accelerations due to gravity and centrifugal force. The ratio AC/AB is the tangent of the angle θ which the water surface at A makes with the axis OX, for this surface must be normal to the resultant AD of the two forces AB and AC. When the ordinate y is increased to $y+\delta y$, the abscissa x is decreased to $x-\delta x$, and hence the value of $\tan\theta$ must be the same as $-\delta y/\delta x$. Accordingly

$$\tan\theta = \frac{AC}{AB} = \frac{v^2}{gx} = 2\frac{y}{x} = -\frac{\delta y}{\delta x}$$

and the integration of this differential equation gives $y = C/x^2$, in which C is the constant of integration. When y equals H, the value of x is r, and hence $C = Hr^2$, and thus

$$y = Hr^2/x^2 \qquad (32)_2$$

is the equation of the curve, which may be called a quadratic hyperbola, the surface of the funnel being then a quadratic hyperboloid. This equation represents the curve at one instant only, for H continually decreases as the water flows out, since the direction of v is not quite horizontal as the investigation assumes. The general phenomena are, however, well explained by this discussion.

Prob. 32. A prismatic vessel has a cross-section of 18 square feet and an orifice in its base has an area of 0.18 square foot. Find the theoretic time for the water level to drop 7 feet, when the head upon the orifice at the beginning is 16 feet.

Art. 33. Computations in Metric Measures

(Art. 22) Using for the acceleration of the mean value 9.80 meters per second per second, formulas $(22)_2$ become

$$V = 4.427\sqrt{h} \qquad h = 0.05102\,V^2 \qquad (33)$$

in which h is in meters and V in meters per second. Table 33 shows values of the velocity for given heads, and values of the velocity-head for given velocities.

(Art. 23) For Fig. 23 let the reservoir be one meter in diameter, the load W be 2000 kilograms, and the orifices be 3 meters below the piston. Let the exterior head on A be 1.5 meters, the orifice B be open to the atmosphere, and the orifice C be in air whose pressure is 0.7 kilograms per square centimeter. The area of the piston is 0.7854

Table 33. Velocities and Velocity-heads

Metric Measures

$V=\sqrt{2gh}=4.427\sqrt{h}$				$h=V^2/2g=0.05102\,V^2$			
Head in Meters	Velocity in Meters per Second	Head in Meters	Velocity in Meters per Second	Velocity in Meters per Second	Head in Meters	Velocity in Meters per Second	Head in Meters
0.1	1.432	1	4.427	0.1	0.0005	1	0.0510
0.2	1.980	2	6.262	0.2	0.0020	2	0.2041
0.3	2.425	3	7.668	0.3	0.0046	3	0.4592
0.4	2.799	4	8.854	0.4	0.0082	4	0.8163
0.5	3.131	5	9.900	0.5	0.0123	5	1.276
0.6	3.429	6	10.84	0.6	0.0184	6	1.837
0.7	3.704	7	11.71	0.7	0.0250	7	2.500
0.8	3.960	8	12.52	0.8	0.0327	8	3.265
0.9	4.200	9	13.28	0.9	0.0413	9	4.133
1.0	4.427	10	14.00	1.0	0.0510	10	5.102

square meters, and the head corresponding to the pressure on the upper water surface is

$$h_0 = \frac{p_0}{w} = \frac{2000}{0.7854 \times 1000} = 2.546 \text{ meters.}$$

The head h_1 is 3 meters for the first orifice, 0 for the second, and $-10(1.033-0.7) = -3.33$ meters for the third. The three theoretic velocities of outflow then are

$$V = 4.427\sqrt{3+2.546-1.5} = 8.91 \text{ meters per second,}$$
$$V = 4.427\sqrt{3+2.546-0} = 10.43 \text{ meters per second,}$$
$$V = 4.427\sqrt{3+.546+3.33} = 13.19 \text{ meters per second.}$$

If in this example the liquid be alcohol which weighs 800 kilograms per cubic meter, the head of alcohol corresponding to the pressure of the piston is

$$h_0 = \frac{2000}{0.7854 \times 800} = 3.183 \text{ meters,}$$

and accordingly for discharge into the atmosphere at the depth $h_1 = 3$ meters the velocity is

$$V = 4.427\sqrt{3+3.18} = 11.01 \text{ meters per second,}$$

while for water the velocity was 10.43 meters per second.

(Art. 26) As an illustration of $(26)_2$ let water issue from a pipe 6 centimeters in diameter with a velocity of 4 meters per second. The cross-section is found from Table F to be 0.002827 square meters, and then the theoretic work in kilogram-meters per second is

$$K = 0.05102 \times 1000 \times 0.002827 \times 4^3 = 9.23$$

which is 0.123 metric horse-power. If the velocity is 16 meters per second, the stream will furnish 7.87 horse-powers.

(Art. 30) The area a is in square meters, the velocity V in meters per second, and the discharge Q in cubic meters per second. Thus if a pipe 20 centimeters in diameter discharges 0.15 cubic meters per second, the area of the cross-section is 0.03142 square meters and the mean velocity is $0.15/0.03142=4.77$ meters per second.

(Art. 31) In Fig. 31*a*, suppose the sections a_1 and a_2 to be 0.06 and 0.12 square meters, and the depths of their centers below the water level of the reservoir to be 4.5 and 5.5 meters. Let 0.24 cubic meters per second be discharged from the pipe, then from $(31)_1$ the mean velocities in a_1 and a_2 are 4.0 and 2.0 meters per second. The velocity-heads are then 0.82 meters for a_1 and 0.20 meters for a_2, so that during the flow the pressure-head at A is $4.5 - 0.82 = 3.68$ meters and that at B is $5.5 - 0.20 = 5.30$ meters.

Prob. 33*a*. What theoretic velocities are produced by heads of 0.1, 0.01, and 0.001 meter? What is the velocity-head of a jet, 7.5 centimeters in diameter, which discharges 500 liters per second?

Prob. 33*b*. A prismatic vessel has a cross-section of 1.5 square meters and an orifice in its base has an area of 150 square centimeters. Compute the theoretic time for the water level to drop 3 meters when the head at the beginning is 4 meters.

Prob. 33*c*. A small turbine wheel using 3 cubic meters of water per minute under a head of $10\frac{1}{2}$ meters is found to deliver 5.1 metric horse-powers. Compute the efficiency of the wheel.

Prob. 33*d*. In an inclined tube there are two sections of diameters 10 and 20 centimeters, the second section being 1.536 meters higher than the first. The velocity in the first section is 6 meters per second and the pressure-head is 7.045 meters. Find the pressure-head for the second section.

CHAPTER 4

INSTRUMENTS AND OBSERVATIONS

ART. 34. GENERAL CONSIDERATIONS

Some of the most important practical problems of Hydraulics are those involving the measurement of the amount of water discharged in one second from an orifice, pipe, or conduit under given conditions. The theoretic formulas of the last chapter furnish the basis of most of these methods, and in the chapters following this one are given coefficients derived from experience which enable those formulas to be applied to practical conditions. These coefficients have been determined by measuring heads, pressures, or velocities with certain instruments, and also the amount of water actually discharged, and then comparing the theoretic results with the actual ones. It is the main object of this chapter to describe the instruments used for this purpose, and a few remarks concerning advantageous methods for the discussion of the observations will also be made.

The engineer's steel tape, level, and transit are indispensable tools in many practical hydraulic problems. For example, two reservoirs M and N, connected by a pipe line, may be several miles apart. To ascertain the difference in elevation of their water surfaces lines of levels may be run and bench marks established near each reservoir, as also at other points along the pipe line. From the bench marks at the reservoirs there can be set up simple board gages, so that simultaneous readings can be taken at any time to find the difference in elevation. From the bench marks along the pipe line a profile of the same can be plotted for use in the discussion. With the transit

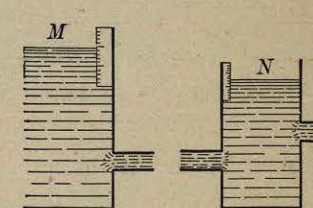

Fig. 34*a*.