TREATISE ON HYDRAULICS

CHAPTER 1

FUNDAMENTAL DATA

ARTICLE 1. UNITS OF MEASURE

The unit of linear measure universally used in English and American hydraulic literature is the foot, which is defined as one-third of the standard yard. For some minor purposes, such as the designation of the diameters of orifices and pipes, the inch is employed, but inches should always be reduced to feet for use in hydraulic formulas. The unit of superficial measure is usually the square foot, except for the expression of the intensity of pressures, when the square inch is more commonly employed.

TABLE 1*a*. INCHES REDUCED TO FEET

Inches	Feet	Inches	Feet	Square Inches	Square Feet	Cubic Inches	Cubic Feet
1/8	0.0104	3	0.2500	10	0.6944	1000	0.5787
1/4	.0208	4	.3333	20	1.3889	2000	1.1574
3/8	.0313	5	.4167	30	2.0833	3000	1.7361
1/2	.0417	6	.5000	40	2.6777	4000	2.3148
5/8	.0521	7	.5833	50	3.4722	5000	2.8935
3/4	.0625	8	.6667	60	4.1667	6000	3.4722
7/8	.0729	9	.7500	70	4.5500	7000	4.0509
1	.0833	10	.8333	80	5.3555	8000	4.6296
2	.1667	11	.9167	90	6.2500	9000	5.2083

The units of volume employed in measuring water are the cubic foot and the gallon, but the latter must always be reduced to cubic feet for use in hydraulic formulas. In Great Britain and its colonies the Imperial gallon is used, but in the United States the old English gallon has continued to be employed, and the former is 20 percent larger than the latter. The following are the relations between the cubic foot and the two gallons:

1 cubic foot = 6.2321 Imp. gallons = 7.481 U. S. gallons
1 Imp. gallon = 0.1605 cubic feet = 1.200 U. S. gallons
1 U. S. gallon = 0.1337 cubic feet = 0.833 Imp. gallons

In this book the word "gallon" will always mean the United States gallon of 231 cubic inches, unless otherwise stated.

TABLE 1*b*. GALLONS AND CUBIC FEET

Cubic Feet	U. S. Gallons	U. S. Gallons	Cubic Feet	Cubic Feet	Imperial Gallons	Imperial Gallons	Cubic Feet
1	7.481	1	0.1337	1	6.232	1	0.16046
2	14.961	2	0.2674	2	12.464	2	0.3209
3	22.442	3	0.4010	3	18.696	3	0.4814
4	28.922	4	0.5347	4	24.928	4	0.6418
5	37.403	5	0.6684	5	32.160	5	0.8023
6	44.883	6	0.8021	6	37.393	6	0.9628
7	52.364	7	0.9358	7	43.625	7	1.1232
8	59.844	8	1.0695	8	49.857	8	1.2837
9	67.325	9	1.2031	9	56.089	9	1.4442
10	74.805	10	1.3368	10	62.321	10	1.6046

The unit of force is the pound, or the force exerted by gravity at the surface of the earth on a mass of matter called the avoirdupois pound. This unit is also used in measuring weights and pressures of water. The intensity of pressure is measured in pounds per square foot or in pounds per square inch, as may be most convenient, and sometimes in atmospheres. Gages for recording the pressure of water are usually graduated to read pounds per square inch.

The unit of time to be used in all hydraulic formulas is the second, although in numerical problems the time is often stated in minutes, hours, or days. Velocity or speed is defined as the space passed over by a body in one second, under the condition of uniform motion, so that velocities are to be always expressed in feet per second, or are to be reduced to these units if stated in

miles per hour or otherwise. Acceleration is the velocity gained in one second, and it is measured in feet per second per second.

The unit of work is the foot-pound; that is, one pound lifted through a vertical distance of one foot. Energy is work which can be done; for example, a moving body has the ability to do a certain amount of work by virtue of its quantity of matter and its velocity, and this is called kinetic energy. Again, water at the top of a fall has the ability to do a certain amount of work by virtue of its quantity and its height above the foot of the fall, and this is called potential energy. Potential energy changes into kinetic energy as the water drops, and kinetic energy is either changed into heat or may be transformed, by means of a water motor, into useful work. Power is work done, or energy capable of being transformed into work, in a specified time, and the unit for its measure is the horse-power, which is 550 foot-pounds per second.

In French and German literature the metric system of measures is employed, and this is far more convenient than the English one in hydraulic computations. This system is understood and more or less used in all countries, and its universal adoption will probably occur during the present century, but the time has not yet come when an American engineering book can be prepared wholly in metric measures. This treatise will, therefore, mainly use the English units described above, but at the close of most of the chapters hydraulic data, tables, and empirical formulas will be given in metric measures. At the end of the volume will be found tables giving fundamental hydraulic constants and equivalents in each system of the principal units in the other system.

Problem 1. When one cubic foot of water, weighing 62½ pounds, falls each second through a vertical height of 11 feet, what horse-power can be developed by a hydraulic motor which utilizes 80 percent of the energy?

ART. 2. PHYSICAL PROPERTIES OF WATER

At ordinary temperatures pure water is a colorless liquid which possesses almost perfect fluidity; that is, its particles have the capacity of moving over each other, so that the slightest disturbance of equilibrium causes a flow. It is a consequence of this property that the surface of still water is always level; also, if several vessels or tubes be connected, as in Fig. 2, and water be poured into one of them, it rises in the others until, when equilibrium ensues, the free surfaces are in the same level plane.

The free surface of water is in a different molecular condition from the other portions, its particles being drawn together by stronger attractive forces, so as to form what may be called the "skin of the water," upon which insects may walk or a needle be caused to float. The skin is not immediately pierced by a sharp point which moves slowly upward toward it, but a slight elevation occurs, and this property enables precise determinations of the level of still water to be made by the hook gage (Art. 35).

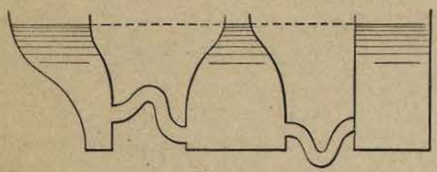

Fig. 2.

At about 32° Fahrenheit a great alteration in the molecular constitution of water occurs, and ice is formed. If a quantity of water be kept in a perfectly quiet condition, it is found that its temperature can be reduced to 20° or even to 15° Fahrenheit, before congelation takes place, but at the moment when this occurs the temperature rises to 32°. The freezing-point is hence not constant, but the melting-point of ice is always at the same temperature of 32° Fahrenheit or 0° centigrade.

While water freezes at 32° Fahrenheit, yet its maximum density is reached at 39°.3 Fahrenheit. At this latter temperature its specific gravity is 1.0 while at 32° it is 0.99987. As the temperature rises above that of maximum density the specific gravity of water steadily grows smaller until the boiling-point is reached at 212° Fahrenheit when its specific gravity is 0.95865. To the occurrence of the maximum density at a temperature above the freezing-point is to be attributed the fortunate circumstance that ponds and streams do not freeze solid from the bottom up.

Ice, as a rule, forms upon the surface of the water in a solid sheet. The rapidity with which such ice forms is dependent on the temperature and decreases with the thickness of the ice-sheet.

The coefficient of linear expansion of ice varies from 0.0000408 to 0.0000197 as the temperature varies from + 30° Fahrenheit to − 30° Fahrenheit.* Under certain conditions a rise in temperature may cause a considerable expansion, and if the sheet is a heavy one and expansion is prevented, the pressure brought to bear on any resisting surface becomes very great. A second variety of ice called frazil or slush ice is formed in rapidly flowing water when the temperature of the air is materially below the freezing-point. This ice is formed in the shape of small needles which are carried along and deposited in quiet water below. Accumulations of frazil to a depth of 80 feet have been known.* A third variety, known as anchor ice, may of itself be formed directly on the bed and sides of a rapidly flowing stream or be increased in volume by accretions of frazil. In cold countries the design of hydraulic structures must take into account all of these three kinds of ice.

Water is a solvent of high efficiency, and is therefore never found pure in nature. Descending in the form of rain, it absorbs dust and gaseous impurities from the atmosphere; flowing over the surface of the earth it absorbs organic and mineral substances. These affect its weight only slightly as long as it remains fresh, but when it has reached the sea and becomes salt, its weight is increased more than 2 percent. The flow of water through orifices is only in a very slight degree affected by the impurities held in solution, but in the flow through pipes they often cause incrustation or corrosion which increases the roughness of the surface and diminishes the velocity.

The capacity of water for heat, the latent heat evolved when it freezes, and that absorbed when it is transformed into steam need not be considered for the purposes of hydraulic investigations. Other physical properties, such as its variation in volume with the temperature, its compressibility, and its capacity for transmitting pressures, are discussed in the following pages. The laws which govern its pressure, flow, and energy under various circumstances belong to the science of Hydraulics and form the subject-matter of this volume.

Prob. 2. How many degrees centigrade are equivalent to − 40° Fahrenheit? How many degrees Fahrenheit are equivalent to − 40° centigrade and how many to + 40° centigrade?

* Barnes's Ice Formation (New York, 1906), pp. 106, 226.

Art. 3. The Weight of Water

The weight of water per unit of volume depends upon the temperature and upon its degree of purity. The following approximate values are, however, those generally employed except when great precision is required:

1 cubic foot of water weighs 62.5 pounds
1 U. S. gallon of water weighs 8.355 pounds

These values will be used in this book, unless otherwise stated, in the solution of the examples and problems.

The weight per unit of volume of pure distilled water is the greatest at the temperature of its maximum density, 39°.3 Fahrenheit, and least at the boiling-point. For ordinary computations the variation in weight due to temperature is not considered, but in tests of the efficiency of hydraulic motors and of pumps it should be regarded. The following table contains the weights of one cubic foot of pure water at different temperatures as deduced by Hamilton Smith from the experiments of Rosetti.*

Table 3. Weight of Distilled Water

Temperature Fahrenheit	Pounds per Cubic Foot	Temperature Fahrenheit	Pounds per Cubic Foot	Temperature Fahrenheit	Pounds per Cubic Foot
32°	62.42	95°	62.06	160°	61.01
35	62.42	100	62.00	165	60.90
39.3	62.424	105	61.93	170	60.80
45	62.42	110	61.86	175	60.69
50	62.41	115	61.79	180	60.59
55	62.39	120	61.72	185	60.48
60	62.37	125	61.64	190	60.36
65	62.34	130	61.55	195	60.25
70	62.30	135	61.47	200	60.14
75	62.26	140	61.39	205	60.02
80	62.22	145	61.30	210	59.89
85	62.17	150	61.20	212	59.84
90	62.12	155	61.11		

* Hamilton Smith, Jr., Hydraulics: The Flow of Water through Orifices, over Weirs, and through Open Conduits and Pipes (London and New York, 1886), p. 14.

Waters of rivers, springs, and lakes hold in suspension and solution inorganic matters which cause the weight per unit of volume to be slightly greater than for pure water. River waters are usually between 62.3 and 62.6 pounds per cubic foot, depending upon the amount of impurities and on the temperature, while the water of some mineral springs has been found to be as high as 62.7. It appears that, in the absence of specific information regarding a particular water, the weight 62.5 pounds per cubic foot is a fair approximate value to use. It also has the advantage of being a convenient number in computations, for 62.5 pounds is 1000 ounces, or $\frac{1000}{16}$ is the equivalent of 62.5.

Brackish and salt waters are always much heavier than fresh water. For the Gulf of Mexico the weight per cubic foot is about 63.9, for the oceans about 64.1, while for the Dead Sea there is stated the value 73 pounds per cubic foot. For Great Salt Lake the weight of water varies from 69 to 76 pounds per cubic foot.* The weight of ice per cubic foot varies from 57.2 to 57.5 pounds. The sewage of American cities is impure water which weighs from 62.4 to 62.7 pounds per cubic foot, but the sewage of European cities is somewhat heavier on account of the smaller amount of water that is turned into the sewers.

Prob. 3. How many gallons of water are contained in a pipe 3 inches in diameter and 12 feet long? How many pounds of water are contained in a pipe 6 inches in diameter and 12 feet long?

Art. 4. Atmospheric Pressure

Torricelli in 1643 discovered that the atmospheric pressure would cause mercury to rise in a tube from which the air had been exhausted. This instrument is called the mercury barometer, and owing to the great density of mercury the height of the column required to balance the atmospheric pressure is only about 30 inches. When water is used in the vacuum tube, the height of the column is about 34 feet. In both cases the weight of the barometric column is equal to the weight of a column of air of the same cross-section as that of the tube, both columns being measured upward from the common surface of contact.

* Science, Oct. 21, 1910.

The atmosphere exerts its pressure with varying intensity as indicated by the readings of the mercury barometer. At and near the sea level the average reading is 30 inches, and as mercury weighs 0.49 pounds per cubic inch at common temperatures, the average atmospheric pressure is taken to be 30 × 0.49 or 14.7 pounds per square inch. The pressure of one atmosphere is therefore defined to be a pressure of 14.7 pounds per square inch. Then a pressure of two atmospheres is 29.4 pounds per square inch. And conversely, a pressure of 100 pounds per square inch may be expressed as a pressure of 6.8 atmospheres.

Pascal in 1646 carried a mercury barometer to the top of a mountain and found that the height of the mercury column decreased as he ascended. It was thus definitely proved that the cause of the ascent of the liquid in the vacuum tube was due to the pressure of the air. Since mercury is 13.6 times heavier than water, a column of water should rise to a height of 30 × 13.6 = 408 inches = 34 feet under the pressure of one atmosphere, and this was also found to be the case. A water barometer is impracticable for use in measuring atmospheric pressures, but it is convenient to know its approximate height corresponding to a given height of the mercury barometer. Table 4 shows heights of the mercury and water barometers, with the corresponding pressures in pounds per square inch and in atmospheres. It also gives, in the fifth column, values from the vertical scale of altitudes used in barometric leveling which show approximate elevations above sea level corresponding to barometer readings, provided that the reading at sea level is 30 inches. In the last column are approximate boiling-points of water corresponding to the readings of the mercury barometer.

TABLE 4. ATMOSPHERIC PRESSURE

Mercury Barometer Inches	Pressure Pounds per Square Inch	Pressure Atmospheres	Water Barometer Feet	Elevations Feet	Boiling-point of Water Fahrenheit
31	15.2	1.03	35.1	− 890	213°.9
30	14.7	1.00	34.0	0	212 .2
29	14.2	0.97	32.9	+ 920	210 .4
28	13.7	0.93	31.7	1880	208 .7
27	13.2	0.90	30.6	2870	206 .9
26	12.7	0.86	29.5	3900	205 .0
25	12.2	0.83	28.3	4970	203 .1
24	11.7	0.80	27.2	6080	201 .1
23	11.3	0.76	26.1	7240	199 .0
22	10.8	0.72	24.9	8455	196 .9
21	10.3	0.69	23.8	9720	194 .7
20	9.8	0.67	22.7	11050	192 .4

The atmospheric pressure must be taken into account in many computations on the flow of water in tubes and pipes. It is this pressure that causes water to flow in syphons and to rise in tubes from which the air has been exhausted. By virtue of this pressure the suction pump is rendered possible, and all forms of injector pumps depend upon it to a certain degree. On a planet without an atmosphere many of the phenomena of hydraulics would be quite different from those observed on this earth.

Prob. 4. A mercury barometer reads 30.25 inches at the foot of a hill, and at the same time another barometer reads 28.56 inches at the top of the hill. What is the difference in height between the two stations?

ART. 5. COMPRESSIBILITY OF WATER

The popular opinion that water is incompressible is not justified by experiments, which show in fact that it is more compressible than iron or even timber within the elastic limit. These experiments indicate that the amount of compression is directly proportional to the applied pressure, and that water is perfectly elastic, recovering its original form on the removal of the pressure. The decrease in the unit of volume caused by a pressure of one atmosphere varies, according to the experiments of Grassi, from 0.000051 at 35° Fahrenheit to 0.000045 at 80° Fahrenheit.* As a mean 0.00005 may be taken for this cubical unit-compression.

A vertical column of water accordingly increases in density from the surface downward. If its weight at the surface be 62.5 pounds per cubic foot, at a depth of 34 feet the weight of a cubic foot will be

$$62.5(1 + 0.00005) = 62.503 \text{ pounds},$$

* Grassi, Annales de chemie et physique, 1851, vol. 31, p. 437.

and at a depth of 340 feet a cubic foot will weigh

$$62.5(1+0.0005)=62.53 \text{ pounds.}$$

The variation in weight, due to compressibility, is hence too small to be regarded in hydrostatic computations.

The modulus of elasticity of volume for water is the ratio of the unit-stress to the cubical unit-compression, or

$$E=\frac{14.7}{0.00005}=294\,000 \text{ pounds per square inch.}$$

The modulus of elasticity of volume for steel, when subjected to uniform hydrostatic pressure, is the same as the common modulus due to stress in one direction only, or $E = 30\,000\,000$ pounds per square inch. Hence water is about 100 times more compressible than steel.

The velocity of sound or stress in any substance is given by the formula $u = \sqrt{Eg/w}$, where w is the weight of a cubic unit of the material weighed by a spring balance at the place where the acceleration of gravity is g (Art. 6). For water having $w = 62.4$ pounds per cubic foot at a place where $g = 32.2$ feet per second per second, and $E = 42\,300\,000$ pounds per square foot, this formula gives $u = 4670$ feet per second for the velocity of sound, which agrees well with the results of experiments.

In order to deduce the above formula for the velocity of stress it is necessary to use some of the fundamental principles of elementary mechanics and of the mechanics of elastic bodies. Let a free rigid body of weight W be acted upon for one second by a constant force F and let f be the velocity of the body at the end of one second. Let g be the velocity gained in one second by W when falling under the action of the constant force of gravity. Then, since forces are proportional to their accelerations, $F=W \cdot f/g$, and during the second of time the body has moved the distance $\frac{1}{2}f$. Now, consider a long elastic bar of the length u, so that a force applied at one end will be felt at the other end in one second, it being propagated by virtue of the elasticity of the material. Let A be the area of the cross-section of the bar and E the modulus of elasticity of the material. When a constant compressive force F is applied to the bar, the shortening ultimately produced is $2\,Fu/AE$,* but if this be done for one second only the elongation is only half this amount, since the first increment of stress is just reaching the other end of the bar at the end of the second. The center of gravity of the bar has then moved through the distance $\frac{1}{2}\,Fu/AE$, and its velocity v is Fu/AE. If w is this weight of a cubic unit of the material, the weight W is wAu. Inserting these values of v and W in the above equation, there is found

$$\frac{F}{wAu}=\frac{Fu}{AEg} \qquad \text{whence} \qquad u=\sqrt{\frac{Eg}{w}} \tag{5}$$

which is the formula for the propagation of sound or stress in elastic materials first established by Newton.

Prob. 5. Compute the velocity of sound in distilled water at 35° and also at 80° Fahrenheit.

Art. 6. Acceleration Due to Gravity

The motion of water in river channels, and its flow through orifices and pipes, is produced by the force of gravity. This force is proportional to the acceleration of the velocity of a body falling freely in a vacuum; that is, to the increase in velocity in one second. Acceleration is measured in feet per second per second, so that its numerical value represents the number of feet per second which have been gained in one second. The letter g is used to denote the acceleration of a falling body near the surface of the earth. In pure mechanics g is found in all formulas relating to falling bodies; for instance, if a body falls from rest through the height h, it attains in a vacuum a velocity equal to $\sqrt{2gh}$. In hydraulics g is found in all formulas which express the laws of flow of water under the influence of gravity.

The quantity of 32.2 feet per second per second is an approximate value of g which is often used in hydraulic formulas. It is, however, well known that the force of gravity is not of constant intensity over the earth's surface, but is greater at the poles than at the equator, and also greater at the sea level than on high mountains. The following formula of Peirce, which is partly theoretical and partly empirical, gives g in feet per second per

* Merriman's Mechanics of Material (New York, 1911), pp. 25, 325.

second for any latitude l, and any elevation e above the sea level, e being in feet:

$$g = 32.0894(1 + 0.0052375 \sin^2 l)(1 - 0.0000000957e) \quad (6)_1$$

and from this its value may be computed for any locality.

The greatest value of g is at the sea level at the pole, and for this locality $l = 90°$, $e = 0$, whence $g = 32.258$. The least value of g is on high mountains at the equator; for this there may be taken $l = 0°$, $e = 10\,000$ feet, whence $g = 32.059$. The mean of these is the value of the acceleration used in this book, unless otherwise stated, namely,

$$g = 32.16 \text{ feet per second per second,}$$

and from this the mean values of the frequently occurring quantities $\sqrt{2g}$ and $1/2g$ are found to be

$$\sqrt{2g} = 8.020, \qquad 1/2g = 0.01555. \quad (6)_2$$

If greater precision be required, which will sometimes be the case, g can be computed from the above formula for the particular latitude and elevation. Table 6 gives multiples of the quantities g, $2g$, $1/2g$, and $\sqrt{2g}$ which will often be useful in numerical computations.

TABLE 6. ACCELERATION OF GRAVITY

No.	Multiples of g	Multiples of $2g$	Multiples of $1/2g$	Multiples of $\sqrt{2g}$	No
1	32.16	64.32	0.01555	8.02	1
2	64.32	128.6	0.03109	16.04	2
3	96.48	193.0	0.04664	24.06	3
4	128.6	257.3	0.06219	32.08	4
5	160.8	321.6	0.07774	40.10	5
6	193.0	385.9	0.09328	48.12	6
7	225.1	450.2	0.1088	56.14	7
8	257.3	514.5	0.1244	64.16	8
9	289.4	578.9	0.1399	72.18	9
10	321.6	643.2	0.1555	80.20	10

Prob. 6. Compute to four significant figures the values of g and $\sqrt{2g}$ for the latitude of $40°36'$ and the elevation 400 feet. Also for the same latitude and the elevation 4000 feet.

ART. 7. HISTORICAL NOTES

Hydraulics is that branch of the mechanics of fluids which treats of water in motion, while Hydrostatics treats of water at rest. These two branches are sometimes regarded as a part of Hydromechanics, the name of the mechanics of fluids and gases. While the main purpose of this book is to treat of water in motion, the most important principles of hydrostatics will also be discussed, since these are necessary for a complete development of the laws of flow. The word "Hydraulics" is hence here used as closely synonymous with the hydromechanics of water.

Hydraulics is a modern science which is still far from perfect. Archimedes, about 250 B.C., established a few of the principles of hydrostatics and showed that the weight of an immersed body is less than its weight in air by the weight of the water that it displaces. Chain and bucket pumps were used at this period by the Egyptians, and the force pump was invented by Ctesibius about 120 B.C. The Romans built aqueducts as early as 300 B.C., and later used earthen and lead pipes to convey water from them to their houses. They knew that water would rise in a lead pipe to the same level as in the aqueduct and that a slope was necessary to cause flow in the latter, but had no conception of such a simple quantity as a cubic foot per minute. Even this slight knowledge was lost after the destruction of Rome, 475 A.D., and Europe, for a thousand years sunk in barbarism, made no scientific inquiries until the Renaissance period began.

Galileo, in 1630, studied the subject of the flotation of bodies in water, and a little later his pupils Castelli and Torricelli made notable discoveries, the former on the flow of water in rivers and the latter on the height of a jet issuing from an orifice. Pascal, about 1650, extended Torricelli's researches on the influence of atmospheric pressure in causing liquids to rise in a vacuum. Mariotte, about 1680, considered the influence of friction in retarding the flow in pipes and channels, and Newton, in 1685, observed the contraction of a jet issuing from an orifice.