case surfaces are given whose centers of gravity are difficult to determine, they should be divided into simpler surfaces, and then the total normal pressure is the sum of the normal pressures on the separate surfaces.

The normal pressure on the base of a vessel filled with water is equal to the weight of a cylinder of water whose base is the base of the vessel, and whose height is the depth of water. Only in the case of a vertical cylinder does this become equal to the weight of the water, for the pressure on the base of a vessel depends upon the depth of water and not upon the shape of the vessel. Also in the case of a dam, the depth of the water and not the size of the pond, determines the amount of pressure.

When a surface is plane, the total normal pressure is the resultant of all the parallel pressures acting upon it. This is not true for curved surfaces; for, as the pressures have different directions, their resultant is not equal to their numerical sum, but must be obtained by the rules for the composition of forces. For example, when a sphere of diameter d is filled with water, the total normal pressure as found by the formula (15) is

$$P = w \cdot \pi d^2 \cdot \tfrac{1}{2} d = \tfrac{1}{2} w \pi d^3$$

but the resultant pressure is nothing, for the elementary normal pressures act in all directions so that no tendency to motion exists. The weight of water in this sphere is $\frac{1}{6} w \pi d^3$, or one-third of the total normal pressure, and the direction of this is vertical.

Prob. 15. An ellipse, with major and minor axes equal to 12 and 8 feet, is submerged so that one extremity of the major axis is 3.5 and the other 8.5 feet below the water surface. Find the normal pressure on one side.

Art. 16. Pressure in a Given Direction

The pressure against an immersed plane surface in a given direction may be found by obtaining the normal pressure by Art. 15 and computing its component in the required direction, or by means of the following theorem:

The horizontal pressure on any plane surface is equal to the normal pressure on its vertical projection; the vertical pressure is equal to the normal pressure on its horizontal projection; and the pressure in any direction is equal to the normal pressure on a projection perpendicular to that direction.

To prove this let A be the area of the given surface, represented by AA in Fig. 16a, and P the normal pressure upon it, or $P = wAh$. Now let it be required to find the pressure P' in a direction making an angle θ with the normal to the given plane. Draw $A'A'$ perpendicular to the direction of P', and let A' be the area of the projection of A upon it. The value of P' then is

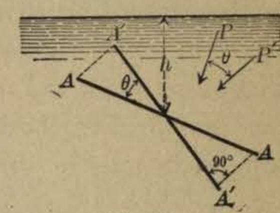

Fig. 16a.

$$P' = P \cos\theta = wAh \cos\theta$$

But $A \cos\theta$ is the value of A' by the construction. Hence

$$P' = wA'h \qquad (16)$$

and the theorem is thus demonstrated.

This theorem does not in general apply to curved surfaces. But in cases where the head of water is so great that the pressure may be regarded as uniform it is also true for curved surfaces. For instance, consider a cylinder or sphere subjected on every elementary area to the unit-pressure p due to the high head h, and let it be required to find the pressure in the direction shown by q_1, q_2, and q_3 in Fig. 16b. The pressures p_1, p_2, p_3, etc., on the elementary areas a_1, a_2, a_3, etc., have the values

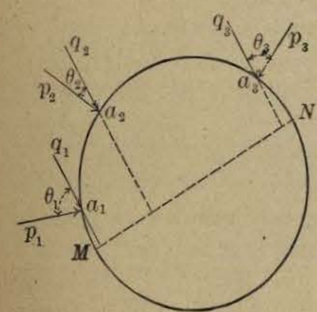

Fig. 16b.

$$p_1 = pa_1, \qquad p_2 = pa_2, \qquad p_3 = pa_3, \text{ etc.,}$$

and the components of these in the given direction are

$$q_1 = pa_1 \cos\theta_1, \qquad q_2 = pa_2 \cos\theta_2, \qquad q_3 = pa_3 \cos\theta_3, \text{ etc.,}$$

whence the total pressure P' in the given direction is

$$P' = p(a_1 \cos\theta_1 + a_2 \cos\theta_2 + a_3 \cos\theta_3 + \text{etc.})$$

But the quantity in the parenthesis is the projection of the given surface upon a plane perpendicular to the given direction, or MN. Hence there results

$$P' = p \times \text{area}\, MN$$

which is the same rule as for plane surfaces.

For the case of a water pipe let p be the interior pressure per square inch, t its thickness, and d its diameter in inches. Then for a length of one inch the force tending to rupture the pipe longitudinally is pd. The tensile unit-stress S in the walls of the pipe acting over the area $2t$ constitutes the resisting force $2tS$. Since these forces are equal, it follows that $2St = pd$ is the fundamental equation for the discussion of the strength of water pipes under static water pressure. For example, when the tensile strength of cast iron is 20 000 pounds per square inch, the unit-pressure p required to burst a pipe 24 inches in diameter and 0.75 inches thick is 1250 pounds per square inch, which corresponds to a head of 2880 feet.

Prob. 16. A circular plate 5 feet in diameter is immersed so that the head on its center is 18 feet, its plane making an angle of 30° with the vertical. Compute the horizontal and vertical pressures upon one side of it.

Art. 17. Center of Pressure on Rectangles

The center of pressure on a surface immersed in water is the point of application of the resultant of all the normal pressures upon it. The simplest case is the following:

> When a rectangle is placed with one end in the water surface, the center of pressure is distant from that end two-thirds of the length of the rectangle.

This theorem will be proved by the help of the graphical illustration shown in Fig. 17*a*. The rectangle, which in practice might be a board, is placed with its breadth perpendicular to the plane of the drawing, so that AB represents its edge. It is required to find the center of pressure C. For any head h the unit-

pressure is wh (Art. 15), and hence the unit-pressures on one side of AB may be graphically represented by arrows which form a triangle. Now when a force P equal to the total pressure is applied on the other side of the rectangle to balance these unit-pressures, it must be placed opposite to the center of gravity of the triangle. Therefore AC equals two-thirds of AB, and the rule is proved. The head on C is evidently also two-thirds of the head on B.

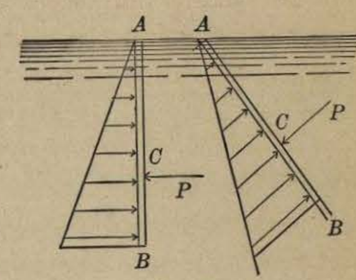

Fig. 17*a*.

Another case is that shown in Fig. 17*b*, where the rectangle, whose length is B_1B_2, is wholly immersed, the head on B_1 being h_1, and on B_2 being h_2. Let $AB_1 = b_1$, $AC = y$, and $AB_2 = b_2$. Now the normal pressure P_1, on AB_1 is applied at the distance $\frac{2}{3}\,b_1$ from A, and the normal pressure P_2 on AB_2 is applied at the distance $\frac{2}{3}\,b_2$ from A. The normal pressure P on B_1B_2 is the difference of P_1 and P_2, or $P = P_2 - P_1$. Also by taking moments about A as an axis,

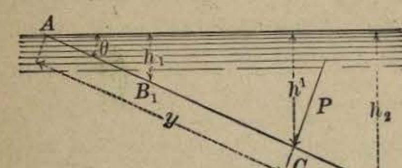

Fig. 17*b*.

$$P \times y = P_2 \times \tfrac{2}{3}\,b_2 - P_1 \times \tfrac{2}{3}\,b_1$$

Now, by Art. 15, the normal pressures P_2 and P_1 for a rectangle one unit in breadth are $P_2 = \frac{1}{2}\,wb_2h_2$ and $P_1 = \frac{1}{2}\,wb_1h_1$, whence the total normal pressure is $P = \frac{1}{2}\,w(b_2h_2 - b_1h_1)$, and accordingly the center of pressure is given by

$$y = \tfrac{2}{3} \cdot \frac{b_2{}^2h_2 - b_1{}^2h_1}{b_2h_2 - b_1h_1}$$

When θ is the angle of inclination of the plane to the water surface, the values of h_2 and h_1 are $b_2 \sin\theta$ and $b_1 \sin\theta$. Accordingly the expression becomes

$$y = \tfrac{2}{3} \cdot \frac{b_2{}^3 - b_1{}^3}{b_2{}^2 - b_1{}^2} \qquad (17)_1$$

Again, if h' is the head on the center of pressure, $y = h' \operatorname{cosec}\theta$, $b_2 = h_2 \operatorname{cosec}\theta$, and $b_1 = h_1 \operatorname{cosec}\theta$. These inserted in the last equation give

$$h' = \tfrac{2}{3} \cdot \frac{h_2^3 - h_1^3}{h_2^2 - h_1^2} \qquad (17)_2$$

These formulas are very convenient for computation, since the squares and cubes may be taken from tables.

If h_1 equals h_2, the above formula becomes indeterminate, which is due to the existence of the common factor $h_2 - h_1$ in both numerator and denominator of the fraction; dividing out this common factor, it becomes

$$h' = \tfrac{2}{3} \cdot \frac{h_2^2 + h_2h_1 + h_1^2}{h_2 + h_1} = \tfrac{2}{3}\left(h_2 + h_1 - \frac{h_2h_1}{h_2 + h_1}\right)$$

from which, if $h_2 = h_1 = h$, there is found the result $h' = h$.

Prob. 17. In Fig. 17*a* let the length of AB be 8.5 feet and its inclination to the vertical be 45 degrees. Find the depth of the center of pressure.

Art. 18. General Rule for Center of Pressure

For any plane surface immersed in a liquid, the center of pressure may be found by the following rule:

> Find the moment of inertia of the surface and its statical moment, both with reference to an axis situated at the intersection of the plane of the surface with the water level. Divide the former by the latter and the quotient is the perpendicular distance from that axis to the center of pressure.

The demonstration is analogous to that in the last article. Let B_1B_2 in Fig. 17*b* be the trace of the plane surface, which itself is perpendicular to the plane of the drawing, and C be the center of pressure, at a distance y from A where the plane of the surface intersects the water level. Let a_1, a_2, a_3, etc., be elementary areas of the surface, and h_1, h_2, h_3, etc., the heads upon them, which produce the normal elementary pressures, wa_1h_1, wa_2h_2, wa_3h_3, etc. Let y_1, y_2, y_3, etc., be the distances from A to these elementary areas. Then taking the point A as a center of moments, the definition of center of pressure gives the equation

$$(wa_1h_1 + wa_2h_2 + wa_3h_3 + \text{etc.})\, y = wa_1h_1y_1 + wa_2h_2y_2 + wa_3h_3y_3 + \text{etc.}$$

Now let θ be the angle of inclination of the surface to the water level; then $h_1 = y_1 \sin\theta$, $h_2 = y_2 \sin\theta$, $h_3 = y_3 \sin\theta$, etc. Hence, inserting these values, the expression for y is

$$y = \frac{a_1y_1^2 + a_2y_2^2 + a_3y_3^2 + \text{etc.}}{a_1y_1 + a_2y_2 + a_3y_3 + \text{etc.}}$$

The numerator of this fraction is the sum of the products obtained by multiplying each element of the surface by the square of its distance from the axis, which is called the moment of inertia of the surface. The denominator is the sum of the products obtained by multiplying each element of the surface by its distance from the axis, which is called the statical moment of the surface. Therefore

$$y = \frac{\text{moment of inertia}}{\text{statical moment}} = \frac{I'}{S} \qquad (18)$$

is the general rule for finding the position of the center of pressure of an immersed plane surface.

The statical moment of a surface is simply its area multiplied by the distance of its center of gravity from the given axis. The moments of inertia of plane surfaces with reference to an axis through the center of gravity are deduced in works on theoretical mechanics; the following are a few values, the axis being parallel to the base of the rectangle or triangle:

for a rectangle of base b and depth d,	$I = \frac{1}{12} bd^3$
for a triangle of base b and altitude d,	$I = \frac{1}{36} bd^3$
for a circle with diameter d,	$I = \frac{1}{64} \pi d^4$

To find from these the moment of inertia with reference to a parallel axis, the well-known formula $I' = I + Ak^2$ is to be used, where A is the area of the surface, k the distance from the given axis to the center of gravity of the surface, and I' the moment of inertia required.

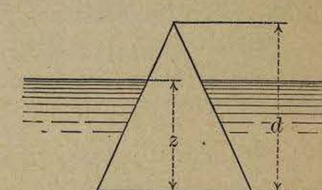

Fig. 18.

For example, let it be required to find the center of pressure of a vertical circle immersed so that the head on its center is equal to its radius. The area of the circle is $\frac{1}{4} \pi d^2$, and its statical moment with reference to the upper edge is $\frac{1}{4}\pi d^2 \times \frac{1}{2}d$. Then from (18)

$$y = \frac{\frac{1}{64}\pi d^4 + \frac{1}{4}\pi d^2 \cdot \frac{1}{4}d^2}{\frac{1}{4}\pi d^2 \cdot \frac{1}{2}d} = \frac{5}{8}d$$

or the center of pressure is at a distance $\frac{1}{8}d$ below the center of the circle.

Prob. 18. Find the depth of flotation for the triangle in Fig. 18. Also find the position of the center of pressure upon it in terms of z.

Art. 19. Pressures on Gates and Dams

In the case of an immersed plane the water presses equally upon both sides so that no disturbance of the equilibrium results from the pressure. But in case the water is at different levels on opposite sides of the surface the opposing pressures are unequal. For example, the cross-section of a self-acting tide-gate, built to drain a salt marsh, is shown in Fig. 19*a*. On the ocean side there is a head of h_1 above the sill, which gives for every linear foot of the gate the horizontal pressure

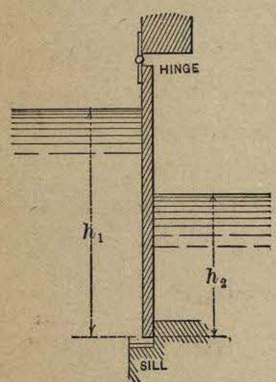

Fig. 19*a*.

$$P_1 = w \times h_1 \times \tfrac{1}{2}h_1 = \tfrac{1}{2}wh_1^2$$

which is applied at the distance $\frac{1}{3}h_1$ above the sill. On the other side the head on the sill is h_2, which gives the horizontal pressure $P_2 = \frac{1}{2}wh_2^2$ acting in the opposite direction to that of P_1. The resultant horizontal pressure is

$$P = P_1 - P_2 = \tfrac{1}{2}w(h_1^2 - h_2^2)$$

and if z be the distance of the point of application of P above the sill, the equation of moments is

$$Pz = P_1 \times \tfrac{1}{3}h_1 - P_2 \times \tfrac{1}{3}h_2$$

from which z can be computed. For example, if h_1 is 7 feet and h_2 is 4 feet, the resultant pressure on one linear foot of the gate is found to be 1031 pounds and its point of application to be 2.82 feet above the sill. The action of this gate in resisting the water pressure is like that of a beam under its load, the two points of

support being at the sill and the hinge. If h is the height of the gate, the reaction at the hinge is Pz/h, and from the above expression for Pz it is seen that this reaction has its greatest value when h_1 becomes equal to h and h_2 is zero. In the case of the vertical gate of a canal lock, which swings horizontally like a door, a similar problem arises and a similar conclusion results.

When the water level behind a masonry dam is lower than its top, as in Fig. 19*b*, the water pressure on the back is normal to the plane AB and for computations this may be resolved into

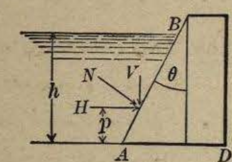
Fig. 19*b*.

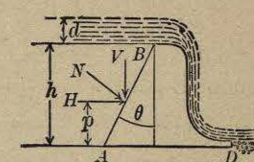
Fig. 19*c*.

horizontal and vertical components. Let h be the height of water above the base, θ the angle which the back makes with the vertical, then from Arts. 15–16 the values of these pressures, for one linear unit of the dam, are

$$\text{Normal Pressure } N = w \cdot h\sec\theta \cdot \tfrac{1}{2}h = \tfrac{1}{2}wh^2\sec\theta$$

$$\text{Horizontal Component } H = N\cos\theta = \tfrac{1}{2}wh^2$$

$$\text{Vertical Component } V = N\sin\theta = \tfrac{1}{2}wh^2\tan\theta$$

and from Art. 17 the point of application of these pressures is at a distance $\frac{1}{3}h$ above the base. Except in the case of hollow dams only the horizontal component H need usually be considered, since the neglect of V is on the side of safety.

When the water runs over the top of a dam, as in Fig. 19*c*, let h be the height of the dam and d the depth of water on its crest. Then

$$\text{Normal Pressure } N = w \cdot h\sec\theta \cdot (d + \tfrac{1}{2}h) = \tfrac{1}{2}wh(h+2d)\sec\theta$$

$$\text{Horizontal Component } H = N\cos\theta = \tfrac{1}{2}wh(h+2d)$$

$$\text{Vertical Component } V = N\sin\theta = \tfrac{1}{2}wh(h+2d)\tan\theta$$

and, from Art. 17, the point of application above the base BD is

$$p = \frac{h+3d}{h+2d} \cdot \tfrac{1}{3}h$$

when $d = 0$, these expressions for H and p become $\frac{1}{2}wh^2$ and $\frac{1}{3}d$. If d is infinite, the value of p reduces to $\frac{1}{2}h$ and hence in no case can the pressure N be applied as high as the middle of the height of the dam. Unless the dam be hollow or θ be greater than $30°$ it will usually be proper to neglect V and to consider only H.

It is not the place here to enter into the discussion of the subject of the design of masonry dams, but two ways in which they are liable to fail may be noted. The first is that of sliding along a horizontal joint, as BD; here the horizontal component of the thrust overcomes the resisting force of friction acting along the joint. If W is the weight of masonry above the joint, and f the coefficient of friction, the resisting friction is fW, and the dam will slide if the horizontal component of the pressure is equal to or greater than this. The condition for failure by sliding then is $H = fW$. For example, consider a masonry dam of rectangular cross-section which is 4 feet wide and h feet high, the water being level with its top. Let its weight per cubic foot be 140 pounds, and let it be required to find the height h for which it would fail by sliding along the base, the coefficient of friction being 0.70. The horizontal water pressure is $\frac{1}{2} \times 62.5 \times h^2$ and the resisting friction is $0.7 \times 140 \times 4 \times h$. Placing these equal, there is found for the height of the dam $h = 12.5$ feet.

The second method of failure of a masonry dam is by overturning, or by rotating about the toe D. This occurs when the moment of H equals the moment of W with respect to D, or if p and q are the lever arms dropped from D upon the directions of H and W, the condition for failure by rotation is $Hp = Wq$. For example, when it is required to find the height of the above rectangular dam so that it will fail by rotation, the lever arms p and q are $\frac{1}{3}h$ and 2 feet, and the equation of moments with respect to the toe of the dam is

$$\tfrac{1}{2} \times 62.5 \times h^2 \times \tfrac{1}{3}h = 140 \times 4 \times h \times 2$$

from which there is found $h = 10.4$ feet. The horizontal water pressure for one linear foot of the dam at the instant of failure is $\frac{1}{2}wh^2 = 3380$ pounds.

In the case of an overfall dam, as in Fig. 19c, the falling sheet of water produces a partial vacuum when air cannot freely enter behind it, and thus the force H, tending to produce sliding, is increased. In the design of a dam consideration must also be given to the upward pressure of that water which gains access either beneath its foundation

or directly into its mass. This upward pressure is equivalent to a loss of weight due to percolating water, as was described in Art. 12.

Prob. 19. A water pipe passing through a masonry dam is closed by a cast-iron circular valve AB, which is hinged at A, and which can be raised by a vertical chain BC. The diameter of the valve is 3 feet, its plane makes an angle of $27°$ with the vertical, and the depth of its center below the water level is 10.5 feet. Compute the normal water pressure P, and the distance of the center of pressure from the hinge A. Disregarding the weight of the valve and chain, compute the force F required to open the valve. When the weight of the chain is 23 pounds and that of the valve 180 pounds, compute the force F.

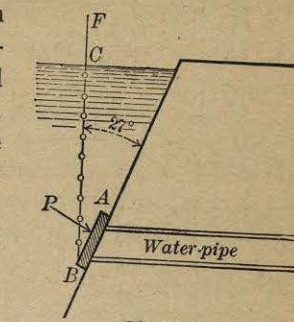

Fig. 19d.

Art. 20. Hydrostatics in Metric Measures

(Art. 11) When the head h is in meters and the unit-pressure p is in kilograms per square meter, the formulas $(11)_1$ become

$$p = 1000h \qquad h = 0.001p$$

In engineering practice p is usually taken in kilograms per square centimeter, while h is expressed in meters. Then

$$p = 0.1h \qquad h = 10p \tag{20}$$

Stated in words these practical rules are:

1 meter head corresponds to 0.1 kilogram per square centimeter
1 kilogram per square centimeter corresponds to 10 meters head

These values depend upon the assumption that 1000 kilograms is the weight of a cubic meter of water, and hence results derived from them are liable to an uncertainty in the third or fourth significant figure, as Table 20 shows.

The atmospheric pressure of 1.033 kilograms per square centimeter is to be added to the pressure due to the head whenever it is necessary to regard the absolute pressure. For example, if the air is exhausted from a small globe so that its pressure is only 0.32 kilogram per square centimeter and it be submerged in water to a depth of 86 meters, the absolute pressure per square centimeter on the globe is $0.1 \times 86 + 1.033 = 9.633$ kilograms, and the resultant effective pressure per square centimeter is $9.633 - 0.32 = 9.313$ kilograms.

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TABLE 20. HEADS AND PRESSURES

Metric Measures

Head in Meters	Pressure in Kilograms per Square Centimeter		Pressure in Kilograms per Square Centimeter	Head in Meters	
	$w = 1000$	$w = 997$		$w = 1000$	$w = 997$
1	0.1	0.0997	1	10	10.03
2	0.2	0.1994	2	20	20.06
3	0.3	0.2991	3	30	30.09
4	0.4	0.3988	4	40	40.12
5	0.5	0.4985	5	50	50.15
6	0.6	0.5982	6	60	60.18
7	0.7	0.6979	7	70	70.21
8	0.8	0.7976	8	80	80.24
9	0.9	0.8973	9	90	90.27
10	1.0	0.9970	10	100	100.30

(Art. 12) The specific gravity of a substance is expressed by the same number as the weight of a cubic centimeter in grams, or the weight of a cubic decimeter in kilograms, or the weight of a cubic meter in metric tons. Thus, if the specific gravity of stone is 2.4, a cubic meter weighs 2.4 metric tons or 2400 kilograms. A bar one square centimeter in cross-section and one meter long contains 100 cubic centimeters; hence if such a bar be of steel having a specific gravity of 7.9, it weighs 790 grams or 0.79 kilogram in air, while in water it weighs 690 grams or 0.69 kilogram.

(Art. 15) Here h is to be taken in meters, A in square meters, and w as 1000 kilograms per cubic meter; then P will be in kilograms.

(Art. 16) For a water pipe let p be the interior pressure in kilograms per square centimeter and d its diameter in centimeters. Then for a length of one centimeter the force tending to rupture the pipe longitudinally is pd. Let S be the stress in kilograms per square centimeter in the walls of the pipe; this acts over the area $2t$, if t be the thickness. As these forces are equal, the equation $2St = pd$ is to be used for the investigation of water pipes. For example, let it be required to find what head will burst a cast-iron pipe 60 centimeters in diameter and 2 centimeters thick; the tensile strength of the material being 1400 kilograms per square centimeter. Using the equation, the value of p is found to be 93.3 kilograms per square centimeter and then, from Art. 9, the required head h is 933 meters.

(Art. 19) Consider a rectangular masonry dam which weighs 2400 kilograms per cubic meter and which is 1.4 meters thick. First, let it be required to find the height of water for which it would fail by sliding, the coefficient of friction being 0.75. The horizontal water-pressure is $\frac{1}{2} \times 1000 \times h^2$, and the resisting friction is $0.75 \times 2400 \times 1.4 \times h$; placing these equal, there is found $h = 5.04$ meters. Secondly, to find the height for which failure will occur by rotation, the equation of moments is

$$\tfrac{1}{2} \times 1000 \times h^2 \times \tfrac{1}{3} h = 2400 \times 1.4 \times h \times 0.75$$

from which there is found $h = 3.89$ meters. The horizontal water-pressure for one linear meter of this dam is $\frac{1}{2} wh^2 = 7560$ kilograms.

Prob. 20*a*. In a hydrostatic press one-half of a metric horse-power is applied to the small piston. The diameter of the large piston is 30 centimeters and it moves 2 centimeters per minute. Compute the pressure in the liquid.

Prob. 20*b*. What is the specific gravity of dry hydraulic cement of which 20.6 cubic centimeters weigh 63.2 grams? If a cube of stone 12.4 centimeters on each edge weighs 4.88 kilograms, what is its specific gravity?

Prob. 20*c*. In Fig. 19*a* let the head on one side of the gate be 2.5 and on the other side 0.6 meters above the sill. Find the resultant pressure for one linear meter of the gate and the distance of its point of application above the sill.