. 125). Accordingly the following is an expression for the actual discharge:

$$q = c\cdot\tfrac{2}{3}\sqrt{2g}\cdot b\,(H+nh)^{\frac{3}{2}} \qquad (61)_2$$

in which c is the coefficient of discharge whose value is always less than unity, and n is a number which lies between 1.0 and 1.5. For the English system of measures a mean value of $\sqrt{2g}$ is 8.020, but a more precise value can be found from $(6)_1$ for any locality.

The above formulas are not in all respects perfectly satisfactory, and indeed many others have been proposed, one of these being derived from $(50)_4$ by making $h_0 = h$, $h_2 = H$, and $h_1 = 0$. The actual discharge differs, however, so much from the theoretical that the final dependence must be upon the coefficients deduced from experiment, and hence any fairly reasonable formula may be used within the limits for which its coefficients have been established. In spite of the objections which may be raised against all forms of formulas, the fact remains that the measurement of water by weirs is one of the most convenient methods, and for many conditions the most precise method. If the quantity is so small as to pass through a circular orifice less than one foot in diameter, then the orifice is more precise than the weir. For the continuous measurement of water passing through large pipes the Venturi meter gives the best results. With proper precautions the probable error in measurements of discharge by weirs should be less than two or three percent.

Prob. 61. Show by using formula $(61)_1$ that an error of about one-half of one percent results in the computed discharge if an error of 0.001 feet is made in reading the head when H = 0.3 feet.

Art. 62. Velocity of Approach

The head h which produces the velocity v is expressed by $v^2/2g$, and in the case of a weir, the velocity of approach v is due to a fall from the height h; thus the velocity-head is

$$h = v^2/2g = 0.01555\,v^2$$

and when v is known, h can be computed. One way of finding v is to observe the time of passage of a float through a given distance; but this is not a precise method. The usual method is to compute v from an approximate value of the discharge, which is itself first computed by regarding v, and hence h, as zero. This determination is rendered possible by the fact that v is usually small, and hence that h is quite small as compared with H.

Let B be the breadth of the cross-section of the feeding canal at the place where the readings of the hook are taken, and let G be its depth below the crest (Fig. 61). The area of that cross-section then is

$$A = B\,(G+H)$$

The mean velocity in this section now is

$$v = q'/A$$

in which the discharge q' is found from the formula

$$q' = c\,\tfrac{2}{3}\sqrt{2g}\cdot bH^{\frac{3}{2}}$$

This value of q' is an approximation to the actual discharge; from it v is found, and then h, after which the discharge q can be computed. If thought necessary, h may be recomputed by using q instead of q'; but this will rarely be necessary.

For example, a small weir with end contractions, which was used in the hydraulic laboratory of Lehigh University prior to 1896, had B = 7.82 feet and G = 2.5 feet. The length of the weir b was adjustable according to the quantity of water delivered by the stream. On April 10, 1888, the value of b was 1.330 feet, and values of H ranged from 0.429 to 0.388 feet.

It is required to find the velocity v and the head h, when H = 0.429 feet. Here the coefficient c is 0.602 (Table 63); hence the approximate discharge per second is

$$q' = 0.602 \times \tfrac{2}{3} \times 8.02 \times 1.33 \times 0.429^{\frac{3}{2}}$$

or

$$q' = 1.203 \text{ cubic feet per second.}$$

The mean velocity of approach then is

$$v = \frac{1.203}{(2.5+0.4)\,7.82} = 0.053 \text{ feet per second,}$$

and the head h producing this velocity is

$$h = 0.01555 \times 0.053^2 = 0.00004 \text{ feet,}$$

which is too small to be regarded, since the hook gage used determined the heads only to thousandths of a foot.

The head h may be directly expressed in terms of the discharge by substituting for v its value q/A; thus

$$h = 0.01555\,(q/A)^2 \qquad (62)_1$$

and when q is approximately known, this expression will be found a very convenient one for computing the value of the head corresponding to the velocity of approach.

The head h may be directly computed, when it is small compared with H, from the formula

$$h = H\left(\frac{2\,cHb}{3\,(H+G)B}\right)^2 \qquad (62)_2$$

To deduce this, let the above values of A and q' be inserted in the equation $v = q'/A$, and then v be placed in $h = v^2/2g$. This is a convenient expression for logarithmic computation.

With a weir opening of given size under a given head H, the velocity of approach is less the greater the area of the section of the feeding canal, and it is desirable in building a weir to make this area large so that the velocity v may be small. For large weirs, and particularly for those without end contractions, v is sometimes as large as one foot per second, giving h = 0.0155 feet, and these should be regarded as the highest values allowable if precision of measurement is required.

Prob. 62. Fteley and Stearns' large suppressed weir had the following dimensions: $b = B$ = 18.996 feet, G = 6.55 feet, and the greatest measured head was 1.6038 feet. Taking c = 0.622, compute the velocity of approach and its velocity-head.

Art. 63. Weirs with End Contractions

Let b be the breadth of the notch or length of the weir, H the head above the crest measured by the hook gage, and c an experimental coefficient. Then, when there is no velocity of approach, the discharge per second is

$$q = c \cdot \tfrac{2}{3}\sqrt{2g} \cdot bH^{\frac{3}{2}} \qquad (63)_1$$

But when the mean velocity of approach at the section where the hook is placed is v, let h be the head which would produce this velocity as computed by $(62)_2$. Then the discharge is

$$q = c \cdot \tfrac{2}{3}\sqrt{2g} \cdot b\,(H + 1.4\,h)^{\frac{3}{2}} \qquad (63)_2$$

The quantity $H + 1.4\,h$ is called the effective head on the crest, and, as shown in the last article, the velocity-head h is usually small compared with the head H.

Table 63 contains values of the coefficient of discharge c as deduced by Hamilton Smith, from a discussion of the experiments made by Lesbros, Francis, Fteley and Stearns, and others on standard weirs.* In these experiments q was determined by actual measurement in a tank of large size, and the other quantities being observed, the coefficient c was computed. Values of c for different lengths of weir and for different heads were thus obtained, and after plotting them mean curves were drawn from which immediate values were taken. The heads in the first column are the effective heads $H + 1.4\,h$; but as h is small, little error can result in using H as the argument with which to enter the table in selecting a coefficient.

It is seen from the table that the coefficient c increases with the length of the weir, which is due to the fact that the end contractions are independent of the length. The coefficient also

* Hamilton Smith, Hydraulics, 1884, p. 132.

increases as the head on the crest diminishes. The table also shows that the greatest variation in the coefficients occurs under small heads, which are hence to be avoided in order to secure accurate measurements of discharge.

TABLE 63. COEFFICIENTS FOR CONTRACTED WEIRS

Effective Head in Feet	Length of Weir in Feet						
	0.66	1	2	3	5	10	19
0.1	0.632	0.639	0.646	0.652	0.653	0.655	0.656
0.15	.619	.625	.634	.638	.640	.641	.642
0.2	.611	.618	.626	.630	.631	.633	.634
0.25	.605	.612	.621	.624	.626	.628	.629
0.3	.601	.608	.616	.619	.621	.624	.625
0.4	.595	.601	.609	.613	.615	.618	.620
0.5	.590	.596	.605	.608	.611	.615	.617
0.6	.587	.593	.601	.605	.608	.613	.615
0.7		.590	.598	.603	.606	.612	.614
0.8			.595	.600	.604	.611	.613
0.9			.592	.598	.603	.609	.612
1.0			.590	.595	.601	.608	.611
1.2			.585	.591	.597	.605	.610
1.4			.580	.587	.594	.602	.609
1.6				.582	.591	.600	.607

Interpolation may be made in this table for heads and lengths of weirs intermediate between the values given, regarding the coefficient to vary uniformly between the values given. When coefficients are frequently required for a weir of given length, it will be best to make out a special table for that weir and to diagram the results to a large scale on cross-section paper, so that interpolation for different heads can be more readily made.

As an example of the use of the formulas and Table 63, let it be required to find the discharge per second over a weir 4 feet long when the head H is 0.457 feet, there being no velocity of approach. From the table the coefficient of discharge is 0.614 for H = 0.4 and 0.6095 for H = 0.5, which gives about 0.612 when H = 0.457. Then the discharge per second is

$$q = 0.612 \times \tfrac{2}{3} \times 8.02 \times 4 \times 0.457^{\frac{3}{2}} = 4.04 \text{ cubic feet.}$$

V.

If the width of the feeding canal be 7 feet, and its depth below the crest be 1.5 feet, the velocity-head is

$$h = 0.01555\left(\frac{4.04}{7 \times 1.96}\right)^2 = 0.00134 \text{ feet.}$$

The effective head now becomes $H + 1.4\,h = 0.459$ feet, and the discharge per second over the weir is

$$q = 0.612 \times \tfrac{2}{3} \times 8.02 \times 4 \times 0.459^{\frac{3}{2}} = 4.07 \text{ cubic feet.}$$

It is to be observed that the reliability of these computed discharges depends upon the precision of the observed quantities and upon the coefficient c; this is probably liable to an error of one or two units in the third decimal place, which is equivalent to a probable error of about three-tenths of one per cent. On the whole, regarding the inaccuracies of observation, a probable error of one per cent should at least be inferred, so that the value q = 4.07 cubic feet per second should strictly be written q = 4.07 ± 0.04; that is, the discharge per second has 4.07 cubic feet for its most probable value, and it is as likely to be between the values 4.03 and 4.11 as to be outside of those limits.

When velocity of approach is considered, an excellent method of computing the discharge is to expand the parenthesis of $(63)_2$ in a series and use only two terms of the expansion, thus

$$q = c \cdot \tfrac{2}{3}\sqrt{2g} \cdot bH^{\frac{3}{2}}\left(1 + 2.1\frac{h}{H}\right) \qquad (63)_3$$

in which h/H is computed from the expression $(2\,cHb/3\,(H + G)\,B)^2$, where B is the breadth of the feeding canal and G is the distance of the bottom of the canal below the level of the crest (Fig. 61). For example, in the case of the last paragraph h/H is found from the numerical data to be 0.00297, whence the quantity in the parenthesis is 1.00624 and the discharge is 4.04 × 1.00624 = 4.07 cubic feet per second. It is seen that this method requires less numerical work than that of the one explained above.

In very precise work the value of the acceleration g should be computed from formula $(6)_1$ for the particular latitude and elevation above sea level where the weir is located.

Prob. 63. A weir in north latitude 40° 24′ and 395 feet above sea level has a length of 2.5 feet. Compute the discharges over it, the feeding canal having the width 6 feet and the depth below crest 1.6 feet, when the heads on the crest are 0.314, 0.315, and 0.316 feet.

Art. 64. Weirs without End Contractions

For weirs without end contractions, or suppressed weirs as they are often called, when there is no velocity of approach, the discharge per second is

$$q = c \cdot \tfrac{2}{3}\sqrt{2g} \cdot bH^{\frac{3}{2}} \qquad (64)_1$$

and when there is velocity of approach,

$$q = c \cdot \tfrac{2}{3}\sqrt{2g} \cdot b(H + 1\tfrac{1}{3}h)^{\frac{3}{2}} \qquad (64)_2$$

Here the notation is the same as in the last article, and c is to be taken from Table 64, which gives the coefficients of discharge as deduced by Smith, in 1888.

Table 64. Coefficients for Suppressed Weirs

Effective Head in Feet	Length of Weir in Feet						
	19	10	7	5	4	3	2
0.1	0.657	0.658	0.658	0.659			
0.15	.643	.644	.645	.645	0.647	0.649	0.652
0.2	.635	.637	.637	.638	.641	.642	.645
0.25	.630	.632	.633	.634	.636	.638	.641
0.3	.626	.628	.629	.631	.633	.636	.639
0.4	.621	.623	.625	.628	.630	.633	.636
0.5	.619	.621	.624	.627	.630	.633	.637
0.6	.618	.620	.623	.627	.630	.634	.638
0.7	.618	.620	.624	.628	.631	.635	.640
0.8	.618	.621	.625	.629	.633	.637	.643
0.9	.619	.622	.627	.631	.635	.639	.645
1.0	.619	.624	.628	.633	.637	.641	.648
1.2	.620	.626	.632	.636	.641	.646	
1.4	.622	.629	.634	.640	.644		
1.6	.623	.631	.637	.642	.647		

It is seen that the coefficients for suppressed weirs are greater than for those with end contractions; this of course should be the case, since contractions diminish the discharge. They decrease

with the length of the weir, while those for contracted weirs increase with the length. Their greatest variation occurs under low heads, where they rapidly increase as the head diminishes. It should be observed that these coefficients are not reliable for lengths of weirs under 4 feet, owing to the few experiments which have been made for short suppressed weirs. Hence, for small quantities of water, weirs with end contractions should be built in preference to suppressed weirs. For a weir of infinite length it would be immaterial whether end contractions exist or not; hence for such a case the coefficients lie between the values for the 19-foot weir in Table 63 and those for the 19-foot weir in Table 64.

For a numerical illustration a suppressed weir having the same dimensions as in the example of the last article will be used, namely, $b = 4$ feet, $G = 1.5$ feet, and $H = 0.457$ feet. The coefficient is found from Table 64 to be 0.630; then for no velocity of approach the discharge per second is

$$q = 0.630 \times \tfrac{2}{3} \times 8.02 \times 4 \times 0.457^{\frac{3}{2}} = 4.16 \text{ cubic feet.}$$

Here the width B is also 4 feet; the head corresponding to the velocity of approach then is by $(62)_1$

$$h = 0.01555\left(\frac{4.16}{4 \times 1.96}\right)^2 = 0.0044 \text{ feet,}$$

and the effective head on the crest is

$$H + 1\tfrac{1}{3}h = 0.463 \text{ feet,}$$

from which the discharge per second is

$$q = 0.630 \times \tfrac{2}{3} \times 8.02 \times 4 \times 0.463^{\frac{3}{2}} = 4.24 \text{ cubic feet.}$$

This shows that the velocity of approach exerts a greater influence upon the discharge than in the case of a weir with end contractions.

When velocity of approach exists, a good method of computation is to expand the parenthesis of $(64)_2$ in a series and use only two terms of the expansion thus,

$$q = c \cdot \tfrac{2}{3}\sqrt{2g} \cdot bH^{\frac{3}{2}}\left(1 + 2.0\frac{h}{H}\right)$$