The energy of a jet is the same whether its direction of motion be vertical, horizontal, or inclined, and per second it is always Wh, where h is the velocity-head corresponding to actual velocity v, and W is the weight of water delivered per second. The energy should not be computed from the theoretical velocity V, as this is usually greater than the actual velocity.

Prob. 26. When water issues from a pipe with a velocity of 3 feet per second, its kinetic energy is sufficient to generate 1.3 horse-powers. What is the horse-power when the velocity becomes 6 feet per second?

Art. 27. Impulse and Reaction of a Jet

When a stream or jet is in motion, delivering W pounds of water per second with the uniform velocity v, that motion may be regarded as produced by a constant force F, which has acted upon W for one second and then ceased. In this second the velocity of W has increased from 0 to v, and the space $\frac{1}{2}v$ has been described. Consequently the work $F \times \frac{1}{2}v$ has been imparted to the water by the force F. But the kinetic energy of the moving water is $W \cdot v^2/2g$, and hence by the law of conservation of energy $F \times \frac{1}{2}v = W \times v^2/2g$, from which the constant force is

$$F = W \cdot v/g \qquad (27)_1$$

This value of F is called the "impulse" of the jet. As W is in pounds per second, v in feet per second, and g in feet per second per second, the value of F is in pounds.

In theoretical mechanics, the term "impulse" is used in a slightly different sense, namely, as force multiplied by time. In hydraulics, however, W is not pounds, but pounds per second, and thus the impulse is simply pounds. The force F is to be regarded as a continuous impulsive pressure acting at the origin of the jet in the direction of the motion. For, by the definition, F acts for one second upon the W pounds of water which pass a given section; but in the next second W pounds also pass, and the same is the case for each second following. This impulse will be exerted as a pressure upon any surface which is placed in the path of the jet.

The reaction of a jet upon a vessel occurs when water flows from an orifice. This reaction must be equal in value and opposite in direction to the impulse, as in all cases of stress action and reaction are equal. In the direction of the jet the impulse produces motion, in the opposite direction it produces an equal pressure which tends to move the vessel backward. The force of reaction of a jet is hence equal to the impulse but opposite in direction. For example (Fig. 27), let a vessel containing water be suspended at A so that it can swing freely, and let an orifice be opened in its side at B. The head of water at B causes a pressure which acts toward the left and causes W pounds of water to move during every second with the velocity of v feet per second, and which also acts toward the right and causes the vessel to swing out of the vertical; the first of these forces is the impulse, and the second is the reaction of the jet. If a force R be applied on the right of a vessel so as to prevent the swinging, its value is

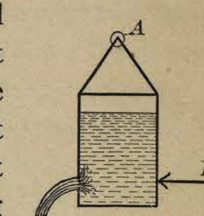

Fig. 27.

$$R = F = W \cdot v/g \qquad (27)_2$$

and this is the formula for the reaction of the jet.

The impulse or reaction of a jet issuing from an orifice is double the hydrostatic pressure on the area of the orifice. Let h be the head of water, a the area of the orifice, and w the weight of a cubic unit of water; then, by Art. 15, the normal pressure when the orifice is closed is wah. When the orifice is opened, the weight of water issuing per second is $W = wav$, and hence the impulse or reaction of the jet is

$$R = F = wav \cdot v/g = 2wa \cdot v^2/2g = 2wah$$

which is double the hydrostatic pressure. This theoretic conclusion has been verified by many experiments (Art. 144).

When a jet impinges normally on a plane, it produces a dynamic pressure on that plane equal to the impulse F, since the force required to stop W pounds of water in one second is the same as that required to put it in motion. Again, if a stream moving with the velocity v is retarded so that its velocity becomes v_2, the impulse in the first instant is $W \cdot v_1/g$, and in the second $W \cdot v_2/g$. The difference of these, or

$$F_1 - F_2 = W(v_1 - v_2)/g \qquad (27)_3$$

is a measure of the dynamic pressure which has been developed. It is by virtue of the pressure due to change of velocity that turbine wheels and other hydraulic motors transform the kinetic energy of moving water into useful work.

Prob. 27. If a stream of water 3 inches in diameter issues from an orifice in a direction inclined downward 26° to the horizon with a velocity of 15 feet per second, find its horizontal reaction on the vessel.

Art. 28. Absolute and Relative Velocities

Absolute velocity is defined in this book as that with respect to the surface of the earth, and relative velocity as that with respect to a body moving on the earth. Thus absolute velocity is that seen by a spectator who is on the earth, and relative velocity is that seen by one who is on the moving body. For instance, if a body is dropped by a person who is on a moving railroad car, it appears to a person standing outside to move obliquely, but to one on the car it appears to move vertically. On a car in uniform motion all the laws of mechanics prevail exactly as if it were at rest; hence if a body of weight W is dropped through a height h, it acquires a theoretic vertical velocity of $\sqrt{2gh}$ with respect to the car. But if the horizontal velocity of the car is u, the kinetic energy of the body at the moment of letting it fall is $W \cdot u^2/2g$ and its potential energy is Wh, so that, neglecting frictional resistances, its total energy after falling through the height h is the sum of these, and accordingly its absolute velocity with respect to the earth is $\sqrt{2gh + u^2}$.

When a vessel containing water with a free surface, as in Fig. 28a, has an orifice under the head h and is in motion in a straight line with the uniform absolute velocity u, the theoretic velocity of flow relative to the vessel is $V = \sqrt{2gh}$, or the same as its absolute velocity if the vessel were at rest, for no accelerating forces exist to change the direction or the value of g. The abso-

lute velocity of flow, however, may be greater or less than V, depending upon the value of u and its direction. To illustrate, take the case of a vessel in uniform horizontal motion from which water is flowing through three orifices. At A the direction of V is horizontal, and as the vessel is moving in the opposite direction with the velocity u, the absolute velocity of the water as it leaves the orifice is $v = V - u$. It is also plain, if the orifice is in front of the vessel and the direction of V is horizontal, that the absolute velocity of the water as it leaves the orifice is $v = V + u$.

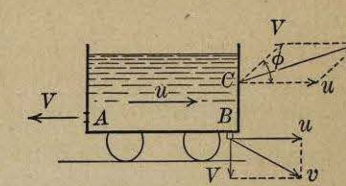

Fig. 28a.

Again, at B is an orifice from which the water issues vertically with respect to the vessel with the relative velocity V, while at the same time the orifice moves horizontally with the absolute velocity u. Forming the parallelogram, the absolute velocity v is seen to be the resultant of the velocities V and u, or

$$v = \sqrt{V^2 + u^2}$$

Lastly, at C is shown an orifice in the front of the vessel so arranged that the direction of the relative velocity V makes an angle ϕ with the horizontal. From C draw Cu to represent the velocity u, and CV to represent V, and complete the parallelogram as shown; then Cv, the resultant of u and V, is the absolute velocity with which the water leaves the orifice. From the triangle Cuv

$$v = \sqrt{V^2 + u^2 + 2uV\cos\phi} \qquad (28)$$

In this, if $\phi = 0$, the absolute velocity v becomes $V + u$, as before shown for an orifice in the front; if $\phi = 90°$, it becomes the same as when the water issues vertically from the orifice in the base; and if $\phi = 180°$, the value of v is $V - u$ as before found for an orifice in the rear end.

Another case is that of a revolving vessel having an opening from which the water issues horizontally with the relative velocity V, while the orifice is moving horizontally with the absolute velocity u. Fig. 28b shows this case, β being the angle which V makes with the reverse direction of u, and here also

$$v = \sqrt{V^2 + u^2 - 2uV\cos\beta}$$

is the absolute velocity of the water as it leaves the vessel. In all cases the absolute velocity of a body leaving a moving surface is the diagonal of a parallelogram, one side of which is the velocity of the body relative to the surface and the other side is the absolute velocity of that surface.

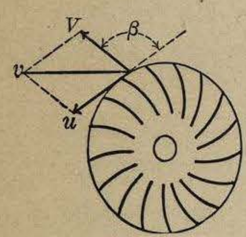

Fig. 28b.

When a vessel moves with a motion which is accelerated or retarded, this affects the value of g, and the reasoning of the preceding articles does not give the correct value of V. For instance, when a vessel moves vertically upward with an acceleration f, the relative velocity of flow from an orifice in it is $V = \sqrt{2(g+f)h}$, and if u be the velocity of the vessel at any instant, the absolute downward velocity of flow is $V - u$. Again, when it moves downward with the acceleration f, the relative velocity of flow is $V = \sqrt{2(g-f)h}$ and the absolute is $V + u$. If the downward acceleration is g, the vessel is freely falling and V will be zero, since both vessel and water are alike accelerated and there is then no pressure on the base.

Prob. 28. In Fig. 28a let the orifice at A be under a head of 5.5 feet and its height above the earth be 7.5 feet, while the car moves with a velocity of 40 miles per hour. Compute the relative velocity V, the absolute velocity v, and the absolute velocity of the jet as it strikes the earth.

Art. 29. Flow from a Revolving Vessel

Water in a vessel at rest on the surface of the earth is acted upon only by the vertical force of gravity, and hence its surface is a horizontal plane. Water in a revolving vessel is acted upon by centrifugal force as well as by gravity, and it is observed that its surface assumes a curved shape. The simplest case is that of a cylindrical vessel rotating with uniform velocity about its

vertical axis, and it will be shown that here the water surface is that of a paraboloid.

Let BC be the vertical axis of the vessel, h the depth of water in it when at rest, and h_1 and h_2 the least and greatest depths of water in it when in motion. Let G be any point on the surface of the water at the horizontal distance x from the axis, and let y be the vertical distance of G above the lowest point C. The head of water on any point E in the base is EG or $h_1 + y$. Now this head y is caused by the velocity u with which the point G revolves around the axis, or, in other words, the position of G above C is due to the energy of rotation. Thus if W is the weight of a particle of water at G, the potential energy Wy equals the kinetic energy $Wu^2/2g$, and hence $y = u^2/2g$.

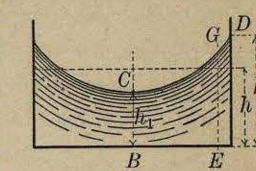

Fig. 29a.

Let n be the number of revolutions made by the vessel and water in one second. Then $u = 2\pi x \cdot n$, and hence

$$y = u^2/2g = 2\pi^2 n^2 x^2/g$$

which is the equation of a common parabola with respect to rectangular axes having an origin at its vertex C. The surface of revolution is hence a paraboloid.

Since the volume of a paraboloid is one-half that of its circumscribing cylinder, and since the same quantity of water is in the vessel when in motion as when at rest, it is plain that in the figure $\frac{1}{2}(h_2 - h_1)$ equals $h - h_1$. Consequently $h - h_1$ equals $h_2 - h$, or the elevation of the water surface at D above its original level is equal to its depression at C. If r be the radius of the vessel, the height $h_2 - h_1$ is, from the above equation, $2\pi^2 n^2 r^2/g$, and hence the distances $h - h_1$ and $h_2 - h$ are each equal to $\pi^2 n^2 r^2/g$. The head at the middle of the base of the vessel during the motion is now $h_1 = h - \pi^2 n^2 r^2/g$ and the head at any point E is $h_1 + y = h + (2x^2 - r^2)\pi^2 n^2/g$.

The theoretic velocity of flow from the small orifice in the base is that due to the head $h_1 + y$, or

$$V = \sqrt{2g(h_1 + y)} = \sqrt{2gh + 2\pi^2 n^2 (2x^2 - r^2)}$$

which is less than $\sqrt{2gh}$ when x^2 is less than $\frac{1}{2}r^2$, and greater when x^2 is greater than $\frac{1}{2}r^2$. For example, let $r = 1$ foot and $h = 3$ feet, then $V = 13.9$ feet per second when the vessel is at rest. But if it is rotating three times per second around its axis with uniform speed, the velocity from an orifice in the center of the base, where $x = 0$, is 3.9 feet per second, while the velocity from an orifice at the circumference of the base, where $x = 1$ foot, is 19.2 feet per second. At this speed the water is depressed 2.76 feet below its original level at the center and elevated the same amount above that level around the sides of the vessel.

In the case of a closed vessel where the paraboloid cannot form, the velocity of flow from all orifices, except one at the axis, is increased by the rotation. Thus in Fig. 29*b*, if the vessel is at rest and the head on the base is h, the velocity of flow from all small orifices in the base is $\sqrt{2gh}$. But if the vessel is revolved about the vertical axis BC, so that an orifice at E has the velocity u around that axis, then the pressure-head at E is $h + u^2/2g$, and accordingly

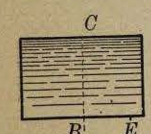

Fig. 29*b*.

$$V = \sqrt{2gh + u^2} \qquad (29)$$

is the theoretic velocity of flow from an orifice at E. This formula is an important one in the discussion of hydraulic motors. Here, as before, the value of u may be expressed as $2\pi xn$, when x is the distance of E from the axis and n is the number of revolutions per second. As an example, let a closed vessel full of water be revolved about an axis 120 times per minute, and let it be required to find the theoretic velocity of flow from an orifice $1\frac{1}{2}$ feet from the axis, the head on which is 4 feet when the vessel is at rest. The velocity u is found to be 18.85 feet per second, and then the theoretic velocity of flow from the orifice is 24.8 feet per second, whereas it is only 16 feet per second when the vessel is at rest.

The velocity V in both these cases is a relative velocity, for the pressure at the moving orifice produces a velocity with respect to the vessel. The absolute velocity, or that with respect to the earth, is greater than the relative velocity when the stream issues from an orifice in the base, for the orifice moves horizontally with the absolute velocity u and the stream moves downward with the relative velocity V, and hence the absolute velocity of the stream is $\sqrt{V^2 + u^2}$. When the stream issues from an orifice in the side of the vessel upon which the head is h, formula (29) gives its relative velocity, and then the absolute velocity is found by (28).

Prob. 29. A cylindrical vessel 2 feet in diameter and 3 feet deep is three-fourths full of water, and is revolved about its vertical axis so that the water is just on the point of overflowing around the upper edge. Find the number of revolutions per minute. Find the relative velocity of flow from an orifice in the base at a distance of 0.75 foot from the axis. Show that the velocity from all orifices within 0.707 foot of the axis is less than if the vessel were at rest.

Art. 30. Theoretic Discharge

The term "discharge" means the volume of water flowing in one second from a pipe or orifice, and the letter Q will designate the theoretic discharge; that is, the discharge as computed without considering the losses due to frictional resistances. When all the filaments of water issue from the pipe or orifice with the same velocity, the quantity of water issuing in one second is equal to the volume of a prism having a base equal to the cross-section of the stream and a length equal to the velocity. If this area is a and the theoretic velocity is V, then $Q = aV$ is the theoretic discharge. Taking a in square feet and V in feet per second, the discharge Q is in cubic feet per second.

For a small orifice on which the head h has the same value for all parts of the opening, the theoretic discharge is

$$Q = aV = a\sqrt{2gh} \qquad (30)$$

and in English measures $Q = 8.02a\sqrt{h}$. For example, let a circular orifice 3 inches in diameter be under a head of 10.5 feet, and let it be required to compute Q. Here 3 inches = 0.25 foot and from Table F the area of the circle is 0.04909 square foot. From Art. 22 the theoretic velocity V is $8.02 \times \sqrt{10.5} = 25.99$ feet per second. Accordingly the theoretic discharge is $0.04909 \times 25.99 = 1.28$ cubic feet per second.

The above formula for Q applies strictly only to horizontal orifices upon which the head h is constant, but it will be seen later that its error for vertical orifices is less than one-half of one percent when h is greater than double the depth of the orifice. Horizontal orifices are but little used, as it is more convenient in practice to arrange an opening in the side of a vessel than in its base. In applying the above formula to a vertical orifice, h is taken as the vertical distance from its center to the free-water surface. Vertical orifices where the head h is small are discussed in Arts. 47 and 48.

Since the theoretic velocity is always greater than the actual velocity, the theoretic discharge is a limit which can never be reached under actual conditions. Theoretically the discharge is independent of the shape of the orifice, so that a square orifice of area a gives the same theoretic discharge as a circular orifice of area a; it will be seen in Chap. 5 that this is not quite true for the actual discharge.

In this chapter it is supposed that the velocity of a jet is the same in all parts of the cross-section, as this would be the case if h has the same value throughout the section were it not for the retarding influence of friction. Actually, however, the filaments of water near the edges of the orifice move slower than those near the center. If q be the actual discharge from any orifice and v the mean velocity in the area a, then $q = av$, or the equation $v = q/a$ may be regarded as a definition of the term "mean velocity." The theoretic mean velocity is $2\sqrt{gh}$, but the actual mean velocity is slightly smaller, as will be seen in Chap. 5.

Formula (30) may be used for computing h when Q and a are given, and it shows that the theoretic head required to deliver a given discharge varies inversely as the square of the area of the orifice.

Prob. 30*a*. Compute the theoretic head required to deliver 300 gallons of water per minute through an orifice 3 inches in diameter.

Prob. 30*b*. A vessel one foot square has a small orifice in the base. What is the theoretic velocity of flow from this orifice when the vessel contains 125 pounds of mercury? Also when it contains 250 pounds of water?

Art. 31. Steady Flow in Smooth Pipes

When water flows through a pipe of varying cross-section and all sections are filled with water, the same quantity of water passes each section in one second. This is called the case of steady flow. Let q be this quantity of water and let v_1, v_2, v_3 be the mean velocities in three sections whose areas are a_1, a_2, a_3. Then

$$q = a_1v_1 = a_2v_2 = a_3v_3 \qquad (31)_1$$

This is called the condition for steady flow or the equation of continuity, and it shows that the velocities at different sections vary inversely as the areas of those sections. If v be the velocity at the end of the pipe where the area is a, then also $q = av$. When the discharge q and the areas of the cross-sections have been measured, the mean velocities may be computed.

When a pipe is filled with water at rest, the pressure at any point depends only upon the head of water above that point. But when the water is in motion, it is a fact of observation that the pressure becomes less than that due to the head. The unit-pressure in any case may be measured by the height of a column of water. Thus if water be at rest in the case shown in Fig. 31a, and small tubes be inserted at the sections whose areas are a_1 and a_2, the water will rise in each tube to the same level as that of the water surface in the reservoir, and the pressures in the sections

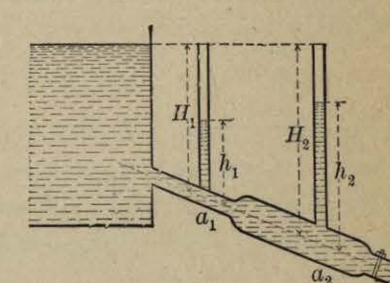

Fig. 31*a*.

will be those due to the hydrostatic heads H_1 and H_2. But if the valve at the right be opened, the water levels in the small tubes will sink and the mean pressures in the two sections will be those due to the pressure-heads h_1 and h_2.

Let W be the weight of water flowing in each second through each section of the pipe, and let v_1 and v_2 be the mean velocity in the section a_1 and a_2. When this water was at rest, the potential energy of pressure in the section a_1 was WH_1; when it is in motion, the energy in the section is the pressure energy Wh_1 plus the kinetic energy $W \cdot v_1^2/2g$. If no losses of energy due to friction or impact have occurred, the energy in the two cases must be equal. The same reasoning applies to the section a_2, and hence

$$H_1 = h_1 + \frac{v_1^2}{2g} \qquad \text{and} \qquad H_2 = h_2 + \frac{v_2^2}{2g} \qquad (31)_2$$

These equations exhibit the law of steady flow first deduced by Daniel Bernouilli in 1738, and hence often called Bernouilli's theorem; it may be stated in words as follows:

> At any section of a tube or pipe, under steady flow without friction, the pressure-head plus the velocity-head is equal to the hydrostatic head that obtains when there is no flow.

This theorem of theoretical hydraulics is of great importance in practice, although it has been deduced for mean velocities and mean pressure-heads, while actually the velocity and the pressure are not the same for all points of the cross-section.

The pressure-head at any section hence decreases when the velocity of the water increases. To illustrate, let the depths of the centers of a_1 and a_2 be 6 and 8 feet below the water level, and let their areas be 1.2 and 2.4 square feet. Let the discharge of the pipe be 14.4 cubic feet per second. Then from $(31)_1$ the mean velocity in a_1 is $v_1 = 14.4/1.2 = 12$ feet per second, which corresponds to a velocity head of $0.01555v^2 = 2.24$ feet, and consequently from $(31)_2$ the pressure-head in a_1 is $6.0 - 2.24 = 3.76$ feet. For the section a_2 the velocity is 6 feet per second and the velocity head is 0.56 feet, so that the pressure-head there is $8.0 - 0.56 = 7.44$ feet.

The theorem of $(31)_2$ may be also applied to the jet issuing from the end of the pipe. Outside the pipe there can be no pressure, and if h be the hydrostatic head and V the velocity, the equation gives $h = V^2/2g$, or $V = \sqrt{2gh}$; that is, if frictional resistances be not considered, the theoretic velocity of flow from the end of a pipe is that due to the hydrostatic head upon it. In Chap. 8 it will be seen that the actual velocity is much smaller

than this, for a large part of the head h is expended in overcoming friction in the pipe.

A negative pressure may occur if the velocity-head becomes greater than the hydrostatic head, for $(31)_2$ shows that h_1 is negative when $v_1^2/2g$ exceeds H_1. A case of this kind is given in Fig. 31*b*, where the section at A is so small that the velocity is greater than that due to the head H_1, so that if a tube be inserted at A, no water runs out; but if the tube be carried downward into a vessel of water, there will be lifted a column CD whose height is that of the negative pressure-head h_1. For example, let the cross-section of A be 0.4 square feet, and its head h be 4.1 feet, while 8 cubic feet per second are discharged from the orifice below. Then the velocity at A is 20 feet per second, and the corresponding velocity-head is 6.22 feet. The pressure head at A then is, from the theorem of formula $(31)_2$,

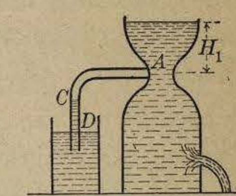

Fig. 31*b*.

$$h_1 = 4.1 - 6.22 = -2.12 \text{ feet}$$

and accordingly there exists at A an inward pressure

$$p_1 = -2.12 \times 0.434 = -0.92 \text{ pounds per square inch}$$

This negative pressure will sustain a column of water CD whose height is 2.12 feet. When the small vessel is placed so that its water level is less than 2.12 feet below A, water will be constantly drawn from the smaller to the larger vessel. This is the principle of the action of the injector-pump.

Prob. 31. In a horizontal tube there are two sections of diameters 1.0 and 1.5 feet. The velocity in the first section is 6.32 feet per second, and the pressure-head is 21.57 feet. Find the pressure-head for the second section if no energy is lost between the sections.

Art. 32. Emptying a Vessel

Let the depth of water in a vessel be H; it is required to determine the theoretic time of emptying it through an orifice in the base whose area is a. Let Y be the area of the water surface when the depth of water is y; let δt be the time during which the water level falls the distance δy. During this time the quantity of water $Y \cdot \delta y$ passes through the orifice. But the discharge in one second under the constant head y is $a\sqrt{2gy}$, and hence the discharge in the time δt is $a\delta t\sqrt{2gy}$. Equating these two expressions, there is found the general formula which gives the time for the water surface to drop the distance δy,

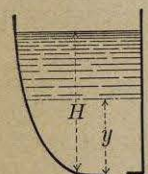

Fig. 32*a*.

$$\delta t = \frac{Y\delta y}{a\sqrt{2gy}} \qquad (32)_1$$

The time of emptying any vessel is now determined by inserting for Y its value in terms of y, and then integrating between the limits H and 0.

For a cylinder or prism the cross-section Y has the constant value A, and the formula becomes

$$\delta t = \frac{Ay^{-\frac{1}{2}}\,\delta y}{a\sqrt{2g}}$$

the integration of which, between limits H and h, gives

$$t = \frac{2A}{a\sqrt{2g}}(\sqrt{H} - \sqrt{h})$$

as the theoretic time for the head H to fall to h. If $h = 0$, this formula gives the time of emptying the vessel. If the head were maintained constant, the uniform discharge per second would be $a\sqrt{2gH}$, and the time of discharging a quantity equal to the capacity of the vessel is AH divided by $a\sqrt{2gH}$, which is one-half of the time required to empty it.

To find the time of emptying a hemispherical bowl of radius r through a small orifice at its lowest point, let x be the radius of the cross-section Y; then $x^2 + (r - y)^2 = r^2$ is the equation of the circle, from which the area Y is $\pi(2ry - y^2)$. Then

$$\delta t = \frac{\pi}{a\sqrt{2g}}(2ry^{\frac{1}{2}} - y^{\frac{3}{2}})\,\delta y$$

and by integration between the limits r and 0

$$t = 14\pi r^{\frac{5}{2}}/15\,a\sqrt{2g}$$

which is the theoretic time required to empty the bowl.

The most important application of these principles is in the case of the right prism or cylinder, and here the formula for the time is modified in practice by introducing a coefficient, as may be seen in Art. 58. The theoretic time found by the above formula is always too small, since frictional resistances have not been considered. Moreover, the formula does not strictly apply when the head is very small, owing to a whirling motion that occurs and which tends to increase the theoretic time.

Venturi, in 1798, first described the phenomena of this whirl.* When the head becomes less than about three diameters of the orifice, the water is observed in whirling motion, the velocity being greatest near the vertical axis through the center of the orifice, and as the head decreases a funnel is formed through the middle of the issuing stream. The direction of this whirl, as seen from above, may be either clockwise or contraclockwise, depending on initial motions in the water or on irregularities in the vessel or orifice, but under ideal conditions it should be clockwise in the southern hemisphere of the earth and contraclockwise in the northern hemisphere, this being the effect of the earth's rotation. Fig. 32*b* represents a vertical section of this funnel, on which A is any point having the coördinates x and y with respect to the rectangular axes OX and OY. The axis OY is drawn through the center of the orifice, and OX is tangent to the level water surface at a distance H above the bottom of the vessel. Let r be the radius of the funnel in the plane of the orifice. It is required to find the relation between x, y, H, and r, or the equation of the curve shown in the figure.

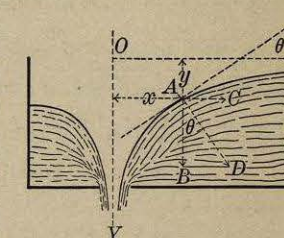

Fig. 32*b*.

An approximate solution may be made by supposing that the particle of water at A is moving nearly horizontally around the axis OY with the velocity v; this velocity must be due to the head y, whence $v^2 = 2gy$. This particle is acted upon by the downward force AB, due to gravity, and by the horizontal force AC, due to centrifugal action, and they are proportional to g and v^2/x, these being the

* Tredgold's Tracts on Hydraulics (London, 1799 and 1826) gives a translation of the memoir of Venturi.