increases and as the head increases. Comparing this table with Table 47*a* it is seen that the coefficient of discharge for a square is always slightly larger than that for a circle having a diameter equal to the side of the square. The values above the horizontal lines in the last three columns are to be used in the exact formula $(48)_2$ when precision is required, and all other values in the approximate formula $(48)_1$.

There are few recorded experiments on large square orifices. Ellis measured the discharge from a vertical orifice 2 feet square* and deduced the following coefficients for use in the approximate formula:

for $h = 2.07$ feet, $c = 0.611$
for $h = 3.05$ feet, $c = 0.597$
for $h = 3.54$ feet, $c = 0.604$

which indicate that a mean value of 0.60 may be used for large square orifices under low heads.

Prob. 48. Find from the table the coefficient for an orifice 3 inches square when the head on its center is 1.8 feet.

Art. 49. Rectangular Vertical Orifices

The theoretic formulas of Art. 48 apply to rectangles of width b and depth d, and the approximate formula for computing the actual discharge is

$$q = cbd\sqrt{2gh} = 8.02\,cbd\sqrt{h} \tag{49}$$

in which c is the coefficient of discharge, b the width and d the depth of the rectangular orifice, and h the head on its center.

Table 49 gives values of the coefficient c which have been compiled and rearranged from the discussion given by Fanning.† It is seen that the variation of c with the head follows the same law as for circles and squares. It is also seen that for a rectangle of constant breadth the coefficient increases as the depth decreases, from which it is to be inferred that for a rectangle of constant depth the coefficient increases with the breadth,

* Transactions American Society of Civil Engineers, 1876, vol. 5, p. 92.
† Treatise on Water Supply Engineering (New York, 1888), p. 205.

Table 49. Coefficients for Rectangular Orifices 1 Foot Wide

Head h in Feet	Depth of Orifice in Feet						
	0.125	0.25	0.50	0.75	1.0	1.5	2.0
0.4	0.634	0.633	0.622				
0.6	.633	.633	.619	0.614			
0.8	.633	.633	.618	.612	0.608		
1.0	.632	.632	.618	.612	.606	0.626	
1.5	.630	.631	.618	.611	.605	.626	0.628
2.0	.629	.630	.617	.611	.605	.624	.630
2.5	.628	.628	.616	.611	.605	.616	.627
3.0	.627	.627	.615	.610	.605	.614	.619
4.0	.624	.624	.614	.609	.605	.612	.616
6.0	.615	.615	.609	.604	.602	.606	.610
8.0	.609	.607	.603	.602	.601	.602	.604
10.0	.606	.603	.601	.601	.601	.601	.602
20.0				.601	.601	.601	.602

and this is confirmed by other experiments. The value of c for a rectangular orifice is seen to be only slightly larger than that for a square whose side is equal to the depth of the rectangle. All the coefficients in this table are for the above approximate formula, since that formula was used in computing them.

A comparison of the values of c for the orifice one foot square with those in the last article shows that the two sets of coefficients disagree, these being about one percent greater. This is probably due to the less precise character and smaller number of experiments from which they were deduced.

Prob. 49. What constant head is required to discharge 5 cubic feet of water per second through an orifice 3 inches deep and 12 inches long?

Art. 50. Velocity of Approach

It was shown in Art. 24 that the theoretic velocity of flow from an orifice is greater then $\sqrt{2gh}$ when the ratio of the cross-section of the orifice to that of the vessel or tank is not small. The same is true for the actual velocity, but formula $(24)_1$ must be modified because it takes no account of the contraction of the jet. Let v be the velocity at the contracted section of the jet and a' the area of that section; let v_1 be the velocity through the horizontal cross-section A of the vessel; then $a'v = Av_1$. But if a be the area of the orifice and c' the coefficient of contraction, then a' equals ac' and hence $c'av = Av_1$. Now the effective head on the orifice is

$$H = h + \frac{v_1^2}{2g}$$

and the velocity v is given by $c_1\sqrt{2gH}$ where c_1 is the coefficient of velocity. Substituting in the last equation $v^2/2gc_1^2$ for H and $c'va/A$ for v_1, and noting that c_1c' is equal to the coefficient of discharge c, it reduces to

$$v = c_1\sqrt{\frac{2gh}{1 - c^2(a/A)^2}} \qquad (50)_1$$

which is the velocity of the jet at a section distant from the orifice about one-half its diameter. The discharge q is found by multiplying this by the area $c'a$ of that cross-section, whence

$$q = ca\sqrt{\frac{2gh}{1 - c^2(a/A)^2}} = a\sqrt{\frac{2gh}{(1/c)^2 - (a/A)^2}} \qquad (50)_2$$

is the formula for the actual discharge, and this includes no coefficient except that of discharge.

These formulas apply to orifices of any kind, and when c equals unity, they reduce to the theoretic expressions established in Art. 24. When a/A is less than $1/5$, as is almost always the case in practice, the last formula may be written, with sufficient precision,

$$q = (1 + \tfrac{1}{2}(ca/A)^2 ca\sqrt{2gh} \qquad (50)_3$$

For example, let a square tank, 4×4 feet in horizontal cross-section, have a standard square orifice one square foot in area, and let the head on its center be 16 feet. From Table 48 the coefficient of discharge is 0.60, and the formula gives

$$q = (1 + 0.0007) \times 0.60 \times 1 \times 8.02 \times 4 = 19.3 \text{ cubic feet per second}$$

For this case it is seen that the influence of velocity of approach is expressed by the addition of 0.0007 to unity, which is an in-

crease of less than one-tenth of one percent. In general the increase in discharge due to velocity of approach is expressed, when a/A is not greater than $1/5$, by $\frac{1}{2}c^3a(a/A)^2\sqrt{2gh}$.

A common case is that where the vessel or tank is of large horizontal and small vertical cross-section, and where the water approaches the orifice with a horizontal velocity, as in a canal or conduit. Here let A be the area of the vertical cross-section of the vessel, a the area of the orifice, and h the head on its center. Then, if the head h be large compared with the depth of the orifice, the same reasoning applies as in Art. 24, the theoretic velocity is given by $(24)_1$ and the actual discharge by $(50)_2$.

When the head h is not large, let h_1 and h_2 be the heads on the upper and lower edges of the orifice, which is taken as rectangular and of the width b. Let v be the velocity of approach, which is regarded as uniform over the area A. Then by the same reasoning as that in Art. 24, the theoretic velocity in the plane of the orifice at the depth y below the water level is given by $V^2 = 2gy + v^2$. The theoretic discharge through an elementary strip of the length b and the depth δy now is

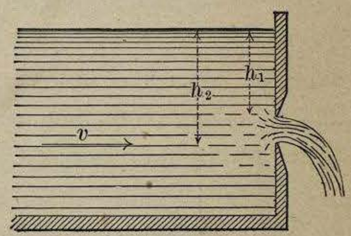

Fig. 50.

$$\delta Q = (2gy + v^2)^{\frac{1}{2}} b\delta y$$

and, by integration between the limits h_2 and h_1, the total theoretic discharge is found. If $v^2/2g$ be replaced by h_0, the head which would cause the velocity v, the theoretic discharge is

$$Q = \tfrac{2}{3} b\sqrt{2g}\,[(h_2 + h_0)^{\frac{3}{2}} - (h_1 + h_0)^{\frac{3}{2}}] \qquad (50)_4$$

and the actual discharge q is found by multiplying this by a coefficient of discharge. When there is no velocity of approach, the formula reduces to that found in Art. 49 for this case.

Prob. 50*a*. When n is a small quantity compared with unity, show that $(1 + n)^{\frac{1}{2}} = 1 + \frac{1}{2}n$, and that $1/(1 + n) = 1 - n$. Deduce formula $(50)_3$ from $(50)_2$.

Prob. 50*b*. In the case of horizontal approach, as seen in Fig. 50, compute the discharge when $b = 4$ feet, $h_2 = 0.8$ feet, $h_1 = 0$, $v = 2.5$ feet per second, and $c = 0.6$.

Art. 51. Submerged Orifices

It is shown in Art. 23 that the effective head h which causes the flow from a submerged orifice is the difference in level between the two water surfaces. The discharge from such an orifice, its inner edge being a sharp definite one, as in Fig. 43*a*, has been found by experiment to be slightly less than when the flow occurs freely into the air, and hence the values of the coefficients of discharge are slightly smaller than those given in Tables 47*a*, 47*b*, 48, 49. For large orifices and large heads the difference is very small, and for orifices one inch square under six inches head it is about 2 percent. In all cases of submerged orifices the discharge is to be found from $q = ca\sqrt{2gh}$.

Table 51 gives values of the coefficient of discharge for submerged orifices as determined from experiments made by Hamilton Smith in 1884. The depth of submergence of the orifices varied from 0.57 to 0.73 foot. As a mean value of the coefficient of discharge for standard submerged orifices 0.6 is frequently used.

Table 51. Coefficients for Submerged Orifices

Effective Head in Feet	Size of Orifice in Feet				
	Circle 0.05	Square 0.05	Circle 0.1	Square 0.1	Rectangle 0.05 × 0.3
0.5	0.615	0.619	0.603	0.608	0.623
1.0	.610	.614	.602	.606	.622
1.5	.607	.612	.600	.605	.621
2.0	.605	.610	.599	.604	.620
2.5	.603	.608	.598	.604	.619
3.0	.602	.607	.598	.604	.618
4.0	.601	.606	.598	.604	

The theoretic discharge from a submerged orifice is the same for the same effective head h, whatever be its distance below water level. The theoretic velocity in all parts of the orifice is the

same, as may be proved from Fig. 51, where the triangles ACD and BCE represent the distribution of pressure on AC and BC when the orifice is closed (Art. 17). Making CF equal to CE and drawing BF, the unit-pressure on BC is seen to have the constant value DF. Now when the orifice is opened, the velocity at any point depends on the unit-pressure there acting as seen by $(23)_1$, and accordingly the theoretic velocity is uniform over the section. For this reason the coefficients of discharge probably vary less with the head than for the previous cases.

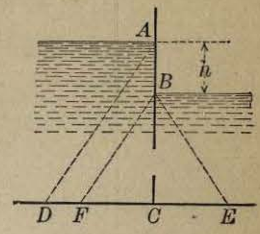

Fig. 51.

Submerged orifices are used for canal-locks, tide-gates, filter-beds, for the discharge of waste water through dams, and for the admission of water from a canal to a power-plant. The inner edges of such orifices are usually rounded, and the coefficient of discharge may then be higher than 0.9 (Art. 53).

Prob. 51. An orifice one inch square in a gate, such as shown in Fig. 19*a*, is 4.1 feet below the higher water level and 3.1 feet below the lower level. Compute the discharge in cubic feet per second, and also in gallons per minute.

Art. 52. Suppression of the Contraction

When a vertical orifice has its lower edge at the bottom of the reservoir, as shown at A in Fig. 52, the particles of water flowing through its lower portion move in lines nearly perpendicular to the plane of the orifice, or the contraction of the jet does not form on the lower side. This is called a case of suppressed or incomplete contraction. The same thing occurs, but in a lesser degree, when the lower edge of the orifice is near the bottom, as shown at B. In like manner, if an orifice be placed so that one of its vertical edges is at or near a side of the reservoir, as at C, the contraction of the jet is suppressed upon one side, and if it be placed at the lower corner of the reservoir suppression occurs both upon one side and the lower part of the jet.

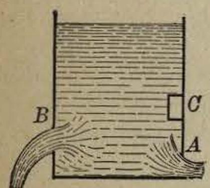

Fig. 52.

The effect of suppressing the contraction is, of course, to increase the cross-section of the jet at the place where full contraction would otherwise occur, and it is found by experiment that the discharge is likewise increased. Experiments also show that more or less suppression of the contraction will occur unless each edge of the orifice is at a distance at least equal to three times its least diameter from the sides or bottom of the reservoir.

The experiments of Lesbros and Bidone furnish the means of estimating the increased discharge caused by suppression of the contraction. They indicate that for square orifices with contraction suppressed on one side the coefficient of discharge is increased about 3.5 percent, and with contraction suppressed on two sides about 7.5 percent. For a rectangular orifice with the contraction suppressed on the bottom edge the percentages are larger, being about 6 or 7 percent when the length of the rectangle is four times its height, and from 8 to 12 percent when the length is twenty times the height. The percentage of increase, moreover, varies with the head, the lowest heads giving the lowest percentages.

It is apparent that suppression of the contraction should be avoided if accurate results are desired. The experiments from which the above conclusions are deduced were made upon small orifices with heads less than 6 feet, and it is not known how they will apply to large orifices under high heads. For a rectangular orifice of length about three times its height, with contraction suppressed on the ends and bottom, the coefficient of discharge is probably about 0.75.

Prob. 52. Compute the probable discharge from a vertical orifice one foot square when the head on its upper edge is 4 feet, the contraction being suppressed on the lower edge. Compute the discharge for the same data when contraction is suppressed on all sides.

Art. 53. Orifices with Rounded Edges

When the inner edge of the orifice is made rounded, as shown in Fig. 53, the contraction of the jet is modified, and the discharge is increased. With a slight degree of rounding, as at A, a partial contraction occurs; but with a more complete rounding, as at C, the particles of water issue perpendicular to the plane

of the orifice and there is no contraction of the jet. If a be the area of the least cross-section of the orifice, and a' that of the jet, the coefficient of contraction as defined in Art. 44 is

$$c' = a'/a \qquad (53)$$

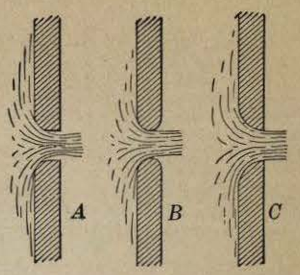

Fig. 53.

For a standard orifice with sharp inner edges (Art. 43) the mean value of c' is 0.62, but for an orifice with rounded edges c' may have any value between 0.62 and 1.0, depending upon the degree of rounding.

The coefficient of discharge c for standard orifices has a mean value of about 0.61; this is increased with rounded edges and may have any value between 0.61 and 1.0. A rounded interior edge in an orifice is therefore always a source of error when the object of the orifice is the measurement of the discharge. If a contract provides that water shall be gaged by standard orifices, care should always be taken that the interior edges do not become rounded either by accident or by design.

Prob. 53. When an orifice with rounded edges has a coefficient of velocity of 0.88 and a coefficient of discharge of 0.75, find the coefficient of contraction of the jet.

Art. 54. Water Measurement by Orifices

In order that water may be accurately measured by the use of orifices many precautions must be taken, some of which have already been noted, but may here be briefly recapitulated. The area of the orifice should be small compared with the size of the reservoir in order that velocity of approach may not exist, or if this cannot be avoided, it should be taken into account by formula $(50)_1$. The inner edge of the orifice must have a definite right-angled corner, and its dimensions are to be accurately determined. If the orifice be in wood, care should be taken that the inner surface be smooth, and that it be kept free from the slime which often accompanies the flow of water, even when apparently clear. That no suppression of the contraction may occur, the edges of the orifice should not be nearer than three times its least dimension to a side of the reservoir.

Orifices under very low heads should be avoided, because slight variations in the head produce relatively large errors, and also because the coefficients of discharge vary more rapidly and are probably not so well determined as for cases where the head is greater than four times the depth. If the head be very low on an orifice, vortices will form which render any estimation of the discharge unreliable.

The measurement of the head, if required with precision, must be made with the hook gage described in Art. 35. For heads greater than two or three feet the readings of an ordinary glass gage placed upon the outside of the reservoir will usually prove sufficient, as this can be read to hundredths of a foot with accuracy. An error of 0.01 foot when the head is 3.00 feet produces an error in the computed discharge of less than two-tenths of one per cent; for, the discharges being proportional to the square roots of the heads, the square root of 3.01 divided by the square root of 3.00 equals 1.0017. For the rude measurements in connection with the miner's inch a common foot-rule will usually suffice.

The effect of temperature upon the discharge remains to be noticed; this is only appreciable with small orifices and under low heads and hence such orifices and heads are not desirable in precise measurements. Unwin found that the discharge was diminished one percent by a rise of 144° in temperature; his orifice was a circle 0.033 feet in diameter under heads ranging from 1.0 to 1.5 feet. Hamilton Smith found that the discharge was diminished one percent by a rise of 55° in temperature; his orifice was a circle 0.02 feet in diameter under heads ranging from 0.56 to 3.2 feet.

The coefficients given in the tables of this chapter may be supposed liable to a probable error of about two units in the third decimal place: thus a coefficient 0.615 should really be written 0.615 ± 0.002; that is, the actual value is as likely to be between 0.613 and 0.617 as to be outside of those limits. The probable error in computed discharges due to the coefficient is hence nearly one-half of one percent. To this are added the errors due to inaccuracy of observation, so that it is thought that the probable error of careful work with standard circular orifices is at least one percent. The computed discharges are hence liable to error in the third significant figure, so that it is useless to carry numerical results beyond three figures when based upon tabular coefficients. As a precise method of measuring small quantities of water, standard orifices take a high rank when the observations are conducted with care.

Prob. 54. If e is a small error in measuring the head h, show that the error in the computed discharge q due to this cause is $qe/2h$.

Art. 55. The Miner's Inch

The miner's inch may be roughly defined to be the quantity of water which will flow from a vertical standard orifice one inch square, when the head on the center of the orifice is $6\frac{1}{2}$ inches. From Table 48 the coefficient of discharge is seen to be about 0.623 and accordingly the actual discharge from the orifice in cubic feet per second is $q = \frac{1}{144} \times 0.623 \times 8.02 \sqrt{6.5/12} = 0.0255$ and the discharge in one minute is $60 \times 0.255 = 1.53$ cubic feet. The mean value of one miner's inch is therefore about 1.5 cubic feet per minute.

The actual value of the miner's inch, however, differs considerably in different localities. Bowie states that in different counties of California it ranges from 1.20 to 1.76 cubic feet per minute.* The reason for these variations is due to the fact that when water is bought for mining or irrigating purposes, a much larger quantity than one miner's inch is required, and hence larger orifices than one square inch are needed. Thus at Smartsville a vertical orifice or module 4 inches deep and 250 inches long, with a head of 7 inches above the top edge, is said to furnish 1000 miner's inches. Again, at Columbia Hill, a module 12 inches deep and $12\frac{3}{4}$ inches wide, with a head of 6 inches above the upper edge, is said to furnish 200 miner's inches. In Montana the customary method of measurement is through a vertical rectangle,

* Treatise on Hydraulic Mining (New York, 1885), p. 268.

1 inch deep, with a head on the center of the orifice of 4 inches, and the number of miner's inches is said to be the same as the number of linear inches in the rectangle; thus under the given head an orifice 1 inch deep and 60 inches long would furnish 60 miner's inches. The discharge of this is said to be about 1.25 cubic feet per minute, or 75 cubic feet per hour.

The following are the values of the miner's inch in different parts of the United States; in California and Montana it is established by law that 40 miner's inches shall be the equivalent of one cubic foot per second, and in Colorado 38.4 miner's inches is the equivalent. In other States and Territories there is no legal value, but by common agreement 50 miner's inches is the equivalent of one cubic foot per second in Arizona, Idaho, Nevada, and Utah; this makes the miner's inch equal to 1.2 cubic feet per minute.

A module is an orifice which is used in selling water, and which under a constant head is to furnish a given number of miner's inches, or a given quantity per second. The size and proportions of modules vary greatly in different localities, but in all cases the important feature to be observed is that the head should be maintained nearly constant in order that the consumer may receive the amount of water for which he bargains, and no more.

The simplest method of maintaining a constant head is by placing the module in a chamber which is provided with a gate that regulates the entrance of water from the main reservoir or canal. This gate is raised or lowered by an inspector once or twice a day so as to keep the surface of the water in the chamber at a given mark. This plan is a costly one, on account of the wages of the inspector, except in works where many modules are used and where a daily inspection is necessary in any event, and it is not well adapted to cases where there are frequent and considerable fluctuations in the water surface of the feeding canal.

Numerous methods have been devised to secure a constant head by automatic appliances; for instance, the gate which admits water into the chamber may be made to rise and fall by means of a float upon the surface; the module itself may be made to decrease in size when the water rises, and to increase when it falls, by a gate or by a tapering plug which moves in and out and whose motion is controlled by a float. In another variety the head on an orifice is kept constant by placing it in the side of a vessel which is movable and whose vertical movement is proportional to the rise or fall of the water in the feeding channel or reservoir. These self-acting contrivances, however, are liable to get out of order, and require to be inspected more or less frequently.* Another method is to have the water flow over the crest of a weir as soon as it reaches a certain height.†

The use of the miner's inch, or of a module, as a standard for selling water, is awkward and confusing, and for the sake of uniformity it is greatly to be desired that water should always be bought and sold by the cubic foot per second. Only in this way can comparisons readily be made, and the consumer be sure of obtaining exact value for his money.

Prob. 55. When a miner's inch is 1.57 cubic feet per minute, how many miner's inches will be furnished by a module 2 inches deep and 50 inches long with a head of 6 inches above the upper edge?

Art. 56. Loss of Energy or Head

A jet of water flowing from an orifice possesses by virtue of its velocity a certain kinetic energy, which is always less than the theoretic potential energy due to the head (Art. 26). Let h be the head and W the weight of water discharged per second, then the theoretic energy per second or the power of the jet, is

$$K = Wh$$

Let v be the actual velocity of the water at the contracted section of the jet; then the actual energy per second of the water as it passes that section is $k = W \cdot v^2/2g$

Now let c_1 be the coefficient of velocity (Art. 45); then

$$v^2 = c_1^2 \cdot 2gh$$

and accordingly the actual energy of the jet per second is

$$k = c_1^2 Wh$$

* For descriptions of several see Engineering News, Dec. 17, 1908.

† Foote, Transactions American Society Civil Engineers, 1887, vol. 16, p. 134.