the most probable ones that can be derived from the given data. It has the further advantage that all computors will derive the same results, whereas in the graphic method the results will usually differ, because the position of the line drawn on the plot is affected by the different degrees of judgment and experience of the draftsmen. It will be seen from Fig. 42*b* that it is not very easy to determine close values of log m since the plotted points are so far away from the origin.

Prob. 42*a*. In order to rate a certain current meter four observations were taken in still water as follows:

Velocity of the car	0.7	2.4	4.7	9.3 feet per second
Revolutions of meter	18	60	120	240 per minute

Find the values of a and b in the formula $v = a + bn$, both by plotting and by the method of least squares.

Prob. 42*b*. Three observations of horizontal angles are made at the station O, which give $AOB = 62°17'$, $BOC = 20°35'$, $AOC = 82°55'$. Adjust these observations by the method of least squares so that the large angle may be equal to the sum of its parts.

CHAPTER 5

FLOW OF WATER THROUGH ORIFICES

Art. 43. Standard Orifices

Orifices for the measurement of water are usually placed in the vertical side of a vessel or reservoir, but may also be placed in the base. In the former case it is understood that the upper edge of the opening is completely covered with water; and generally the head of water on an orifice is at least three or four times its vertical height. The term "standard orifice" is here used to signify that the opening is so arranged that the water in flowing from it touches only a line, as would be the case in a plate of no thickness. To secure this result the inner edge of the opening has a square corner, which alone is touched by the water. In precise experiments the orifice may be in a metallic plate whose thickness is really small, as at A in the figure, but more commonly it is cut in a board or plank, care being taken that the inner edge is a definite corner. It is usual to bevel the outer edges of the orifice, as at C, so that the escaping jet may by no possibility touch the edges except at the inner corner. The term "orifice in a thin plate" is often used to express the condition that the water shall only touch the edges of the opening along a line. This arrangement may be regarded as a kind of standard apparatus for the measurement of water; for, as will be seen later, the discharge is modified when the inner corner is rounded, and different degrees of rounding give different discharges. The standard arrangements shown in Fig. 43*a* are accordingly always used when water is to be measured by the use of orifices.

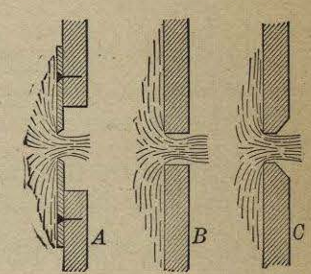

Fig. 43*a*.

The contraction of the jet which is always observed when water issues from a standard orifice, as described above, is a most interesting and important phenomenon. It is due to the circumstance that the particles of water as they approach the orifice move in converging directions, and that these directions continue to converge for a short distance beyond the plane of the orifice. It is this contraction of the jet that causes only the inner corner of the orifice to be touched by the escaping water. The appearance of such a jet under steady flow, issuing from a circular orifice, is that of a clear crystal bar whose beauty claims the admiration of every observer. The convergence due to this cause ceases at a distance from the plane of the orifice of about one-half its diameter. Beyond this section the jet enlarges in size if it be directed upward, but decreases in size if it be directed downward or horizontally.

The contraction of the jet is also observed in the case of rectangular and triangular orifices, its cross-section being similar to that of the orifice until the place of greatest contraction is passed. Fig. 43*b* shows in the top row cross-sections of a jet from a square orifice, in the middle row those from a triangular one, and in the third row those from an elliptical orifice. The left-hand diagram in each case is the cross-section of the jet near the place of greatest contraction, while the following ones are cross-sections at greater distances from the orifice, and the jets are supposed to be moving horizontally or nearly so.

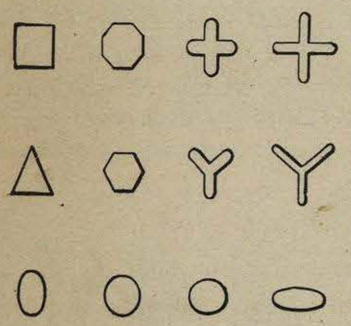

Fig. 43*b*.

Owing to this contraction, the discharge from a standard orifice is always less than the theoretic discharge, which, from Arts. 22 and 30, would be expressed by

$$Q = a\sqrt{2gh} \tag{43}$$

where a is the area of the orifice and h the head above its center. It is evident that the quantity of water passing the plane of the

orifice and that passing the plane of the contracted section in any unit of time are the same, and since there probably can be no appreciable change in the density of the water, there must therefore be an increase in velocity between these two planes. The reasons for such an increase are not fully known. It is not probable that the velocity at the center of the jet changes materially, but rather that the increase occurs in its outer filaments, so that at the contracted section they are all traveling parallel with each other and at the same velocity.*

It is the object of this chapter to determine how the theoretic formulas for orifices given in Chap. 3 are to be modified so that they may be used for the practical purposes of the measurement of water. This is to be done by the discussion of the results of experiments. It will be supposed, unless otherwise stated, that the size of the orifice is small compared with the cross-section of the reservoir, so that the effect of velocity of approach may be neglected (Art. 24).

Prob. 43. At a distance from a circular orifice of one-half its diameter a jet has a diameter of 1 inch and a velocity of 16 feet per second. When it is directed vertically downward, what is the diameter of a section 5 feet lower? When it is directed vertically upward, what is the diameter of a section 5 feet higher?

Art. 44. Coefficient of Contraction

The coefficient of contraction is the number by which the area of the orifice is to be multiplied in order to give the area of the section of the jet at a distance from the plane of the orifice of about one-half its diameter. Thus, if c' be the coefficient of contraction, a the area of the orifice, and a' the area of the contracted section of the jet, then

$$a' = c'a \tag{44}$$

The coefficient of contraction for a standard orifice is evidently always less than unity.

The only direct method of finding the value of c' is to measure by calipers the dimensions of the least cross-section of the jet. The size of the orifice can usually be determined with precision,

* Engineering News, Sept. 27, 1906.

and with care almost an equal precision in measuring the jet. To find c' for a circular orifice let d and d' be the diameters of the sections a and a'; then

$$c' = a'/a = (d'/d)^2$$

Therefore the coefficient of contraction is the square of the ratio of the diameter of the jet to that of the orifice. The first measurements were made by Newton * who found the ratio of d' to d to be 21/25, which gives for c the value 0.73. The experiments of Bossut gave from 0.66 to 0.67; and Michelotti found from 0.57 to 0.624 with a mean of 0.61. Eytelwein gave 0.64 as a mean value, and Weisbach mentions 0.63.

The following mean value will be used in this book, and it should be kept in mind by the student:

$$\text{Coefficient of contraction } c' = 0.62$$

or, in other words, the minimum cross-section of the jet is 62 percent of that of the orifice. This value, however, undoubtedly varies for different forms of orifices and for the same orifice under different heads, but little is known regarding the extent of these variations or the laws that govern them. Probably c' is slightly smaller for circles than for squares, and smaller for squares than for rectangles, particularly if the height of the rectangle is long compared with its width. Probably also c' is larger for low heads than for high heads.

Judd and King in 1906,† using a specially constructed pair of calipers,‡ found the following values for the coefficient of contraction for standard orifices:

Orifice diameter, inches,	0.75	1.00	1.50	2.00	2.50
Coefficient of contraction,	0.6134	0.6115	0.6051	0.6082	0.5955

Prob. 44. The diameter of a circular orifice is 1.995 inches. Three measurements of the diameter of the contracted section of the jet gave 1.55, 1.56, and 1.59 inches. Find the mean coefficient of contraction.

* Philosophiæ Naturalis Principia Mathematica, 1687, Book II, prop. 36.
† Engineering News, Sept. 27, 1906. ‡ Science, March 4, 1904.

Art. 45. Coefficient of Velocity

The coefficient of velocity is the number by which the theoretic velocity of flow from the orifice is to be multiplied in order to give the actual velocity at the least cross-section of the jet. Thus, if c_1 be the coefficient of velocity, V the theoretic velocity due to the head on the center of the orifice, and v the actual velocity at the contracted section, then

$$v = c_1 V = c_1 \sqrt{2gh} \qquad (45)$$

The coefficient of velocity must be less than unity, since the force of gravity cannot generate a greater velocity than that due to the head.

The velocity of flow at the contracted section of the jet cannot be directly measured. To obtain the value of the coefficient of velocity, indirect observations have been taken on the path of the jet. Referring to Art. 25, it will be seen that when a jet flows from an orifice in the vertical side of a vessel, it takes a path whose equation is $y = gx^2/2v^2$, in which x and y are the coordinates of any point of the path measured from vertical and horizontal axes, and v is the velocity at the origin. Now placing for v its value $c_1\sqrt{2gh}$, and solving for c_1, gives

$$c_1 = x/2\sqrt{hy}$$

Therefore c_1 becomes known by the measurement of the head h and the coordinates x and y. In making this experiment it would be well to have a ring, a little larger than the jet, supported by a stiff frame which can be moved until the jet passes through the ring. The flow of water can then be stopped, and the coordinates of the center of the ring determined. By placing the ring at different points of the path different sets of coordinates can be obtained. The value of x should be measured from the contracted section rather than from the orifice, since v is the velocity at the former point and not at the latter.

By this method of the jet Bossut in two experiments found for the coefficient of velocity the values 0.974 and 0.980, Michelotti in three experiments obtained 0.993, 0.998, and 0.983, and Weisbach deduced 0.978. Great precision cannot be obtained in these determinations, nor indeed is it necessary for the purposes of hydraulic investigation that c_1 should be accurately known for standard orifices. As a mean value the following may be kept in the memory:

$$\text{Coefficient of velocity } c_1 = 0.98$$

or, the actual velocity of flow at the contracted section is 98 percent of the theoretic velocity. The value of c_1 for the standard orifice is greater for high than for low heads, and may probably often exceed 0.99.

Another method of finding the coefficient c_1 is to place the orifice horizontal so that the jet will be directed vertically upward, as in Fig. 22. The height to which it rises is the velocity-head $h_0 = v^2/2g$, in which v is the actual velocity $c_1\sqrt{2gh}$. Accordingly, $h_0 = c_1^2 h$, from which c_1 may be computed. For example if, under a head of 23 feet, a jet rises to a height of 22 feet, the coefficient of velocity is

$$c_1 = \sqrt{h_0/h} = \sqrt{22/23} = 0.978$$

This method, however, fails to give good results for high velocities, owing to the resistance of the air, and moreover it is impossible to measure with precision the height h_0.

For a vertical orifice Poncelet and Lesbros found, in 1828, that the coefficient c_1 was sometimes slightly greater than unity, and this was confirmed by Bazin in 1893. This is probably due to the fact that the head is greater for the lower part of the orifice than for the upper part, and hence $\sqrt{2gh}$ does not represent the true theoretic velocity. The same experimenters found no instance of a horizontal orifice where the coefficient exceeded unity.

Since the coefficient of velocity is the ratio between the coefficient of discharge (Art. 46) and the coefficient of contraction, it may be computed from observations on these quantities. Thus Judd and King,* using the average of the coefficients of contraction shown in Art. 44 and the average of the coefficients of discharge shown in Art. 46, found the following:

$$\text{coefficient of velocity} = \frac{\text{coefficient of discharge}}{\text{coefficient of contraction}} = \frac{0.60664}{0.60674} = 0.99983$$

* Engineering News, Sept. 27, 1906.

By traversing the jets with a Pitot tube they also determined the coefficient of velocity to be 0.99993 and showed that the velocity at the contracted area is uniform throughout its cross-section. From the results of these experiments they concluded that the coefficient of velocity is unity and hence adopted the term "frictionless orifice" as descriptive of the particular standard orifices used by them.

Prob. 45. The range of a jet is 13.5 feet on a horizontal plane 2.82 feet below the orifice which is under a head of 14.38 feet. Compute the coefficient of velocity.

Art. 46. Coefficient of Discharge

The coefficient of discharge is the number by which the theoretic discharge is to be multiplied in order to obtain the actual discharge. Thus, if c is the coefficient of discharge, Q the theoretical, and q the actual discharge per second, then

$$q = cQ \qquad (46)_1$$

Here also the coefficient c is a number less than unity.

The coefficient of discharge can be accurately found by allowing the flow from an orifice to fall into a vessel of constant cross-section and measuring the heights of water by the hook gage (Art. 35). Thus q is known, and Q having been computed,

$$c = q/Q \qquad (46)_2$$

For example, a circular orifice of 0.1 foot diameter was kept under a constant head of 4.677 feet; during 5 minutes and $32\frac{1}{5}$ seconds the jet flowed into a measuring vessel which was found to contain 27.28 cubic feet. Here the actual discharge was

$$q = 27.28/332.2 = 0.08212 \text{ cubic feet per second}$$

The theoretic discharge, from formula (30), is

$$Q = \pi \times 0.05^2 \times 8.02\sqrt{4.677} = 0.1361 \text{ cubic feet per second}$$

Then the coefficient of discharge is found to be

$$c = 0.08212/0.1361 = 0.604$$

In this manner thousands of experiments have been made upon different forms of orifices under different heads, for accurate knowledge regarding this coefficient is of great importance in practical hydraulic work.

The following articles contain values of the coefficient of discharge for different kinds of orifices, and it will be seen that in general c is greater for low heads than for high heads, greater for rectangles than for squares, and greater for squares than for circles. Its value ranges from 0.59 to 0.63 or higher, and as a mean to be kept in mind the following value may be stated:

$$\text{Coefficient of discharge } c = 0.61$$

or, the actual discharge from a standard orifice is, on the average, about 61 percent of the theoretic discharge.

The coefficient c may be expressed in terms of the coefficients c' and c_1. Let a and a' be the areas of the orifice and the cross-section of the contracted jet, and Q and q the theoretic and actual discharge per second. Then, since $a'/a = c'$

$$c = \frac{q}{Q} = \frac{a'c_1\sqrt{2gh}}{a\sqrt{2gh}} = \frac{a'}{a}c_1 = c'c_1$$

and therefore the coefficient of discharge is the product of the coefficients of contraction and velocity.

The coefficient of discharge is of greater importance than the coefficients of contraction and velocity, since it is the quantity generally used in making measurements of water. Tabulations of its values for all practical cases are given below.

Prob. 46. The diameter of a contracted circular jet was found to be 0.79 inches, the diameter of the orifice being 1 inch. Under a head of 16 feet the actual discharge per minute was found to be 6.42 cubic feet. Find the coefficient of velocity.

Art. 47. Circular Vertical Orifices

Let a circular orifice of diameter d be in the side of a vessel and let h be the head of water on its center. Then, from Art. 22, the theoretic mean velocity is $\sqrt{2gh}$, and from Art. 30 the theoretic discharge is

$$Q = \tfrac{1}{4}\pi d^2\sqrt{2gh}$$

which applies when h is large compared with d.

To deduce a more exact formula let the radius of the circle be r, and let an elementary strip be drawn at a distance y above

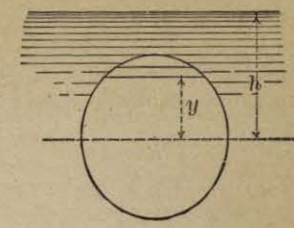

Fig. 47.

the center; the length of this is $2\sqrt{r^2-y^2}$, its area is $2\delta y\sqrt{r^2-y^2}$, and the head upon it is $h-y$. Then the theoretic discharge through this strip is

$$\delta Q = 2\delta y\sqrt{r^2-y^2}\sqrt{2g(h-y)}$$

To integrate this $(h-y)^{\frac{1}{2}}$ is to be expanded by the binomial formula. Then it may be written

$$\delta Q = 2\sqrt{2gh}\left[(r^2-y^2)^{\frac{1}{2}} - \frac{(r^2-y^2)^{\frac{1}{2}}y}{2h} - \frac{(r^2-y^2)^{\frac{1}{2}}y^2}{8h^2} - \text{etc.}\right]\delta y$$

Each term of this expression is now integrable, and taking the limits of y as $+r$ and $-r$ the entire circle is covered, and Q is found. Finally, replacing r by $\frac{1}{2}d$ there results

$$Q = \tfrac{1}{4}\pi d^2\sqrt{2gh}\left[1 - \frac{(d/h)^2}{128} - \frac{5(d/h)^4}{16384} - \text{etc.}\right]$$

which is the theoretic discharge from the circular orifice.

It is plain that this formula gives values which are always less than those found from the approximate formula of the first paragraph. Thus for $h = d$ the quantity in the parenthesis is 0.992 and for $h = 2d$ it is 0.998. Hence the error in using the approximate formula is less than three-tenths of one percent when the head on the center of the orifice is greater than twice its diameter.

For most cases, then, the actual discharge from a circular vertical orifice of area a may be computed from

$$q = c \cdot a\sqrt{2gh} = 8.02\,ca\sqrt{h} \qquad (47)_1$$

in which c is the coefficient of discharge. When h is smaller than two or three times the diameter of the orifice, and when precision is required, then

$$q = [1 - 0.07812(d/h)^2 - 0.000306(d/h)^4]\,8.02\,ca\sqrt{h} \qquad (47)_2$$

is the formula to be used. Here a may be taken from Table F (Art. 205) for the given diameter expressed in feet, h is to be taken in feet, and then q will be in cubic feet per second.

Table 47*a* gives values of c for circular orifices as determined by Hamilton Smith in a discussion of all the best experiments.* They apply only to standard orifices with definite inner edges.

TABLE 47*a*. COEFFICIENTS FOR CIRCULAR VERTICAL ORIFICES

Head h in Feet	Diameter of Orifice in Feet						
	0.02	0.04	0.07	0.1	0.2	0.6	1.0
0.4		0.637	0.624	0.618			
0.6	0.655	.630	.618	.613	0.601	0.593	
0.8	.648	.626	.615	.610	.601	.594	0.590
1.0	.644	.623	.612	.608	.600	.595	.591
1.5	.637	.618	.608	.605	.600	.596	.593
2.0	.632	.614	.607	.604	.599	.597	.595
2.5	.629	.612	.605	.603	.599	.598	.596
3.0	.627	.611	.604	.603	.599	.598	.597
4.0	.623	.609	.603	.602	.599	.597	.596
6.0	.618	.607	.602	.600	.598	.597	.596
8.0	.614	.605	.601	.600	.598	.596	.596
10.0	.611	.603	.599	.598	.597	.596	.595
20.0	.601	.599	.597	.596	.596	.596	.594
50.0	.596	.595	.594	.594	.594	.594	.593
100.0	.593	.592	.592	.592	.592	.592	.592

The table shows that the coefficient of discharge decreases as the size of the orifice increases, and that in general it also decreases as the head increases. In this table the coefficients found above the horizontal lines in the last three columns are to be used in the exact formula $(47)_2$ and all others in the approximate formula $(47)_1$.

For example, let it be required to find the discharge through a standard circular orifice, 2 inches in diameter, under a head of 2.35 feet. First, 2 inches = 0.1667 feet, and by interpolation in Table 47*a* the coefficient c is found to be 0.602. Next, from Table F at the end of this book, the area a is 0.02182 square feet. Then formula $(47)_1$ gives the discharge q as 0.161 cubic feet per second. As the coefficient is probably liable to an error

* Hydraulics (London and New York, 1886), p. 59.

of one or two units in the last figure, the third figure of this value of q is subject to the same uncertainty.

Judd and King * determined in 1906 the following values of the coefficient of discharge for circular vertical orifices:

Orifice diameter, inches	0.75	1.00	1.50	2.00	2.50
Coefficient of discharge	0.611	0.6097	0.6085	0.6083	0.5956

The heads under which the observations were made ranged from 5 to 90 feet and the results showed no appreciable change in the coefficient of discharge due to increased head. For example the following are part of the results found for a 2-inch orifice:

Head in feet =	5.00	9.08	17.79	36.12	57.70	92.01
Coefficient c =	0.6084	0.6083	0.6080	0.6082	0.6081	0.6080

Bilton† in 1907 made a series of experiments on orifices, ranging from 0.025 to 0.75 inches in diameter and determined the following coefficients for varying heads.

TABLE 47*b*. COEFFICIENTS OF DISCHARGE FOR SMALL ORIFICES

Head h in Feet	Diameter of Orifice in Inches								
	0.025	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.75
0.50	0.748	0.722	0.690	0.673	0.665	0.652	0.645	0.644	0.632
1.00	.748	.717	.680	.659	.647	.636	.630	.627	.618
2.00	.748	.708	.666	.642	.630	.624	.621	.618	.613
4.00	.748	.697	.652	.630	.627	.624	.621	.618	.613
6.00	.748	.688	.647	.630	.627	.624	.621	.618	.613
8.00		.683	.645	.630	.627	.624	.621	.618	.613

Experiments made in 1908 by Strickland‡ on standard orifices 1 and 2 inches in diameter gave results for the coefficient of discharge very closely represented by the formula

$$c = 0.5925 + 0.018/h^{\frac{1}{2}}d^{\frac{2}{3}}$$

* Engineering News, Sept. 27, 1906.
† Proceedings Victorian Institute of Engineers, Australia, 1908.
‡ Transactions Canadian Society of Civil Engineers, 1909, vol. 23, p. 198.

where h is in feet and d in inches. Applying this formula to an orifice 2 inches in diameter under a head of 19 feet, c is found to be 0.5951 while the experiments indicated a value of 0.5947.

Prob. 47. Compute the probable actual discharge from a circular orifice 8 inches in diameter, under a head of 15 inches.

ART. 48. SQUARE VERTICAL ORIFICES

If the size of an orifice in the side of a vessel is small compared with the head, the theoretic velocity of the outflowing water may be taken as $\sqrt{2gh}$, where h is the head on the center of the orifice. For a rectangular orifice under this condition the theoretic discharge is

$$Q = bd\sqrt{2gh}$$

where b is the width and d the depth of the orifice. When b is equal to d, the rectangle becomes a square.

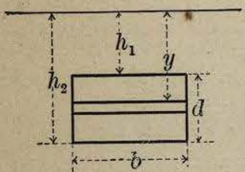

Fig. 48.

To deduce a more exact formula, let h_1 be the head on the upper edge of the orifice and h_2 that on the lower edge. Consider an elementary strip of area $b \cdot \delta y$ at a depth y below the water level. The velocity of flow through this elementary strip is $\sqrt{2gy}$, and the theoretic discharge per second through it is

$$\delta Q = b\delta y\sqrt{2gy}$$

Integrating this between the limits h_2 and h_1, there results

$$Q = \tfrac{2}{3}\, b\sqrt{2g}(h_2^{\frac{3}{2}} - h_1^{\frac{3}{2}})$$

which is the true theoretic discharge from the orifice.

To ascertain the error caused by using the approximate formula, let h be the head on the center of the rectangle; then $h_2 = h + \frac{1}{2}d$ and $h_1 = h - \frac{1}{2}d$. Developing by the binomial formula the values of $h_2^{\frac{3}{2}}$ and $h_1^{\frac{3}{2}}$, the last formula becomes

$$Q = bd\sqrt{2gh}\left[1 - \frac{(d/h)^2}{96} - \frac{(d/h)^4}{2048} - \text{etc.}\right]$$

and this shows that the discharge computed by using the approximate formula is always too great. For $h = d$, the quantity in the parenthesis is 0.989, and for $h = 2d$, it is 0.997. Accordingly, the error of the approximate formula is only three-tenths of one percent when the head on the center of the rectangle is twice the depth of the orifice.

For most cases, then, the actual discharge from a square vertical orifice may be very approximately found from

$$q = c \cdot b^2\sqrt{2gh} = 8.02\, cb^2\sqrt{h} \qquad (48)_1$$

where b is the side of the square and c is the coefficient of discharge. When h is smaller than two or three times the side of the orifice, and when precision is required,

$$q = 5.347\, cb(h_2^{\frac{3}{2}} - h_1^{\frac{3}{2}}) \qquad (48)_2$$

is the formula to be used. The linear quantities are to be taken in feet, and then q will be in cubic feet per second.

Table 48 gives values of the coefficient c for standard square orifices, taken from a more extended one formed by Hamilton Smith in 1886 by the discussion of all the best experiments. It is seen that the coefficient decreases as the size of the orifice

TABLE 48. COEFFICIENTS FOR SQUARE VERTICAL ORIFICES

Head h in Feet	Side of the Square in Feet						
	0.02	0.04	0.07	0.1	0.2	0.6	1.0
0.4		0.643	0.628	0.621			
0.6	0.660	.636	.623	.617	0.605	0.598	
0.8	.652	.631	.620	.615	.605	.600	0.597
1.0	.648	.628	.618	.613	.605	.601	.599
1.5	.641	.622	.614	.610	.605	.602	.601
2.0	.637	.619	.612	.608	.605	.604	.602
2.5	.634	.617	.610	.607	.605	.604	.602
3.0	.632	.616	.609	.607	.605	.604	.603
4.0	.628	.614	.608	.606	.605	.603	.602
6.0	.623	.612	.607	.605	.604	.603	.602
8.0	.619	.610	.606	.605	.604	.603	.602
10.0	.616	.608	.605	.604	.603	.602	.601
20.0	.606	.604	.602	.602	.602	.601	.600
50.0	.602	.601	.601	.600	.600	.599	.599
100.0	.599	.598	.598	.598	.598	.598	.598