aid of the Method of Least Squares the constants of the equation may then be computed and the curve determined (Art. 42). In the case of the small Price meter it has been found that the curve is very closely approximated by two straight lines AB and BC, as shown in Fig. 40c, which is a typical rating curve for this

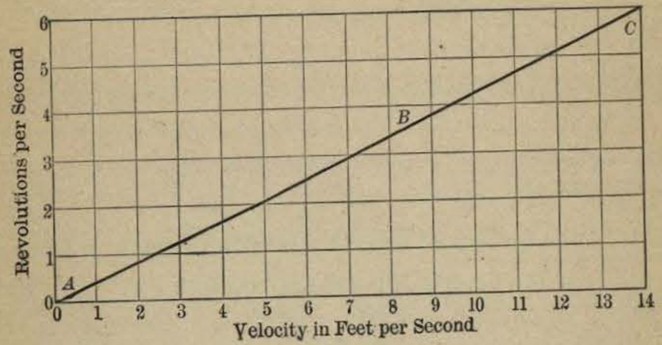

Fig. 40c.

type of meter.* This curve was based on thirty-five observations at different velocities, and practically all of them fell on the line ABC which is also very nearly a straight line.

An examination† of the rating tables of a number of meters has shown that possible errors due to differences in rating are quite small, and that a Price meter in good condition can be used with a standard rating table without serious error for all velocities greater than 0.5 foot per second and then generally within about 2 percent.

While the current meter is an extensively used instrument, there are, as in most other hydraulic work, certain features which are not yet fully understood. These are the differences shown in the results of the ratings of the same meter when held on a rod and when suspended by a cable.‡ It has also been found that the rating of a meter made in still water differs somewhat from that made in running water,‡ but no successful means for making direct running water ratings have as yet been devised. Many good comparisons between current meter gagings and weir measurements have been made, but the current meter

* Transactions American Society of Civil Engineers, 1910, vol. 66, p. 83.
† Transactions American Society of Civil Engineers, 1910, vol. 66, p. 83.
‡ Water Supply and Irrigation Paper, No. 95, U. S. Geological Survey.

velocities in all of them have been relatively low, so that no complete comparison has up to the present been possible.

Prob. 40. In order to rate a certain current meter, three observations were taken in still water, as follows:

Velocity of the car	= 2.0	3.8	7.4 feet per second
Revolutions per minute	= 30	60	120

Plot these observations on cross-section paper and deduce, without using the Method of Least Squares, the relation between V and n in the equation $V = a + bn$.

Art. 41. The Pitot Tube

About 1750 the French hydraulic engineer Pitot invented a device for measuring the velocity in a stream by means of the velocity-head which it will produce. In its simplest form it consists of a bent tube, the mouth of which is placed so as to directly face the current. The water then rises in the vertical part of the tube to a height h above the surface of the flowing stream, and this height is equal to the velocity-head $v^2/2g$, so that the actual velocity v is in practice approximately equal to $\sqrt{2gh}$. As constructed for use in streams, Pitot's apparatus consists of two tubes placed side by side with their submerged mouths at right angles, so that when one is opposed to the current, as seen in Fig. 41b, the other stands normal to it, and the water surface in the latter tube hence is at the same level as that of the stream. Both tubes are provided with cocks which may be closed while the instrument is immersed, and it can be then lifted from the water and the head h be read at leisure. It is found that the actual velocity is always less than $\sqrt{2gh}$, and that a coefficient must be deduced for each instrument by moving it in still water at known velocities. Pitot's tube has the advantage that no time observation is needed to determine the velocity, but it has the disadvantage that the distance h is

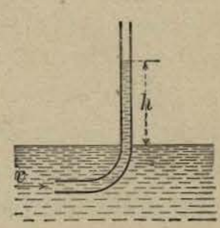

Fig. 41a.

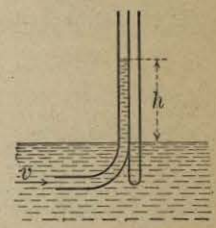

Fig. 41b.

usually very small, so that an error in reading it has a large influence. Although the instrument was improved by Darcy in 1856 and used by him for some stream measurements, it was for a long time regarded as having a low degree of precision.

When using a Pitot tube for measuring the velocity in a stream, the two columns may be raised above the level of the water in the stream and brought to a height convenient for observation by partly exhausting the air from the tubes above the columns. This procedure is analogous to the imposing of an air pressure above the water columns in the case of high heads, as was described in Art. 37.

In 1888 Freeman made experiments on the distribution of velocities in jets from nozzles, in which an improved form of Pitot tube was used.* The point of the tube facing the current was the tip of a stylographic pen, the diameter of the opening being about 0.006 inch. This point was introduced into different parts of the jet and the pressure caused in the tube was measured by a Bourdon pressure gage reading to single pounds. The velocities of the jets were high; for example, in one series of observations on a jet from a $1\frac{1}{8}$-inch nozzle, the gage pressures at the center and near the edge were 51.2 and 18.2 pounds per square inch, which correspond to velocity-heads of 118.2 and 42.0 feet, or to velocities of 87.2 and 52.0 feet per second. By computing the mean velocity of the jet from measurements in concentric rings (Art. 39) and also from the measured discharge, Freeman concluded that any velocity as determined by the tube was smaller than that computed from $v = \sqrt{2gh}$ by less than one percent. This investigation established the fact that the Pitot tube is an instrument of great precision for the measurement of high velocities.

Experiments on the flow of water in pipes, in which Pitot tubes were successfully used, were made in 1897 by Cole at Terre Haute, and in 1898 by Williams, Hubbell, and Fenkell at Detroit.† In the Detroit experiments the tube was introduced into the pipe

* Transactions American Society of Civil Engineers, 1889, vol. 21, p. 413.

† Transactions American Society of Civil Engineers, 1902, vol. 47, pp. 12, 275.

through an opening provided with a stuffing-box, so that the point of the tube might be placed at any desired position. The tubes had openings at their points $\frac{1}{32}$ inch in diameter and other openings of the same size on their sides to admit the static pressure of the water. These latter openings led to a common channel parallel to that leading from the point, and each of these was connected to a rubber hose running to a differential gage, consisting of two parallel glass tubes open at the top, where the difference of head was read on a scale. In order to be able to deduce the velocities in the pipe from the readings of the gage, the Pitot tubes were rated by moving them in still water at known velocities as for the current meter (Art. 40). Thus a coefficient c was derived for each tube for use in the formula $v = c\sqrt{2gh}$. This coefficient was found to range from 0.86 to 0.95 for different tubes, and it varied but little with v.

Many different forms of Pitot tubes have been made and experimented upon. Each of these forms has, in common with the others, the pressure opening which faces the current, though the shape and dimensions of this opening differ materially in the various types. In some of them the static pressure is admitted through a hole in the side of the apparatus, while in others it is admitted through a number of such holes. In another type the tube is made symmetrical with an opening looking downstream. In this case the water column connected with the upstream opening will indicate the velocity head, while that connected with the opening which faces downstream will indicate a pressure less than the static head on account of the negative head induced by the arrangement. The difference between the two columns is thus increased and its reading on the scale rendered more easy, while the proportional error of any reading is also reduced. In Fig. 41c is shown a form of tube used by the U. S. Geological Survey* for the measurement of velocity in small and shallow streams in connection with experiments on the transporting capacity of currents, while in Fig. 41d is shown the type used in connection with the Pitotmeter (Art. 38). In this figure is shown also the method of introducing the tubes into a pipe where the velocity is to be measured.

Some recent comparisons * between the still and moving water ratings of Pitot tubes indicate that there may be a difference between

* Engineering News, Aug. 12, 1909.

the results obtained by these two methods. It is desirable, of course, that every instrument should be rated under conditions similar to those in which it is to be used. One of the ways of rating a Pitot tube

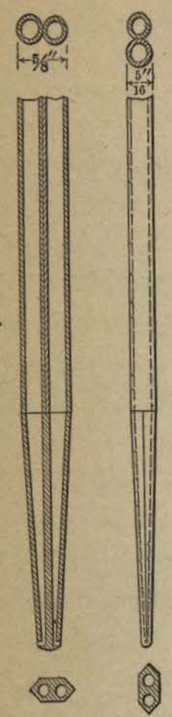

Fig. 41c.

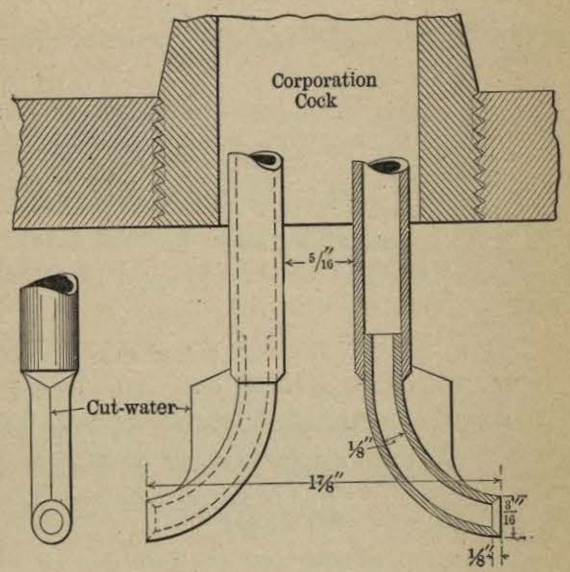

Fig. 41d.

in running water is that suggested and used by Judd and King* who placed the tube used by them at the contracted section of a jet and concluded that its coefficient was 1.00.

Prob. 41. Explain how a well-rated Pitot tube may be used to measure the speed of a boat or ship.

Art. 42. Discussion of Observations

An observation is the recorded result of a measurement. All measurements are affected with errors due to imperfections of the instrument and lack of skill of the observers, and the recorded results contain these errors. Thus, if 6.05, 6.02, 6.01, and 6.04 inches be four observations on the diameter of an orifice, all of

* Engineering News, Sept. 27, 1906.

these cannot be correct, and probably each is in error. The best that can be done is to take the average of these observations, or 6.03 inches, as the most probable result, and to use this in the computations.

An observer is often tempted to reject a measurement when it differs from others, but this can only be allowed when he is convinced that a mistake has been made. A mistake is a large error, due generally to carelessness, and must not be confounded with the small accidental errors of measurement. When a series of observations is placed before a computer, he should never be permitted to reject one of them, unless there is some remark in the note-book which casts doubt upon it.

Graphical methods of discussing and adjusting observations, like that mentioned in Art. 40, are of great value in hydraulic work. As another example, the following observations made by Darcy and Bazin on the flow of water in a rectangular trough, 1.812 meters wide and having the uniform slope 0.049, may be noted. Water was allowed to run through it with varying depths, and for each depth the mean velocity (Art. 39) and the hydraulic mean depth (Art. 112) was determined by measurement. Let v be the mean velocity and r the hydraulic mean depth; then five measurements gave the following observations, v being in meters per second and r in centimeters. Let it be assumed that the

No. =	1	2	3	4	5
v =	1.73	1.98	2.17	2.33	2.46
r =	11.4	14.4	17.0	19.2	21.2

relation between v and r is of the form $v = mr_n$, and let it be required to determine the most probable values of m and n.

For each of these observations a point may be plotted on cross-section paper, taking the values of v as ordinates and those of r as abscissas, and a smooth curve may then be drawn so as to agree as nearly as possible with the points. Such a curve, however, is of little assistance in determining the values of m and n, unless the curve should be a straight line drawn through the origin, in which case it is plain that n is unity and that m is the tangent of the angle that the line makes with axis of abscissas. In this case no straight line can be drawn approximating to the points and passing through the origin, but the plot gives the curve shown in Fig. 22*a*. If, however, the logarithm of each side of the assumed formula be taken, it becomes

$$\log v = n \log r + \log m$$

which represents a straight line if log v be considered as the variable ordinate and log r as the variable abscissa, log m being

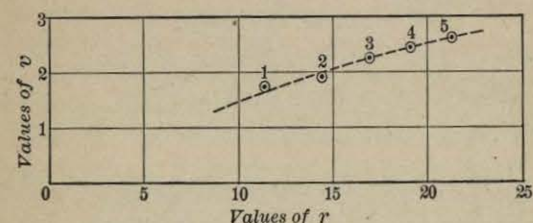

Fig. 42*a*.

the intercept on the axis of ordinates and n the tangent of the angle which the line makes with the axis of abscissas. On plotting the points corresponding to the values of log v and log r, it is seen that a straight line can be drawn closely agreeing with the

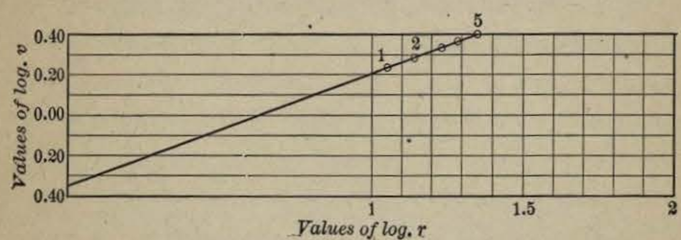

Fig. 42*b*.

points, that this line cuts the axis of ordinates at a distance of about 0.35 below the origin, and that the tangent of the angle made by it with the axis of abscissas is about 0.55. Hence (Fig. 42*b*) $n = 0.55$, $\log m = -0.35 = \bar{1}.65$, or $m = 0.446$; then

$$\log v = 0.55 \log r - 0.35 \quad \text{or} \quad v = 0.446 r^{0.55}$$

is an empirical formula for computing the mean velocity in this trough. Using the above values of r and computing those of v, it is found that the computed and observed results agree fairly,

the former being generally a little smaller, which is due to the fact that only two significant figures are found from the plot.

Whenever a series of plotted points can be closely represented by a straight line on logarithmic section paper, the equation between the variables is an exponential one. Numerous exponential formulas for the flow of water in pipes and channels rest upon the judgment of the investigator in deciding that the plotted points are sufficiently well represented by a straight line.

There is a process, known as the Method of Least Squares, by which the constants of an empirical formula may be obtained from observations with a higher degree of precision than by any graphic method. Its application to the above case will here be given. Let the simultaneous values of log v and log r for each experiment be placed in the logarithmic formula as follows:

for No. 1, $0.238 = 1.057n + \log m$
for No. 2, $0.297 = 1.158n + \log m$
for No. 3, $0.336 = 1.230n + \log m$
for No. 4, $0.367 = 1.283n + \log m$
for No. 5, $0.391 = 1.326n + \log m$

These five equations contain two unknown quantities, n and log m, but no values of these can be found that will exactly satisfy all the equations. The best that can be done is to find the values that have the greatest degree of probability, and these will satisfy the equations with the smallest discrepancies. To do this, let each equation be multiplied by the coefficient of n in that equation and the results be added; also let each equation be multiplied by the coefficient of log m in that equation and the results be added. Thus are found the two normal equations containing the two unknown quantities:

$$1.998 = 7.375n + 6.054 \log m$$
$$1.629 = 6.054n + 5.000 \log m$$

and the solution of these gives $n = 0.571$ and $\log m = -0.366$. Since -0.366 equals $\bar{1}.634$, the value of m is 0.431, and then

$$\log v = 0.571 \log r - 0.366 \quad \text{or} \quad v = 0.431 r^{0.571}$$

is the empirical formula for this particular case.

The Method of Least Squares is usually more laborious than the graphical method, but it has the great advantage that its results are the most probable ones that can be derived from the given data. It has the further advantage that all computors will derive the same results, whereas in the graphic method the results will usually differ, because the position of the line drawn on the plot is affected by the different degrees of judgment and experience of the draftsmen. It will be seen from Fig. 42*b* that it is not very easy to determine close values of log m since the plotted points are so far away from the origin.

Prob. 42*a*. In order to rate a certain current meter four observations were taken in still water as follows:

Velocity of the car	0.7	2.4	4.7	9.3	feet per second
Revolutions of meter	18	60	120	240	per minute

Find the values of a and b in the formula $v = a + bn$, both by plotting and by the method of least squares.

Prob. 42*b*. Three observations of horizontal angles are made at the station O, which give $AOB = 62°17'$, $BOC = 20°35'$, $AOC = 82°55'$. Adjust these observations by the method of least squares so that the large angle may be equal to the sum of its parts.

CHAPTER 5

FLOW OF WATER THROUGH ORIFICES

Art. 43. Standard Orifices

Orifices for the measurement of water are usually placed in the vertical side of a vessel or reservoir, but may also be placed in the base. In the former case it is understood that the upper edge of the opening is completely covered with water; and generally the head of water on an orifice is at least three or four times its vertical height. The term "standard orifice" is here used to signify that the opening is so arranged that the water in flowing from it touches only a line, as would be the case in a plate of no thickness. To secure this result the inner edge of the opening has a square corner, which alone is touched by the water. In precise experiments the orifice may be in a metallic plate whose thickness is really small, as at A in the figure, but more commonly it is cut in a board or plank, care being taken that the inner edge is a definite corner. It is usual to bevel the outer edges of the orifice, as at C, so that the escaping jet may by no possibility touch the edges except at the inner corner. The term "orifice in a thin plate" is often used to express the condition that the water shall only touch the edges of the opening along a line. This arrangement may be regarded as a kind of standard apparatus for the measurement of

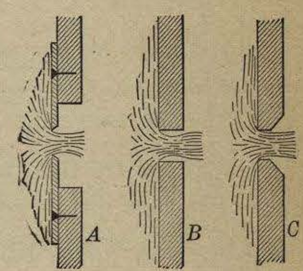

Fig. 43*a*.

water; for, as will be seen later, the discharge is modified when the inner corner is rounded, and different degrees of rounding give different discharges. The standard arrangements shown in Fig. 43*a* are accordingly always used when water is to be measured by the use of orifices.