in a pipe 38 centimeters in diameter and 6 meters long, Table F gives 0.1134 square meter for the sectional area, the volume is then 0.6804 cubic meter, and the weight is 680 kilograms, the fourth figure being omitted because nothing is known about the temperature or purity of the water. In general, hydraulic computations are much easier in the metric than in the English system.

Prob. 9*a*. Compute the acceleration of gravity at Quito, Ecuador, which is in latitude $-0^\circ 13'$ and at an elevation of 2850 meters above sea level.

Prob. 9*b*. What is the pressure in kilograms per square centimeter at the base of a column of water 95.4 meters high?

Prob. 9*c*. Compute the velocity of sound in fresh distilled water at the temperature of 12° centigrade, and also its mean velocity in salt water.

Prob. 9*d*. How many cubic meters of water are contained in a pipe 315 meters long and 15 centimeters in diameter? How many kilograms? How many metric tons?

Prob. 9*e*. What is the boiling-point of water when the mercury barometer reads 735 millimeters? How high will water rise in a vacuum tube at a place where the boiling-point of water is 92° centigrade?

CHAPTER 2

HYDROSTATICS

Art. 10. Transmission of Pressure

One of the most remarkable properties of a fluid is its capacity of transmitting a pressure, applied at one point of the surface of a closed vessel, unchanged in intensity, in all directions, so that the effect of the applied pressure is to cause an equal force per square inch upon all parts of the enclosing surface. Pascal, in 1646, was the first to note that great forces could be produced in this manner; he saw that the total pressure increased proportionally with the area of the surface. Taking a closed barrel filled with water, he inserted a small vertical tube of considerable length tightly into it, and on filling the tube the barrel burst under the great pressure thus produced on its sides, although the weight of the water in the tube was quite small. The first diagram in Fig. 10*a* represents Pascal's barrel, and it is seen that the unit-pressure in the water at B is due to the head AB and independent of the size of the tube AC.

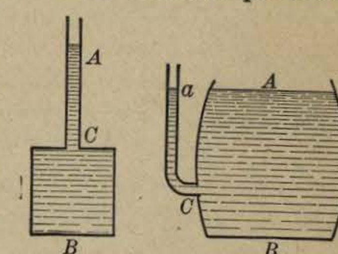

Fig. 10*a*.

Pascal clearly saw that this property of water could be employed in a useful manner in mechanics, but it was not until 1796 that Bramah built the first successful hydraulic press. This machine has two pistons of different sizes, and a force applied to the small piston is transmitted through the fluid and produces an equal unit-pressure at every point on the large piston. The applied force is here multiplied to any required extent, but the work performed by the large piston cannot exceed that imparted to the fluid by the small one. Let a and A be the areas of the small and large pistons, and p the pressure in pounds per square unit applied to a; then the unit-pressure in the fluid is p, and the total pressure on the small piston is pa, while that on the large piston is pA. Let the distances through which the pistons move during one stroke be d and D. Then the imparted work is pad, and the performed work, neglecting frictional resistances, is pAD. Consequently $ad = AD$, and since a is small as compared with A, the distance D must be small compared with d. Here is found an illustration of the popular maxim "What is lost in velocity is gained in force."

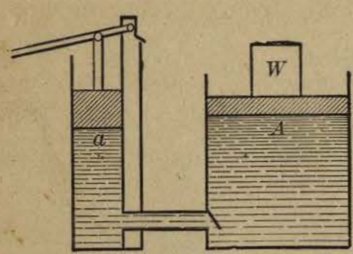

Fig. 10*b*.

Numerous applications of this principle are made in hydraulic presses for compressing materials and forging steel, as also in jacks, accumulators, and hydraulic cranes. The Keely motor, one of the delusions of the nineteenth century, is said to have employed this principle to produce some of its effects; very small pipes, supposed by the spectators to be wires conveying some mysterious force, being used to transmit the pressure of water to a receiver where the total pressure became very great in consequence of greater area.

In consequence of its fluidity the pressure existing at any point in a body of water is exerted in all directions with equal intensity. When water is confined by a bounding surface, as in a vessel, its pressure against that surface must be normal at every point, for if it were inclined, the water would move along the surface. When water has a free surface, the unit-pressure at any depth depends only on that depth and not on the shape of the vessel. Thus in the second diagram of Fig. 10*a* the unit-pressure at C produced by the smaller column of water aC is the same as that caused by the larger column AC, and the total vertical pressure on the upper side of the base B is the product of its area into the unit-pressure caused by the depth AB.

Prob. 10. What is the upward pressure on the lower side of the base B in Fig. 10*a*? Explain why this is less than the downward pressure on the upper side of the base B.

Art. 11. Head and Pressure.

The free surface of water at rest is perpendicular to the direction of the force of gravity, and for bodies of water of small extent this surface may be regarded as a plane. Any depth below this plane is called a "head," or the head upon any point is its vertical depth below the level surface. In Art. 10 it was seen that the unit-pressure at any depth depends only on the head and not on the shape of the vessel. Let h be the head and w the weight of a cubic unit of water; then at the depth h one horizontal square unit bears a pressure equal to the weight of a column of water whose height is h, and whose cross-section is one square unit, or wh. But the pressure at this point is exerted in all directions with equal intensity. The unit-pressure p at the depth h then is wh, and the depth, or head, for a unit-pressure p is p/w, or

$$p = wh \qquad h = p/w \qquad (11)_1$$

If h be expressed in feet and p in pounds per square foot, these formulas become, using the mean value of w,

$$p = 62.5h \qquad h = 0.016p$$

Thus pressure and head are mutually convertible, and in fact one is often used as synonymous with the other, although really each is proportional to the other. Any unit-pressure p can be regarded as produced by a head h, which is frequently called the "pressure head."

In engineering work p is usually taken in pounds per square inch, while h is expressed in feet. Thus the pressure in pounds per square foot is $62.5h$, and the pressure in pounds per square inch is $\frac{1}{144}$ of this, or

$$p = 0.4340h \qquad h = 2.304p \qquad (11)_2$$

These rules may be stated in words as follows:

1 foot head corresponds to 0.434 pounds per square inch;
1 pound per square inch corresponds to 2.304 feet head.

These values, be it remembered, depend upon the assumption that 62.5 pounds is the weight of a cubic foot of water, and hence are liable to variation in the third significant figure (Art. 4). The extent of these variations for fresh water may be seen in Table 11, which gives multiples of the above values, and also the corresponding quantities when the cubic foot is taken as 62.3 pounds.

TABLE 11. HEADS AND PRESSURES

Head in Feet	Pressure in Pounds per Square Inch		Pressure in Pounds per Square Inch	Head in Feet	
	$w = 62.5$	$w = 62.3$		$w = 62.5$	$w = 62.3$
1	0.434	0.433	1	2.304	2.311
2	0.868	0.865	2	4.608	4.623
3	1.302	1.298	3	6.912	6.934
4	1.736	1.731	4	9.216	9.246
5	2.170	2.163	5	11.520	11.557
6	2.604	2.596	6	13.824	13.868
7	3.038	3.028	7	16.128	16.180
8	3.472	3.461	8	18.432	18.491
9	3.906	3.894	9	20.736	20.803
10	4.340	4.326	10	23.040	23.114

The atmospheric pressure, which is about 14.7 pounds per square inch, is transmitted through water, and is to be added to the pressure due to the head whenever it is necessary to regard the absolute pressure. This is important in some investigations on the pumping of water, and in a few other cases where a partial or complete vacuum is produced on one side of a body of water. For example, if the air is exhausted from a small globe, so that its tension is only 6.5 pounds per square inch, and it is submerged in water to a depth of 250 feet, then the absolute pressure on the surface of the globe is

$$p = 0.434 \times 250 + 14.7 = 123.2 \text{ pounds per square inch,}$$

and the resultant effective pressure on that surface is

$$p' = 123.2 - 6.5 = 116.7 \text{ pounds per square inch.}$$

Unless otherwise stated, however, the atmospheric pressure need not be regarded, since under ordinary conditions it acts with equal intensity upon both sides of a submerged surface.

Prob. 11. How many pounds per square inch correspond to a head of 230 feet? How many feet head correspond to a pressure of 100 pounds per square inch?

ART. 12. LOSS OF WEIGHT IN WATER

It is a familiar fact that bodies submerged in water lose part of their weight; a man can carry under water a large stone which would be difficult to lift in air, and timber when submerged has a negative weight or tends to rise to the surface. The following is the law of loss which was discovered by Archimedes, about 250 B.C., when considering the problem of King Hiero's crown:

> The weight of a body submerged in water is less than its weight in air by the weight of a volume of water which is equal to the volume of the body.

To demonstrate this, consider that the submerged body is acted upon by the water pressure in all directions, and that the horizontal components of these pressures must balance. Any vertical elementary prism is subjected to an upward pressure upon its base which is greater than the downward pressure upon its top, since these pressures are due to the heads. Let h_1 be the head on the top of the elementary prism and h_2 that on its base, and a the cross-section of the prism; then the downward press re is wah_1 and the upward pressure is wah_2. The difference of these, $wa(h_2-h_1)$ is the resultant upward water pressure, and this is equal to the weight of a column of water whose cross-section is a and whose height is that of the elementary prism. Extending this theorem to all the elementary prisms, it is concluded that the weight of the body in water is less than its weight in air by the weight of an equal volume of water.

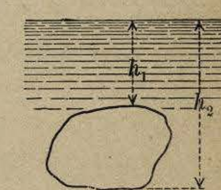

Fig. 12.

It is important to regard this loss of weight in constructions under water. If, for example, a dam of loose stones allows the water to percolate through it, its weight per cubic foot is less than its weight in air, so that it can be more easily moved by horizontal forces. As stone weighs about 150 pounds per cubic foot in air, its weight in water is only about $150 - 62 = 88$ pounds per cubic foot. If a cubic foot of sand, having voids amounting to 40 per cent of its volume, weighs 110 pounds, its loss of weight in water is $0.60 \times 62.5 = 37.5$ pounds, so that its weight in water is $110 - 37.5 = 72.5$ pounds.

The ratio of the weight of a substance to that of an equal volume of water is called the specific gravity of the substance, and this is easily computed from the law of Archimedes after weighing a piece of it in air and then in water; or, if w be the weight of a cubic unit of water and w' the weight of a cubic unit of any substance, the ratio w'/w is the specific gravity of the substance.

Prob. 12. A box containing 1.17 cubic feet weighs 19.3 pounds when empty and 133.5 when filled with sand. It is then found that 29.7 pounds of water can be poured in before overflow occurs. Find the percentage of voids in the sand, the specific gravity of the sand mass, and the specific gravity of a grain of sand.

Art. 13. Depth of Flotation

When a body floats upon water, it is sustained by an upward pressure of the water equal to its own weight, and this pressure is the same as the weight of the volume of water displaced by the body. Let W' be the weight of the floating body in air, and W be the weight of the displaced water; then $W' = W$. Now let z be the depth of flotation of the body; then to find its value for any particular case W' is to be expressed in terms of the linear dimensions of the body, and W in terms of the depth of flotation z. For example, a timber box caisson is $20 \times 10\frac{1}{2}$ feet in outside dimensions and weighs 33 400 pounds. The weight of displaced water in pounds is $62\frac{1}{2} \times 20 \times 10\frac{1}{2} \times z$, and equating this to 33 400 gives $z = 2.54$ feet for the depth of flotation.

To find the depth of flotation for a cylinder lying horizontally, let w' be its weight per cubic unit, l its length, and r the radius of its cross-section. The depth of flotation is DE, or letting θ be the angle ACE, then $z = (1 - \cos\theta)r$. The weight of the cylinder is $W' = \pi r^2 l \cdot w'$, and that of the displaced water is

$$W = (r^2 \operatorname{arc}\theta - r^2 \sin\theta \cos\theta)l \cdot w$$

Equating the values of W and W', and substituting for $\sin\theta \cos\theta$ its equivalent $\frac{1}{2}\sin 2\theta$, there results

$$2 \operatorname{arc}\theta - \sin 2\theta = 2\pi s$$

in which s represents the ratio w'/w or the specific gravity of the material of the cylinder. From this equation θ is to be found by trial for any particular case, and then z is computed. For example, if $w' = 26.5$ pounds per cubic foot, then s is 0.424, and

$$2 \operatorname{arc}\theta - \sin 2\theta - 2.664 = 0$$

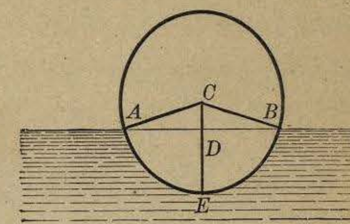

Fig. 13.

To solve this equation, values are to be assumed for θ, until one is found that satisfies it; thus from Table G,

for $\theta = 83°$ $\quad 2.897 - 0.242 - 2.664 = -0.009$

for $\theta = 83\frac{1}{4}$ $\quad 2.906 - 0.234 - 2.664 = +0.008$

Therefore θ lies between 83° and 83° 15′, and is probably about 83° 8′. Hence the depth of flotation is $z = (1 - 0.120)r = 0.88r$, or if the diameter is one foot, the depth of flotation is 0.44 feet.

In a similar way it may be shown that the depth of flotation of a sphere of radius r and specific gravity s is given by the cubic equation $z^3 - 3rz^2 + 4r^3 s = 0$. When $r = 4$ feet and $s = 0.65$, it may be found by trial that $z = 1.21$ feet.

Prob. 13. A wooden stick $1\frac{1}{4}$ inches square and 10 feet long is to be used for a velocity float which is to stand vertically in the water. How many square inches of sheet lead $\frac{1}{32}$ inch thick must be tacked on the sides of this stick so that only 4 inches will project above the water surface? The wood weighs 31.25 and the lead 710 pounds per cubic foot.

Art. 14. Stability of Flotation

The equilibrium of a floating body is stable when it returns to its primitive position after having been slightly moved therefrom by extraneous forces; it is indifferent when it floats in any position, and it is unstable when the slightest force causes it to leave its position of flotation. For instance, a short cylinder with its axis vertical floats in stable equilibrium, but a long cylinder in this position is unstable, and a slight force causes it to fall over and float with its axis horizontal in indifferent equilibrium. It is evident that the equilibrium is the more stable the lower the center of gravity of the body.

The stability depends in any case upon the relative position of the center of gravity of the body and its center of buoyancy, the latter being the center of gravity of the displaced water. Thus in Fig. 14 let G be the center of gravity of the body and let C be its center of buoyancy when in an upright position. Now if an extraneous force causes the body to tip into the position shown, the center of gravity remains at G, but the center of buoyancy moves to D. In this new position of the body it is acted upon by the forces W' and W, which are equal and parallel but opposite in direction. These forces form a couple which tends either to restore the body to the upright position or to cause it to deviate farther from that position. Let the vertical through D be produced to meet the center line CG in M. If M is above G, the equilibrium is stable, as the forces W and W' tend to restore it to its primitive position; if M coincides with G, the equilibrium is indifferent; and if M be below G, the equilibrium is unstable.

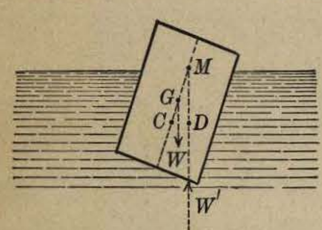

Fig. 14.

The point M is called the "metacenter," and the theorem may be stated that the equilibrium is stable, indifferent, or unstable according as the metacenter is above, coincident with, or below the center of gravity of the body. The measure of the stability of a stable floating body is the moment of the couple formed by the forces W and W'. But GM is proportional to the lever arm of the couple, and hence the quantity $W \times GM$ may be taken as a measure of stability. The stability, therefore, increases with the weight of the body, and with the distance of the metacenter above the center of gravity. (See Art. 189.)

The most important application of these principles is in the design of ships, and usually the problems are of a complex character which can only be solved by tentative methods. The rolling of the ship due to lateral wave action must also receive attention, and for this reason the center of gravity should not be put too low.

Prob. 14. A square prism of uniform specific gravity s has the length h and the cross-section b^2. When this prism is placed in water with its axis vertical, it may be shown that it is in stable, indifferent, or unstable equilibrium according as b^2 is greater, equal to, or less than $6\ h^2s\ (1 - s)$.

Art. 15. Normal Pressure

The total normal pressure on any immersed surface may be found by the following theorem:

> The total normal pressure is equal to the product of the weight of a cubic unit of water, the area of the surface, and the head on its center of gravity.

To prove this let A be the area of the surface, and imagine it to be composed of elementary areas, a_1, a_2, a_3, etc., each of which is so small that the unit-pressure over it may be taken as uniform; let h_1, h_2, h_3, etc., be the heads on these elementary areas, and let w denote the weight of a cubic unit of water. The unit-pressures at the depths h_1, h_2, h_3, etc., are wh_1, wh_2, wh_3, etc. (Art. 11), and hence the normal pressures on the elementary areas, a_1, a_2, a_3, etc., are wa_1h_1, wa_2h_2, wa_3h_3, etc. The total normal pressure P on the entire surface then is

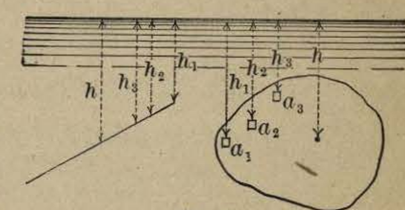

Fig. 15.

$$P = w(a_1h_1 + a_2h_2 + a_3h_3 + \text{etc.})$$

Now let h be the head on the center of gravity of the surface; then, from the definition of the center of gravity,

$$a_1h_1 + a_2h_2 + a_3h_3 + \text{etc.} = Ah$$

Therefore the normal pressure is

$$P = wAh \qquad (15)$$

which proves the theorem as stated.

This rule applies to all surfaces, whether plane, curved, or warped, and however they be situated with reference to the water surface. Thus the total normal pressure upon the surface of an immersed cylinder remains the same whatever be its position, provided the depth of the center of gravity of that surface be kept constant. It is best to take h in feet, A in square feet, and w as 62.5 pounds per cubic foot; then P will be in pounds. In