boiling. It is then set aside over night, or for a few hours, filtered, ignited, and weighed as $BaSO_4$.

Total Sulphur: One gram of the material is weighed out in a large platinum crucible and fused with Na_2CO_3 and a little KNO_3, being careful to avoid contamination from sulphur in the gases from source of heat. This may be done by fitting the crucible in a hole in an asbestos board. The melt is treated in the crucible with boiling water and the liquid poured into a tall, narrow beaker and more hot water added until the mass is disintegrated. The solution is then filtered. The filtrate contained in a No. 4 beaker is to be acidulated with HCl and made up to 250 c.c. with distilled water, boiled, the sulphur precipitated as $BaSO_4$ and allowed to stand over night or for a few hours.

Loss on Ignition: Half a gram of cement is to be weighed out in a platinum crucible, placed in a hole in an asbestos board so that about ⅗ of the crucible projects below, and blasted 15 minutes, preferably with an inclined flame. The loss by weight, which is checked by a second blasting of 5 minutes, is the loss on ignition.

May, 1903:

Recent investigations have shown that large errors in results are often due to the use of impure distilled water and reagents. The analyst should, therefore, test his distilled water by evaporation and his reagents by appropriate tests before proceeding with his work.

APPENDIX II

FORMULAS FOR REINFORCED CONCRETE BEAMS*

Direct working formulas suited to all ordinary cases of reinforced concrete design are presented in Chapter XXI. The analytical methods of deduction, however, are omitted there in order to make the book handier for every day use and are presented in this Appendix.

These formulas cover all the usual conditions occurring in practice and in theoretical treatment of beam design, as follows:

(1) Rectangular beams with steel in bottom, assuming that concrete bears no tensile stress. (See page 751.)

(2) T-shaped section of the beam, for use in combined beam and slab construction. (See p. 754.)

(3) Beam with steel in both top and bottom, for use in connection with the design of a continuous beam at the supports and other special cases. (See p. 757.)

(4) Beam with steel in bottom and concrete assumed to bear tensile stress, for theoretical use in determining accurate stresses at early stages of loading. (See p. 760.)

(5) Beam with compressive stress varying as a parabola, to illustrate a method of computation occasionally used. (See p. 762)

The first three of these analyses are for common use and follow the recommendations of the Joint Committee on Concrete and Reinforced Concrete. This fact has necessitated no changes in the analyses in the first edition of this treatise except in the adoption of the new standard of notation.

As stated in Chapter XXI, the straight line theory,—that is, the theory in which the modulus of elasticity of concrete in compression is assumed to be constant within usual working limits,—is adopted as the standard and the concrete is assumed to bear no tension.

The various other rational formulas† which have been advanced by

*The authors are indebted to Prof. Frank P. McKibben for the formulas in this Appendix which have been especially prepared by him for this Treatise.

†See Christophe's Béton Armé and Morel's Ciments Armé, 1902.

different mathematicians are based upon the same analytical methods of treatment, but on different assumptions of stress. Many have complicated their equations by taking moments about the neutral axis instead of about the centers of tension or compression, but the general principles of the deduction are the same and in accordance with the analyses given below.

It is possible to evolve by calculus a general formula which satisfies all of the various hypotheses,* but the treatment is omitted here and only the more practical demonstrations are given

NOTATION

The same notation is adopted in this Appendix as in Chapter XIV.

h = height of beam.
t = thickness of slab, *i. e.*, thickness of T-flange.
b = breadth of rectangular beam or breadth of flange of T-beam.
b' = breadth of web of T-beam.
p = ratio of cross-section of steel in tension to cross-section of beam above this steel.
p' = ratio of cross-section of steel in compression to cross-section of beam above the steel in tension.
f_c = unit compressive stress in outside fiber of concrete.
f_c' = unit tensile stress, or pull, in outside fiber of concrete.
f_s = unit tensile stress, or pull, in steel.
f_s' = unit compressive stress in steel.
E_c = modulus of elasticity of concrete in compression.
E_c' = modulus of elasticity of concrete in tension.
E_s = modulus of elasticity of steel.
$n = \frac{E_s}{E_c}$
d = distance from outside compressive fiber to center of gravity of steel.
k = ratio of depth of neutral axis to depth of steel in tension.
kd = distance from outside compressive surface to neutral axis in beam in which the depth to steel in tension is d.
z = depth of resultant compression below top.
j = ratio of lever arm of resisting couple to depth d.
$jd = d - z$ = arm of resisting couple.
e = extra thickness of concrete below steel in tension.
d' = depth to center of compressive steel.
M = moment of resistance or bending moment in general.

*See Burr's Materials of Engineering, 1903, p. 633.

ANALYSIS OF RECTANGULAR BEAM

We may represent the stresses in the beam by the diagram shown in Fig. 238, page 751. At any vertical section through the beam the concrete in the upper portion resists the forces which tend to compress it, and the steel in the lower part of the beam resists the forces which tend to stretch and break it in tension. The compressive resistance acts in one direction and the tensile resistance in another direction, as designated by the large arrows in the diagram. The center of tension in the steel is at the center of the bar, or, if there is more than one tier of bars, at the center of gravity of the set of bars. The center of pressure of the concrete passes through the center of gravity of the triangle which represents the compressive stresses.

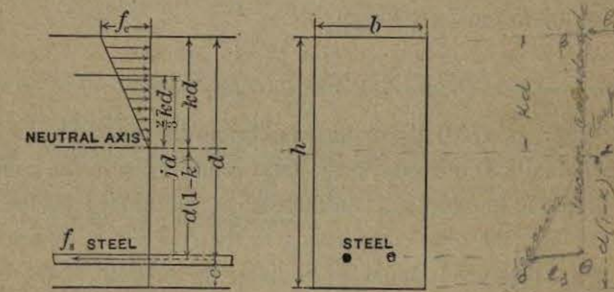


FIG. 238.—Resisting Forces in a Reinforced Concrete Beam. (*See p.* 751.)

The internal resisting forces may be replaced by two forces: the total compression acting in the center of gravity of the triangle, having for its base f_c and its height kd, and the total pull acting in the center of gravity of steel. For equilibrium the sum of all forces must equal zero or the total compression must equal the total pull, so that the forces form a couple. If either tension or compression exceeds its maximum strength, the beam fails. These conditions are assumed to be true only after the point of loading is reached at which the tension is transferred to the steel, as otherwise the tension would be made up of two forces, the tension in the steel and the tension in the concrete, as discussed on page 760 in this Appendix.

The moment of resistance of the couple must be equal to or greater than the bending moment produced by the live and dead loads.

Since it is assumed that a plane section before bending remains a plane section after bending, we have the proportion

$$\frac{\text{deformation in steel}}{\text{deformation in outside compressive concrete fibers}} = \frac{d\,(1-k)}{kd}$$

And since deformation $= \dfrac{\text{stress per square inch}}{\text{modulus of elasticity}}$ we have

$$\frac{\dfrac{f_s}{E_s}}{\dfrac{f_c}{E_c}} = \frac{d\,(1-k)}{kd} \quad \text{or} \quad \frac{f_s}{nf_c} = \frac{1-k}{k} \qquad (1)$$

From which

$$k = \frac{1}{1 + \dfrac{f_s}{nf_c}} \qquad (2)$$

Solving formula (1) for f_c

$$f_c = f_s \frac{k}{n\,(1-k)} \qquad (3)$$

Now, as stated above, for equilibrium the total tension in the steel must be equal and opposite to the total compression in the concrete. The total tension in the steel is its unit tension, f_s, multiplied by the area of the steel, pdb, and the total compression in the concrete is represented by the area of the pressure triangle, $\frac{1}{2} f_c kd$ times the breadth of the beam, b. Equating these two stresses and cancelling out the db which occurs in both,

$$pf_s = \frac{f_c k}{2} \qquad (4)$$

If the value of k in formula (2) be substituted for the k in formula (4), we have

$$p = \frac{1}{2\dfrac{f_s}{f_c}\left(1 + \dfrac{f_s}{nf_c}\right)} \qquad (5)$$

For any given percentage of steel the values of f_s and f_c cannot be assumed independently, as they bear a constant ratio to each other.

Substituting the value of f_c in formula (3) for f_c in formula (4), we have

$$p = \frac{k}{2}\,\frac{k}{(1-k)n}$$

Solving this quadratic equation and adopting the positive sign before the square root,

$$k = -np + \sqrt{2np + (np)^2} \qquad (6)$$

We thus have k in terms of n and p, and from formula (6) the location of the neutral axis may be calculated with any percentage of steel for concrete and steel having known moduli of elasticity.

The moment of resistance is obtained from the couple by taking moments about the center of compression in the concrete, using for the force the total tension in the steel, which, as above, is $pf_s bd$ times the arm (see Fig. 238, p. 751), jd

or

$$M = pf_s jbd^2 \quad \text{and} \quad f_s = \frac{M}{pjbd^2} \qquad (7)$$

The moment of resistance may also be expressed in terms of compression in the concrete by combining equations (4) and (7), or, more directly, by taking moments about the center of the tension in the steel, thus

$$M = \frac{f_c kjbd^2}{2} \quad \text{and} \quad f_c = \frac{2\,M}{kjbd^2} \qquad (8)$$

Values for k with various percentages of steel and moduli of elasticity are given in table 12 on page 521.

The value of the moment of resistance, M, may also be expressed without using k by substituting in formulas (7) and (8) the value of p from formula (5) and the value of k from (2), thus giving

$$M = bd^2 \left[\frac{f_s}{\dfrac{2f_s}{f_c}\left(1 + \dfrac{f_s}{nf_c}\right)} \left(1 - \frac{1}{3\left(1 + \dfrac{f_s}{nf_c}\right)}\right) \right] \qquad (9)$$

or

$$M = bd^2 \left[\frac{f_c}{2\left(1 + \dfrac{f_s}{nf_c}\right)} \left(1 - \frac{1}{3\left(1 + \dfrac{f_s}{nf_c}\right)}\right) \right] \qquad (10)$$

Formula (10) is apparently more complex than (7) and (8), but as the latter require the determination of k, formula (10) is more readily solved unless the table on page 521 is employed.

In the use of formula (10), f_s and f_c must be corresponding values and cannot be assumed independently of each other, since for any given percentage of steel the ratio of f_s to f_c is a constant. (See formula (5), p. 752).

For a given quality of concrete and steel the values of f_s, and f_c and n are constant, so that the term in brackets may be replaced by a constant $\frac{1}{C^2}$

We may thus write in place of formulas (9) and (10) the formula

$$M = \frac{bd^2}{C^2} \qquad (11)$$

where C is a constant for any given concrete and steel. Values of C under different conditions are tabulated on pp. 519 and 520.

Following directly from formula (11)

$$d = C\sqrt{\frac{M}{b}} \qquad (12)$$

In the above formula M represents the bending moment which must be equal to or smaller than the moment of resistance. Also, since in fig. 238, p. 751, $d = h - e$, the formula may be written

$$h = C\sqrt{\frac{M}{b}} + e \qquad (13)$$

from which the required height of the rectangular beam or slab may be directly obtained.

T-SHAPED SECTION OF BEAM

When a reinforced concrete floor slab and beam are built as one piece the slab adds to the strength of the beam by increasing the area which is in compression.

The working formulas for this shape of beam termed a T-beam are given in Chapter XXI, page 423, in sufficient detail for the ordinary design where

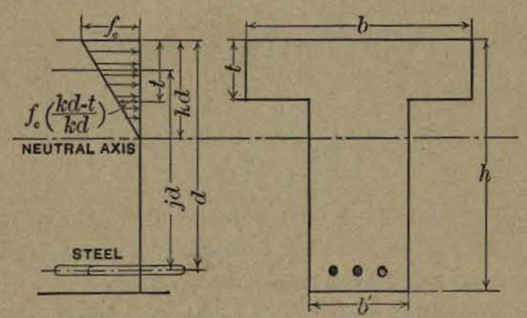


FIG. 239.—Resisting Forces in T-shaped Section of Beam. (*See p. 755.*)

the beam and the slab are assumed to act as a unit. The method of analysis and the formulas deduced are presented below.

These are based upon the assumption that the intensity of the compression in the concrete does not diminish from the web outward towards the

edges of the flange. For a section having a narrow flange, this is practically correct, but with a wide flange, it is probable that the intensity of the compression in the flange diminishes from the web outward so that the breadth of slab should be limited, as indicated on page 424. If this pressure is assumed to decrease either uniformly or otherwise, the formulas may be modified accordingly.

Assuming the compression to be distributed as shown in the diagram, and the steel to take all the tension, the formulas given below may be deduced as in the preceding cases.

Case I. *Neutral Axis Below Flange, $kd > t$.*

Neglect the slight amount of compression in the web below the intersection of the web and flange.

As in the previous case using notation on page 750 and referring to Fig. 239.

$$k = \frac{1}{1 + \frac{f_s}{n f_c}}$$

By equating the forces acting on the section

$$A_s f_s = f_c \frac{2kd - t}{2kd} bt$$

Solving the two above equations for kd and eliminating f_c and f_s

$$kd = \frac{2ndA_s + bt^2}{2nA_s + 2bt} \qquad (14)$$

The position of the resultant compression lies in the center of gravity of the trapezoid, the parallel sides of which are equal to f_c and $f_c\frac{kd-t}{kd}$ and the height to t.

The distance of this center of compression from upper surface of beam is

$$z = \frac{3kd - 2t}{2kd - t}\,\frac{t}{3} \qquad (15)$$

The arm of resisting couple

$$jd = d - z$$

hence

$$M = A_s jd f_s \;(16) \text{ and } M = \frac{2kd - t}{2kd} btjdf_c \qquad (17)$$

or

$$f_s = \frac{M}{A_s jd} \quad (18) \quad \text{and} \quad f_c = \frac{Mkd}{bt(kd - \frac{1}{2}t)\, jd} \qquad (19)$$

From the figure, taking similar triangles, the relation between f_s and f_c is found to be

$$f_c = \frac{f_s}{n}\,\frac{k}{1-k} \qquad (20)$$

The approximate moment arm of resisting couple may be taken as

$$jd = d - \frac{t}{2} \qquad (21)$$

which changes formula (19) to

$$f_s = \frac{M}{A_s\left(d - \frac{t}{2}\right)} \quad \text{(approximate)} \qquad (22)$$

This formula gives for ordinary cases correct and safe results, but should not be used when the flange is small as compared with the stem.

In the above formulas the compression in the stem is neglected. In large beams, where the stem forms the larger part of the compressive area the following formulas derived by the same principles used in derivation of formulas in the previous analysis should be used,

$$kd = \sqrt{\frac{2nd\,A_s + (b-b')\,t^2}{b'} + \left(\frac{n\,A_s + (b-b')\,t}{b'}\right)^2} - \frac{n\,A_s + (b-b')\,t}{b'} \qquad (23)$$

$$z = \frac{(kdt^2 - \frac{2}{3}t^3)\,b + \left[\,(kd-t)^2\left(t + \frac{1}{3}(kd-t)\right)\right]b'}{t\,(2kd - t)\,b + (kd - t)^2\,b'} \qquad (24)$$

Arm of resisting couple

$$jd = d - z \qquad (25)$$

Moment of resistance

$$M = A_s jdf_s \quad (26) \quad M = \frac{f_c}{2kd}\left[(2kd - t)bt + (kd-t)^2 b'\right]jd \qquad (27)$$

Fiber stresses

$$f_s = \frac{M}{A_s jd} \quad (28) \quad and\; f_c = \frac{2Mkd}{\left[(2kd - t)bt + (kd - t)^2 b'\right]jd} \qquad (29)$$

Case II. *Neutral Axis in Flange or at Underside of Flange, $kd \leqq t$*

In this case use the rectangular beam formula, considering the T-beam as a rectangular beam of the same depth, the breadth of which is the breadth of the flange. The percentage is then based on the total area *bd*.

STEEL IN TOP AND BOTTOM OF BEAM, NO TENSION IN CONCRETE

Although the use of steel in the compressive portion of the beam is generally uneconomical, its introduction there is sometimes a necessity for practical reasons. In the ends of a continuous beam the steel in the bottom is usually carried through into the supports, and if the length is enough to provide bond its value in compression may be taken as assisting to resist the negative bending moment.

It is possible to reduce the working formulas to extremely simple form by introducing constants which vary with different conditions, as outlined on page 427, the values for the constants being given in table 8, page 516.

The treatment of a beam subjected to bending and direct stress with the steel in compression is presented in connection with the design of arches on page 563, and these formulas may also be used in other cases of eccentric thrusts.

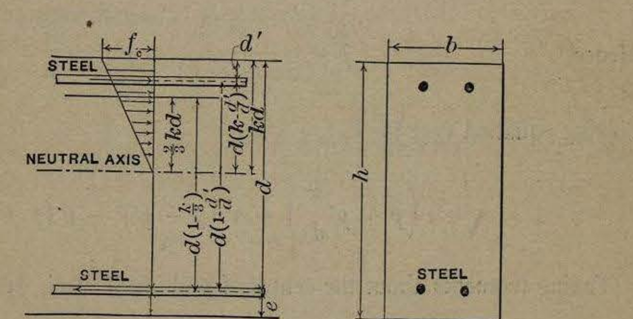


FIG. 240.—Resisting Forces with Steel in Top and Bottom of Beam. (*See p.* 757.)

The analytical treatment of the design of an ordinary beam adopting as usual the assumption of a constant modulus of elasticity and no tension in the concrete, but assuming that the compressive stresses are partially borne by the steel in the compression portion of the beam, is as follows:

FORMULAS.

Deformations, as usual, are assumed to vary directly as distance from neutral axis, hence from Fig. 240, using notation on page 750.

$$\frac{\frac{f_c}{E_s}}{\frac{f_c}{E}} = \frac{d\,(1 - k)}{dk} = \frac{1-k}{k} \qquad \text{Whence}\; k = \frac{1}{1 + \frac{f_s}{nf_c}} \qquad (30)$$

Also,

$$f_s' = f_c \frac{kd - d'}{d - kd} \quad (31) \quad \text{and} \quad f_s' = nf_c \frac{kd - d'}{kd} \tag{32}$$

$$f_s = nf_c \frac{1 - k}{k} \quad (33) \quad \text{and} \quad f_c = \frac{f_s}{n} \frac{k}{1 - k} \tag{34}$$

Equating the horizontal forces acting on the cross-section of the beam we have:

$$bd\left(\frac{f_c k}{2} + p'f_s'\right) = bdpf_s$$

Whence $p = \frac{1}{f_c}\left(\frac{f_c k}{2} + p'f_s'\right) = \frac{1}{f_s}\left(\frac{f_s}{2n}\frac{k^2}{1-k} + p'f_s\frac{kd - d'}{d - kd}\right)$

Hence

$$p = \frac{k^2}{2n(1 - k)} + p'\frac{k - \frac{d'}{d}}{1 - k} \tag{35}$$

Solving equation (35) for k,

$$k = \sqrt{2n\left(p + p'\frac{d'}{d}\right) + n^2(p + p')^2} - n(p + p') \tag{36}$$

Taking moments about the center of pull in the steel, we have

$$M = \frac{bf_c kd}{2}\left(d - \frac{kd}{3}\right) + f_s' p'bd(d - d')$$

$$M = bd^2\left[\frac{f_c k}{2}\left(1 - \frac{k}{3}\right) + f_s' p'\left(1 - \frac{d'}{d}\right)\right]$$

or by eliminating f_s' by means of equation (32),

$$M = f_c bd^2\left[\frac{k}{2}\left(1 - \frac{k}{3}\right) + \frac{np'\left(k - \frac{d'}{d}\right)\left(1 - \frac{d'}{d}\right)}{k}\right] \tag{37}$$

Taking moments about the center of compresssive stress in the steel, we have

$$M = bd^2\left[f_s p\left(1 - \frac{d'}{d}\right) - \frac{f_c k}{2}\left(\frac{k}{3} - \frac{d'}{d}\right)\right]$$

or by eliminating f_c,

$$M = f_s bd^2\left[p\left(1 - \frac{d'}{d}\right) - \frac{k^2}{2n(1 - k)}\left(\frac{k}{3} - \frac{d'}{d}\right)\right] \tag{38}$$

Then taking moments about center of compression in concrete:

$$M = bd^2\left[f_s p\left(1 - \frac{k}{3}\right) + f'_s p'\left(\frac{k}{3} - \frac{d'}{d}\right)\right]$$

or by eliminating f_s,

$$M = f'bd^2\left[p\frac{1 - k}{k - \frac{d'}{d}}\left(1 - \frac{k}{3}\right) + p'\left(\frac{k}{3} - \frac{d'}{d}\right)\right] \tag{39}$$

The values in the square brackets in formulas (37), (38) and (39) are constant for any combination of n, p, p' and $\frac{d'}{d}$.

Substituting

$$C_c = \frac{k}{2}\left(1 - \frac{k}{3}\right) + \frac{np'\left(k - \frac{d'}{d}\right)\left(1 - \frac{d'}{d}\right)}{k} \tag{40}$$

$$C_s = p\left(1 - \frac{d'}{d}\right) - \frac{k^2}{2n(1 - k)}\left(\frac{k}{3} - \frac{d'}{d}\right) \tag{41}$$

$$C'_s = p\frac{1 - k}{k - \frac{d'}{d}}\left(1 - \frac{k}{3}\right) + p'\left(\frac{k}{3} - \frac{d'}{d}\right) \tag{42}$$

$$M = bd^2 f_c C_c \quad (43) \quad \text{and} \quad f_c = \frac{M}{bd^2 C_c} \tag{44}$$

$$M = bd^2 f_s C_s \quad (45) \quad \text{and} \quad f_s = \frac{M}{bd^2 C_s} \tag{46}$$

and

$$M = bd^2 f_s' C_s' \quad (47) \quad \text{and} \quad f_s' = \frac{M}{bd^2 C_s'} \tag{48}$$

and

Values of C_c, C_s, and C_s' for different combinations of n, p, p' and $\frac{d'}{d}$ are given in table 8, page 516.

STEEL IN BOTTOM OF BEAM, CONCRETE BEARING TENSION

In the earlier stages of loading of reinforced concrete beams, the deformation curves (see fig. 130, p. 409) indicate that the concrete actually bears a portion of the pull. Although it is not good practice to consider this pull in the design of beams, but, instead, it is customary to take the working strength as a factor of the ultimate, or nearly the ultimate strength of the beam, the following formulas are useful for determining the actual stresses and for calculating deflections at the earliest stages of loading.

Formulas. Since elongation of steel and concrete at the same point

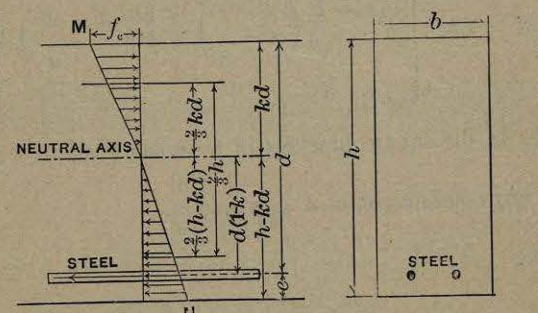


FIG. 241.—Resisting Forces with Concrete Bearing Tension. (See p. 760.)

must be equal, and since cross-sectional planes are assumed to remain plane during bending, we have from Fig. 241, the following equations:

$$\frac{\frac{f_s}{E_s}}{\frac{f_c'}{E_c'}} = \frac{d - kd}{h - kd} \text{ hence } f_s = \frac{E_s}{E_c'} f_c' \frac{d - kd}{h - kd} \tag{49}$$

$$f_c = \frac{E_c}{E_c'} f_c' \frac{kd}{h - kd} \tag{50}$$

$$f_s = \frac{E_s}{E_c} f_c \frac{1 - k}{k} \quad (51) \qquad \text{also } f_c' = \frac{E_c'}{E_c} f_c \frac{h - kd}{kd} \quad (52)$$

Equating horizontal forces on the section we have

$$\frac{bf_c kd}{2} = pf_s bd + \frac{f_c' b (h - kd)}{2} \tag{53}$$

The elimination of f_s and f_c' from (53) gives

$$\frac{kd}{2} = pd \frac{E_s}{E_c} \frac{1 - k}{k} + \frac{E_c'}{E_c} \frac{(h - kd)^2}{2\,kd} \tag{54}$$

From which

$$p = \frac{1}{2(1 - k)} \left[\frac{E_c}{E_s} k^2 - \frac{E_c'}{E_s} \left(\frac{h - kd}{d} \right)^2 \right] \tag{55}$$

Solving equation (55) for k,

$$k = \sqrt{\frac{2p + \frac{E'_c h^2}{E_s d^2}}{\frac{E_c}{E_s} - \frac{E'_c}{E_s}} + \left[\frac{p + \frac{E'_c}{E_s} \frac{h}{d}}{\frac{E_c}{E_s} - \frac{E'_c}{E_s}} \right]^2} - \frac{p + \frac{E'_c}{E_s} \frac{h}{d}}{\frac{E_c}{E_s} - \frac{E'_c}{E_s}} \tag{56}$$

Taking moments about the center of the pull in the concrete, the center of compression in the concrete and the center of pull in the steel respectively, we have the three following equations for the moment of resistance:

$$M = f_s p\, bd \left(d - \frac{kd}{3} - \frac{2h}{3} \right) + \frac{f_c bkd}{2} \frac{2h}{3}$$

$$= f_s bd \left[p \left(d - \frac{kd}{3} - \frac{2h}{3} \right) + \frac{E_c}{E_s} \frac{hk^2}{3(1 - k)} \right] \tag{57}$$

or

$$M = f_s p\, bd \left(d - \frac{kd}{3} \right) + \frac{f_c' b (h - kd)}{2} \frac{2h}{3}$$

$$= f_c' b \left[pd^3 \left(1 - \frac{k}{3} \right) \frac{E_s}{E'_c h} \frac{1 - k}{kd} + \frac{h}{3} (h - kd) \right] \tag{58}$$

or

$$M = \frac{f_c bkd}{2} \left(d - \frac{kd}{3} \right) - \frac{f_c' b (h - kd)}{2} \left(d - \frac{kd}{3} - \frac{2h}{3} \right)$$

$$= \frac{f_c b}{2} \left[kd^2 \left(1 - \frac{k}{3} \right) - \frac{E'_c}{E_c} \frac{(h - kd)^2}{kd} \left(d - \frac{kd}{3} - \frac{2h}{3} \right) \right] \tag{59}$$

If now $E_c' = E_c$, that is, if the modulus of elasticity of concrete is the same in tension as in compression, the line MN becomes straight.

Equation (55) then becomes, letting $\frac{E_s}{E_c} = n$

$$p = \frac{1}{2} \frac{h}{nd^2} \frac{2kd - h}{1 - k} \tag{60}$$

From which

$$k = \frac{h^2 + 2pnd^2}{2dh + 2pnd^2} \qquad (61)$$

Equation (57) is not changed
Equation (58) simply has E_c instead of E_c'
Equation (59) becomes

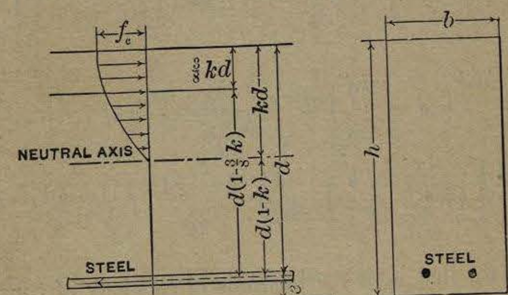
$$M = \frac{f_c b}{2}\left[kd^2\left(1 - \frac{k}{3}\right) - \frac{(h - kd)^2}{kd}\left(d - \frac{kd}{3} - \frac{2h}{3}\right)\right]$$

or

$$M = \frac{f_c bh}{2}\left[2d - \frac{h}{k} - h + \frac{2h^2}{3kd}\right] \qquad (62)$$

COMPRESSIVE STRESS AS A PARABOLA, STEEL IN BOTTOM OF BEAM, NO TENSION IN CONCRETE.

Many experiments upon the compression of concrete show a gradually decreasing modulus of elasticity as the load increases. From the form of the stress deformation curve of these specimens, the stress on the compression side of a beam is sometimes assumed to vary as a parabola instead of as a straight line. This method was first suggested in the United States by Prof. W. Kendrick Hatt.* The formulas which follow present this method of analysis, and permit the comparison† of results by this assumption, with results of the straight line theory adopted by the authors in chapter XXI.



242.—Resisting Forces with Pressure Varying as a Parabola. (See p. 762.)

*Proceedings American Society for Testing Materials, 1902.
†See p. 407 for comparative values by the two theories.

Formulas. As in preceding cases, from Fig. 242, we have

$$\frac{\dfrac{f_s}{E_s}}{\dfrac{f_c}{E_c}} = \frac{d(1 - k)}{kd} = \frac{1 - k}{k}$$

hence

$$k = \frac{1}{1 + \dfrac{f_s}{nf_c}} \qquad (63)$$

from which

$$f_c = \frac{f_s}{n}\,\frac{k}{1 - k} \qquad (64)$$

Equating horizontal forces on the section of the beam we have

$$pbdf_s = \frac{2bf_c kd}{3}\text{, or more simply, } pf_s = \frac{2f_c k}{3} \qquad (65)$$

Substitute the value of k from (63) and we have:

$$p = \frac{2}{3\dfrac{f_s}{f_c}\left(1 + \dfrac{f_s}{nf_c}\right)} \qquad (66)$$

which gives the ratio of steel required for any consistent values of f_s, f_c, E_s, E_c. The position of the neutral axis is dependent upon the per cent of steel and the moduli of elasticity of steel and concrete, and the value of k may be found by substituting in (65) the value of f_s from equation (64). Thus

$$\frac{2f_c k}{3} = pf_c n\,\frac{1 - k}{k} \quad \text{or,} \quad p = \tfrac{2}{3}\,\frac{k^2}{(1 - k)\,n}$$

Solving this quadratic equation and using the positive sign after taking the square root,

$$k = \sqrt{\tfrac{3}{2}np + (\tfrac{3}{4}np)^2} - \tfrac{3}{4}np$$

or in another form,

$$k = \tfrac{3}{4}np\left[\sqrt{\frac{8}{3np} + 1} - 1\right] \qquad (67)$$

The moment of resistance may be found by taking moments about the center of compression in the concrete, thus,

$$M = f_s pbd^2 (1 - \tfrac{3}{8} k) \tag{68}$$

or by taking moments about the center of pull in the steel,

$$M = \tfrac{2}{3} f_c kbd^2 (1 - \tfrac{3}{8} k) \tag{69}$$

Eliminating k from these equations by substituting its value from equation (63), and also substituting the value of p from equation (66), we have

$$M = \tfrac{2}{3} f_s bd^2 \frac{1}{\frac{f_s}{f_c}\left(1 + \frac{f_s}{nf_c}\right)} \left[1 - \frac{3}{8\left(1 + \frac{f_s}{nf_c}\right)}\right] \tag{70}$$

or

$$M = \tfrac{2}{3} f_c bd^2 \frac{1}{1 + \frac{f_s}{nf_c}} \left[1 - \frac{3}{8\left(1 + \frac{f_s}{nf_c}\right)}\right] \tag{71}$$

APPENDIX III

FORMULAS FOR REINFORCED CONCRETE CHIMNEY AND HOLLOW CIRCULAR BEAM DESIGN

The analysis which follows is based upon the several fundamental assumptions adopted in reinforced concrete beam design with the additional assumption that, since the concrete is usually thin as compared to the diameter of the chimney, no appreciable error is involved in assuming all material as concentrated on the mean circumference of the shell. An analysis for shear is also given together with an example of chimney design and review.

The principles involved in the demonstration of the thickness of steel and concrete are taken by permission from the analysis by Messrs. C. Percy Taylor, Charles Glenday, and Oscar Faber.*

The principal formulas given below are quoted in the text, where the general subject of concrete chimneys is discussed, and tables are presented there with the values of constants for use in design.

NOTATION

W = weight in pounds of the chimney above the section under consideration.

M = moment in inch pounds of the wind about that section.

P = total compression in concrete.

T = total tension in steel.

$n = \frac{E_s}{E_c}$ = ratio of modulus of elasticity of steel to that of concrete

f_c = maximum compression in concrete in pounds per square inch (measured at the mean circumference).

f_s = maximum tension in the steel in pounds per square inch.

D = mean diameter of shell in inches.

r = mean radius of shell in inches.

t = total thickness of shell in inches.

t_c = thickness in inches of concrete only.

* Engineering (London), Mar. 13, 1908.