The efficiency of the jet, or the ratio of the actual to the theoretic energy, now is

$$e = k/K = c_1^2 \tag{56}$$

which is a number always less than unity.

For the standard orifice the mean value of c_1 is 0.98, and hence a mean value of c_1^2 is 0.96. The actual energy of a jet from such an orifice is hence about 96 percent of the theoretic energy, and the loss of energy is about 4 percent. This loss is due to the frictional resistance of the edges of the orifice, whereby the energy of pressure or velocity is changed into heat.

In the plane of the standard orifice the velocity is slower than at the contracted section since the area there is greater. If v_1 be this velocity, a the area of the orifice, and a' that of the jet at the contracted section, it is clear that $av_1 = a'v$ or $v_1 = c'v$, where c' is the coefficient of contraction 0.62. The kinetic energy in the plane of the orifice is $W \cdot v_1^2/2g$, or $0.37\ Wv^2/2g$, or $0.37\ Wh$. Thus, in the plane of the orifice 4 percent of the theoretic energy is lost overcoming friction, 37 percent is in the form of kinetic energy, and the remaining 59 percent exists in the form of pressure energy. This 59 percent is transformed into kinetic energy when the water has reached the contracted section.

In hydraulics the terms "energy" and "head" are often used as synonymous, although really energy is proportional to head. Thus the pressure-head that causes the flow is h and the velocity-head of the issuing jet is $v^2/2g$, and these are proportional to the theoretic and effective energies. The lost head h' is the difference of these, or

$$h' = h - \frac{v^2}{2g}$$

and this applies not only to an orifice but to any tube or pipe. Inserting for v^2 its value, this becomes

$$h' = (1 - c_1^2)\,h$$

which gives the lost head in terms of the total head. Inserting for h its value in terms of v reduces this to

$$h' = \left(\frac{1}{c_1^2} - 1\right)\frac{v^2}{2g}$$

which gives the lost head in terms of the velocity-head. Thus, for an orifice whose coefficient of velocity is 0.97 the lost head h' is $0.060\ h$ or $0.063\ v^2/2g$. For the standard orifice the lost head h' is $0.040\ h$ or $0.041\ v^2/2g$. For the standard orifice h' can also be expressed as $0.11\ v_1^2/2g$, where v_1 is the velocity in the plane of the orifice.

Prob. 56. What is the loss of head in an orifice whose coefficient of velocity is unity?

Art. 57. Discharge under a Dropping Head

If a vessel or reservoir receives no inflow of water while an orifice is open, the head drops and the discharge decreases in each successive second. Let H be the head on the orifice at a certain instant, and h the head t seconds later; let A be the area of the uniform horizontal cross-section of the vessel, and a the area of the orifice. Then, the theoretic time t is given by the second formula in Art. 32. To determine the actual time the coefficient of discharge must be introduced. Referring to the demonstration, it is seen that $a\sqrt{2gy}\cdot\delta t$ is the theoretic discharge in the time δt; hence the actual discharge is $c \cdot a\sqrt{2gy}\delta t$, and accordingly a in the above-mentioned formula is to be replaced by ca, or

$$t = \frac{2A}{ca\sqrt{2g}}(\sqrt{H} - \sqrt{h}) \tag{57$_1$}$$

is the practical formula for the time in which the water level drops from H to h. In using this formula c is to be taken from the tables of this chapter, an average value being selected corresponding to the average head.

Experiments have been made to determine the value of c by the help of this formula; the liquid being allowed to flow, A, a, H, h, and t being observed, whence c is computed. In this way c for mercury has been found to be about 0.62.* Only approximate mean values can be found in this manner, since c varies with the head, particularly for small orifices (Art. 47). For a large orifice the time of descent is usually so small that it

* Downing's Elements of Practical Hydraulics (London, 1875), p. 187.

cannot be noted with precision, and the friction of the liquid on the sides of the vessel may also introduce an element of uncertainty. Further, when h is small, a vortex forms which renders the formula unreliable. This experiment has therefore little value except as illustrating and confirming the truth of the theoretic formulas.

The discharge in one second when the head is H at the beginning of that second is found as follows: the above equation may be written in the form

$$\sqrt{H} - tca\sqrt{2g}/2A = \sqrt{h}$$

By squaring both members, transposing, and multiplying by A, this may be reduced to

$$A(H-h) = tca\sqrt{2g}(\sqrt{H} - tca\sqrt{2g}/4A)$$

But the first member of this equation is the quantity discharged in t seconds; therefore the discharge in the first second is

$$q = ca\sqrt{2g}(\sqrt{H} - ca\sqrt{2g}/4A)$$

If $A = \infty$, this becomes $ca\sqrt{2gh}$, which should be the case, for then H would remain constant. At the end of the first second the water level has fallen the amount q/A, so that the head at the beginning of the second second is $H - q/A$.

For example, let an orifice one foot square in a reservoir of 10 square feet section be under a head of 9 feet, and $c = 0.602$. Then the discharge in one second is 13.9 cubic feet, and the head drops to 7.61 feet. The discharge in the next second is 12.7 cubic feet, and the head drops to 6.34 feet.

Prob. 57. Find the time required to discharge 480 gallons of water from an orifice 2 inches in diameter at 8 feet below the water level when the cross-section of the tank is 4×4 feet.

Art. 58. Emptying and Filling a Canal Lock

A canal lock is emptied by opening one or more orifices in the lower gates. Let a be their area and H the head of water on them when the lock is full; let A be the area of the horizontal cross-section of the lock. Then in the first formula of the last

article $h = 0$, and the time of emptying the lock is

$$t = 2A\sqrt{H}/ca\sqrt{2g} \qquad (58)$$

If the discharge be free into the air, H is the distance from the center of the orifice to the level of the water in the lock when filled; but if, as is usually the case, the orifices be below the level of the water in the tail bay, H is the difference in height between the two water levels. The tail bay is regarded as so large compared with the lock that its water level remains constant during the time of emptying.

For example, let it be required to find the time of emptying a canal lock 80 feet long and 20 feet wide through two orifices each of 4 square feet area, the head upon which is 16 feet when the lock is filled. Using for c the value 0.6 for orifices with square inner edges, the formula gives

$$t = \frac{2 \times 80 \times 20 \times 4}{0.6 \times 8 \times 8.02} = 333 \text{ seconds} = 5\tfrac{1}{2} \text{ minutes}$$

If, however, the circumstances be such that c is 0.8, the time is about 250 seconds, or $4\frac{1}{6}$ minutes. It is therefore seen that it is important to arrange the orifices of discharge in canal locks with rounded inner edges.

The filling of the lock is the reverse operation. Here the water in the head bay remains at a constant level, and the discharge through the orifices in the upper gates decreases with the rising head in the lock. Let H be the effective head on the orifices when the lock is empty, and y the effective head at any time t after the beginning of the discharge. The area of the section of the lock being

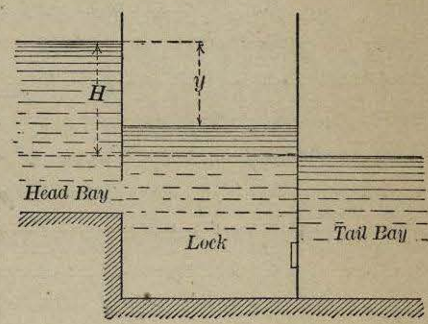

Fig. 58.

A, the quantity $A\delta y$ is discharged in the time δt, and this is equal to $ca\sqrt{2gy}\,\delta t$, if a be the area of the orifices and c the coefficient of discharge. Hence the same expression as (58) results, and the times of filling and emptying a lock are equal if the orifices are of the same dimensions and under the same heads. The area required for the orifices may be found for any case from (58) when A, H, t, and c are given.

Prob. 58. A lock 90 feet long and 20 feet wide, with a lift of 12 feet, contains a boat weighing 500 net tons. When the lock is emptied in order to lower the boat, how much water flows from the lower orifices? If the cross-section of these orifices is 12.3 square feet and $c = 0.7$, what is the time of emptying?

Art. 59. Computations in Metric Measures

Most of the formulas of this chapter are rational and may be used in all systems of measures. The coefficients of contraction, velocity, and discharge are abstract numbers, which are the same in all systems, like the constants of mathematics. In the metric system the area a is to be taken in square meters, the head h in meters, $\sqrt{2g}$ as 4.427, and then the discharge q will be in cubic meters per second.

(Art. 47) For standard circular vertical orifices the formulas $(47)_1$ and $(47)_2$ apply to the metric system if 8.02 be replaced by 4.427. In using these the coefficient c may be taken from Table 59*a* which has been adapted to metric arguments from Table 47. For example, if

Table 59*a*. Coefficients for Circular Vertical Orifices

Arguments in Metric Measures

Head h in Meters	Diameter of Orifice in Centimeters					
	1	2	3	6	18	30
0.1	0.642	0.626	0.619			
0.2	.639	.619	.613	0.601	0.593	
0.3	.634	.613	.608	.600	.595	0.591
0.5	.626	.609	.605	.600	.596	.593
0.7	.620	.607	.603	.599	.598	.596
1.	.619	.605	.602	.599	.598	.597
1.5	.614	.604	.601	.598	.597	.596
2.	.611	.603	.600	.597	.596	.596
3.	.607	.600	.598	.597	.596	.595
6.	.600	.597	.596	.596	.596	.594
15.	.596	.595	.594	.594	.594	.593
30.	.593	.592	.592	.592	.592	.592

the diameter of the orifice is 2.5 centimeters and the head on its center is 0.6 meters, interpolation in the table gives the value of c as 0.606.

(Art. 48) For standard square vertical orifices the formulas $(48)_1$ and $(48)_2$ are changed to the metric system by substituting 4.427 for 8.02 and 2.951 for 5.347. Table 59*b* gives values of the coefficient c for arguments in metric measures.

Table 59*b*. Coefficients for Square Vertical Orifices

Arguments in Metric Measures

Head h in Meters	Side of the Square in Centimeters					
	1	2	3	6	12	30
0.1	0.652	0.632	0.622			
0.2	.648	.624	.617	0.605	0.598	
0.3	.636	.619	.613	.605	.601	0.599
0.5	.628	.618	.610	.605	.602	.601
0.7	.625	.612	.607	.605	.604	.602
1.0	.620	.610	.607	.605	.604	.603
1.5	.618	.609	.606	.604	.603	.602
2.	.614	.608	.605	.604	.603	.602
3.	.611	.606	.604	.603	.602	.601
6.	.605	.603	.602	.602	.601	.600
15.	.601	.601	.600	.600	.599	.599
30.	.598	.598	.598	.598	598	.598

(Art. 49) Table 49 has not been transformed into one with metric arguments, as it applies only to the special case where the rectangular orifice is one foot wide. If the heads in the first column are changed into meters, by writing 0.12 meters for 0.4 feet, 0.18 meters for 0.6 feet, etc., and the numbers at the top are changed into centimeters by writing 3.8 centimeters for 0.125 feet, 7.6 centimeters for 0.25 feet, etc., the table will be ready for use with metric arguments for rectangular orifices 30.5 centimeters wide.

(Art. 55) The miner's inch, when the head on the center of the orifice is 16.5 centimeters, is 0.0433 cubic meters or 43.3 liters per minute.

(Art. 58) In using (58) in the metric system, a and A are to be taken in square meters, H in meters, g as 9.80 meters per second per second, and $\sqrt{2g}$ as 4.427; q will then be found in cubic meters.

Prob. 59*a*. Michelotti found the range of a jet to be 6.25 meters on a horizontal plane 1.41 meters below the vertical orifice, which was under a head of 7.19 meters. Compute the coefficient of velocity.

Prob. 59*b*. An orifice 3 centimeters square was under a constant head of 4 meters, and during 230 seconds the jet flowed into a tank which was found to contain 1122 liters. Show that the coefficient of discharge was 0.612.

Prob. 59*c*. Find from the table the coefficient of discharge for a standard circular orifice 2.5 centimeters in diameter under a head of 2.5 meters.

Prob. 59*d*. Compute the discharge through a standard orifice 7.5 centimeters square under a head of 8 meters.

Prob. 59*e*. Compute the time required to empty a canal lock 7 meters wide and 32 meters long through an orifice of 0.9 square meters area, the head on the center of the orifice being 5.1 meters when the lock is filled.

CHAPTER 6

FLOW OF WATER OVER WEIRS

Art. 60. Standard Weirs

A weir is a notch in the top of the vertical side of a vessel or reservoir through which water flows. The notch is generally rectangular, and the word "weir" will be used to designate a rectangular notch unless otherwise specified, the lower edge of the rectangle being truly horizontal, and its sides vertical. The lower edge of the rectangle is called the "crest" of the weir. In

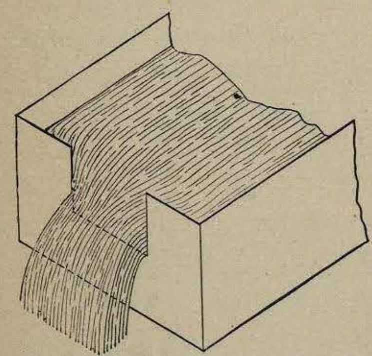

Fig. 60*a*.

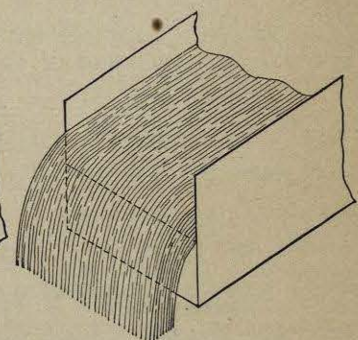

Fig. 60*b*.

Fig. 60*a* is shown the outline of the most usual form, where the vertical edges of the notch are sufficiently removed from the sides of the reservoir or feeding canal, so that the sides of the stream may be fully contracted; this is called a weir with end contractions. In the form of Fig. 60*b* the edges of the notch are coincident with the sides of the feeding canal, so that the filaments of water along the sides pass over without being deflected from the vertical planes in which they move; this is called a weir without end contractions, or with end contractions suppressed. Both