A retaining wall is especially subject to temperature stresses. To locate the stresses at specially prepared joints, contraction joints may be placed at stated intervals. In an unreinforced wall, a spacing of 20 to 30 feet between joints is necessary to prevent intermediate cracks. By introducing steel to prevent the formation of visible cracks, no joints are necessary. Steel reinforcement for shrinkage and temperature contraction is treated on page 499.

EXAMPLE OF T-SHAPED RETAINING WALL

Example 1. Design a retaining wall 12 ft. high above ground to support a sand filling. Angle of internal friction of sand, which weighs 100 lb. per cu. ft. is 35°, and the fill slopes back at the same angle. Working stresses: for the 1 : 2½ : 5 concrete in compression, $f_c = 500$ lb. per sq. in.; steel in tension, $f_s = 16\,000$ lb. per sq. in.; ratio of moduli of elasticity, $n = 15$; allowable shear involving diagonal tension, $v = 32$ lb. per sq. in.; bond of steel to concrete, $u = 80$ lb. per sq. in.

Solution. If base is imbedded 4 ft. to protect from frost, and if the footing is assumed 18 inches thick, total height of wall is 16 ft. and height of stem 14 ft. 6 in. The design is shown in Fig. 216, page 669.

Upright Slab. Earth pressure on stem from formula (2), page 664, taking value of C_p from the table, $P_1 = 0.41 \times 100 \times 14.5^2 = 8600$ lb. This acts at ⅓ the height. Horizontal component, $H_1 = P_1 \cos 35° = 7040$ lb., and since the weight of wall and vertical component of earth pressure do not affect the vertical slab, the moment, $M = 7040 \times \frac{1}{3} \times 14.5 \times 12 = 408\,000$ in lb.

Thickness of vertical slab at bottom, using formula (9), page 421, and table of constants, page 519, and adding 1.7 in. to the depth to steel to properly imbed it, is $d + 1.7 = 0.29 \times 0.118\sqrt{408000} + 1.7 = 23.5$. Ratio of steel is $p = 0.005$ (to correspond to working stresses), hence area of steel is $A_s = 1.31$ sq. in. per foot of length of wall. This is satisfied by ⅝ in. round bars placed vertically 5.5 in. on centers. (See table, p. 507.) The thickness of wall at top may be selected as 12 in. The moment decreases from the bottom upwards so the steel may be reduced as shown in Fig. 216, page 669.

Since total shear, $V = 7040$ lb., unit shear involving diagonal tension, is

$$v = \frac{7040}{12 \times 21.8 \times 0.894} = 30 \text{ lb. per sq. in.}$$ (See p. 447.) As this does not exceed working stress, no stirrups are needed.

Bond stress is $u = \dfrac{7040}{21.8 \times .894 \times 2.18 \times 2.75} = 60$ lb. per sq. in. (see p. 457).

Length of bar to imbed in footing to prevent pulling out is $50 \times \frac{7}{8} = 43.8$ in. (see Table on page 454), hence the vertical bars must extend into the base this distance, or else be provided with bent ends (see page 466).

* A table of dimensions and reinforcement for *T*-shaped and for counterfort retaining walls of different heights, compiled by Sanford E. Thompson, is given in "Concrete in Railroad Construction," published by The Atlas Portland Cement Co.

To obtain this bond, the vertical rods frequently are bent into the right cantilever of the footing. If instead they are bent to run into the left cantilever, they may form the horizontal reinforcement there, as shown in Fig. 216.

Footing. In a correctly designed wall the resultant force should intersect the base within the middle third of its length. This determines the ratio of length of footing to height of wall, and can be obtained only by trial for any particular case. A study of different conditions shows that this ratio is gen-

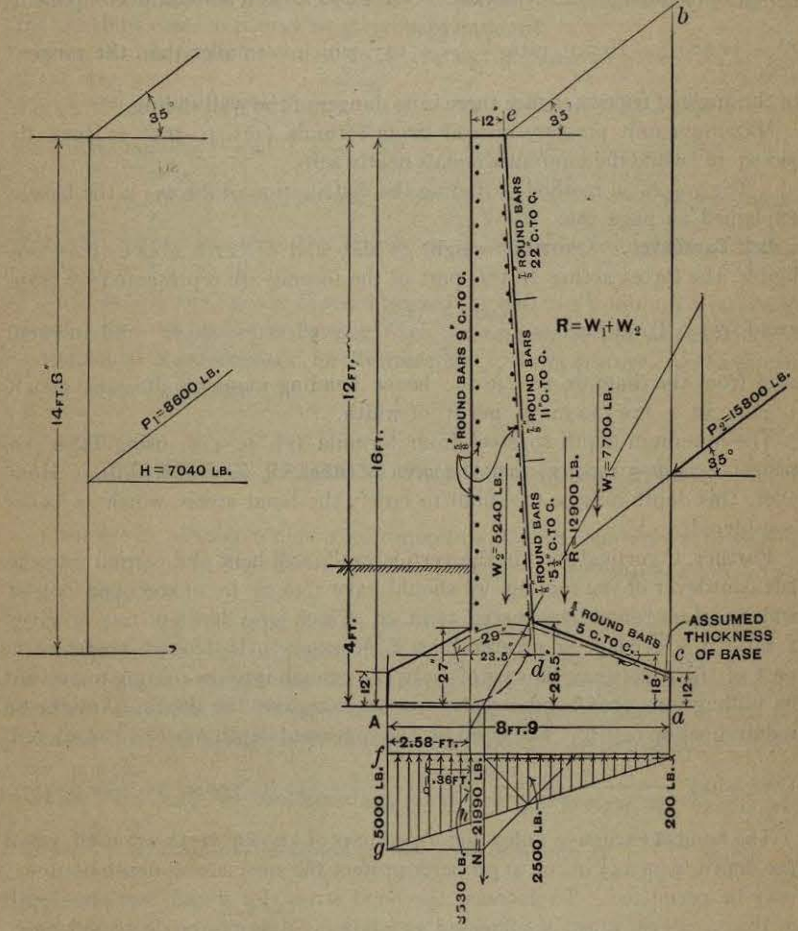

FIG. 216.—Design of T-shaped Retaining Wall. (*See p.* 669.)

erally 0.4 to 0.6, depending upon the inclination of earth pressure, the weight of the fill, and finally upon the ratio between the length of the projecting toe and the total length of the base. The length of base best suited for our example was found after several trials to be 8 ft. 9 in.

The forces acting on the footing are P_2, the earth pressure on the plane ab, W_1, the weight of prism of sand, $bcde$, and W_2, the weight of the retaining wall itself. The distance from the toe to the line of action of the resultant R

of W_1 and W_2 may be obtained as follows: Find center of gravity of earth and center of gravity of concrete; multiply the distance from A to these centers of gravity by the respective weight, and thus obtain the statical moment. Divide the sum of these moments by the sum of the weights, $W_1 + W_2$, and the location of the center of gravity of the combined weight is obtained. The line of pressure drawn for P and R intersects the base just inside of the middle third.

Normal component of resultant, $N = 21\,990$ lb. and horizontal component, $H = 12\,900$ lb. Hence, ratio $\frac{H}{N} = 0.587$, which is smaller than the tangent of the angle of friction, hence there is no danger of the wall sliding.

Maximum unit pressure on soil (from formula (36), p. 562) is 5000 lb. per sq. ft., while the minimum equals nearly zero.

The graphical method of finding the distribution of forces on the base is explained on page 586.

Left Cantilever. Omitting weight of slab and of earth above it as negligible, the forces acting on this part of the footing are represented by trapezoid *fghi*. Total force is $\frac{5000 + 3530}{2} \times 2.58 = 11\,000$ lb. and moment arm from the diagram is 1.36 ft.; hence bending moment, $M = 11\,000 \times 1.36 \times 12 = 179\,500$ in. lb. per ft. of width.

The minimum depth to steel from formula (1), p. 418, using Table 10, page 519, is $d = 14.5$ in., and the area of steel, $A_s = 0.868$ sq. in. However, this depth may be too small to satisfy the bond stress, which is below considered.

Further, if vertical steel in the vertical wall is all bent and carried into the left cantilever of the footing, we should have 1.30 sq. in. of steel per foot of width or $\frac{7}{8}$ in. round bars spaced $5\frac{1}{2}$ in. cc., which for a depth of 14.5 in. gives a ratio $p = 0.0075$, or greater than is necessary. If desired, therefore, a part of this steel may be carried only far enough into the footing to prevent its pulling out, or if bond stress were not excessive, the depth, d, might be reduced below $14\frac{1}{2}$ in. The bond for the suggested depth must be considered.

Unit bond, $u = \frac{11000}{14.5 \times 0.9 \times 2.75 \times 2.18} = 140$ lb. per sq. in. (see p. 457).

The bond is excessive unless deformed bars of known worth are used, when the depth, d, of $14\frac{1}{2}$ in., or to properly protect the steel a total depth of 16 in., may be permitted. To decrease the bond stress, for round bars the depth of the cantilever must be increased as follows: Assume the decreased ratio, p, for the increased section of concrete at $p = 0.0045$. Then the corresponding values from Table 12, page 521, $k = .300$, $j = .900$.

From page 457 $u = \frac{V}{jd\Sigma o}$ hence $d = \frac{V}{uj\Sigma o}$. Substituting values,

$$d = \frac{11000}{80 \times 0.9 \times .218 \times 2.75} = 25.5 \text{ in., and total depth 27 in.}$$

The depth of beam must be increased to 27 in. in order to decrease the bond stress to 80 lb. per sq. in.

Right Cantilever. It is evident from Fig. 216, page 669, that three forces act on the right cantilever: the upward pressure of the soil, the downward weight of the earth filling, and the vertical component of the earth pressure. The resultant of these forces acts downward, hence the moment is negative.

The computations for amount of steel and the shear and bond stresses are similar to that for the left cantilever.

The length of imbedment necessary to prevent slipping is not treated in the previous case, so it may be given here in detail.

Area of concrete, $A = 12 \times 27 = 324$ sq. in.; area of steel, $A_s = 1.07$ sq. in. and ratio of steel, $p = \frac{1.07}{324} = 0.0033$. From table 10, p. 519 find the corresponding k and j, $k = .268$, $j = .911$. From formula (8), p. 420, since $M = 329\,000$ inch pounds, $f_s = \frac{329\,000}{27 \times .91 \times 1.07} = 12\,500$ pounds. For this stress in steel, the length of imbedment from table on page 454 is $39 \times \frac{3}{4} = 29$ in.

Both cantilevers may be tapered toward the end to a minimum practicable depth, since the moments decrease from the support to zero at the end.

Horizontal Reinforcement for Temperature. Temperature reinforcement is treated on page 499.

EXAMPLE OF RETAINING WALL WITH COUNTERFORTS

Example 2. Design a reinforced concrete wall with counterforts to support a sand filling 20 ft. high above ground, using same assumptions as in Example 1, page 668.

Solution. In this type of wall the vertical slab acts as a slab supported by the counterforts, the principal steel being horizontal. The projecting toe of the footing is a cantilever and the footing below the earth is a slab supported by the counterforts. The counterforts tie the imbedded footing to the vertical slab and act as cantilevers fixed to the footing. Design is shown in Fig. 217, p. 672.

The slabs may be considered as partly continuous, using the moment $M = \frac{wl^2}{10}$. If carefully designed for negative moment $M = \frac{wl^2}{12}$ might be permissible. (See p. 428.)

Instead of forming a projecting toe as a cantilever, it is sometimes more economical when the projection is large to introduce small buttresses and construct this part of the footing also as a partly continuous slab.

The first step in the operation of design is to determine the length of base and the relation between the projecting toe and the base by trial, the allowable pressure on the soil and the minimum angle of inclination of the resultant earth pressure being the determining factors. The method is the same as for a T-type wall, as outlined on page 670.

Spacing of Counterforts. The spacing of counterforts or ribs may be found on the basis of minimum material*, from which 8 feet may be adopted.

Vertical Wall. The vertical wall must be considered in narrow horizontal strips as slabs supported by the counterforts, partly continuous, and loaded uniformly. The earth pressure changes with the height, so that the pressure upon the different strips decreases from the bottom up. The pressure against the bottom strip as given on page 672 is 1480 lb. per sq. ft., or 123 lb. per ft. of width for 1-inch of height. Using $M = \frac{wl^2}{10}$, $M = \frac{123 \times 64 \times 12}{10} =$ 9500 inch pounds per inch of width. Hence (p. 418) $d = .118\sqrt{9500} = 11.5$ in.; thickness of wall is thus 13 in., and area of steel, $A_s = 0.005 \times 11.5 \times 12 = 0.69$ sq. in. per ft. of height. Round bars ⅜ in. diameter spaced 5½ inches on centers may be used.

For convenience in construction the thickness of the wall may be made uniform, and the spacing of rods increased with the decreasing earth pressure, as shown on the drawing. The negative bending moment may be provided for by introducing short rods in front of buttresses, or by bending the rods. (p. 428.)

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* For full discussion, see "The Design of Retaining Walls," by H. A. Petterson, Engineering Record, Vol. LVII, 1908, p. 777; for practical purposes the following demonstration illustrates the necessary steps. Use notation page 529, also let x = spacing of buttresses in feet; Q = the maximum horizontal unit pressure on vertical wall, which occurs at the bottom of the wall. Q, from formula (3), page 664, is 1480 lb. per sq. ft. Taking a strip of the vertical slab one ft. in height, whose span is the spacing of the counterforts, the bending moment is then $M = \frac{1480 \times x^2 \times 12}{10} = 1780x^2$; the depth to steel, (p. 421), $d = .29 \times .118\sqrt{1780}\,x = 1.43x$, and the volume per foot of length of wall is $\frac{1.43x}{12} \times 1 \times 22 = 2.6x$ cu. ft. Maximum unit weight acting on horizontal footing slab is 5 325 pounds per sq. ft. Hence $M = \frac{5\,325 \times 12 \times x^2}{10}$, $d = .29 \times .118\sqrt{5325 \times 1.2x^2} = 2.72x$, and volume per foot of length of wall is $\frac{2.72x}{12} \times 1 \times 8.25 = 1.9x$

The thickness below steel is a constant for any spacing and therefore need not be considered in fixing the volume.

Assume the thickness of counterfort as 16 in., and volume will be $\frac{22 \times 8.25 \times 16}{2 \times 12} = 121$ cu. ft., and for one foot of length of wall, $\frac{121}{x}$. Because of the greater cost, per unit of volume, of the counterforts over that of the slab work in a wall of this type, the quantity representing the counterfort volume may be increased by, say, 100%. The expression for this quantity then becomes $\frac{121}{x} \times 2$. Hence total volume, $Q = 2.6x + 1.9x + \frac{121}{x} \times 2$

or $Q = 4.5x + \frac{242}{x}$ and $\frac{dQ}{dx} = 4.5 - \frac{242}{x^2} = 0$ (for minimum, first derivative equals zero).

$$x = \sqrt{\frac{242}{4.5}} = 7.3 \text{ ft.}$$ For practical purposes, say 8 ft.

Horizontal Footing Slab. This slab may be considered as composed of narrow strips uniformly loaded and supported by the counterforts. The loading is the difference between the weight of the earth above it plus the vertical component of the earth pressure, and the upward pressure of the soil. As indicated in the drawing, this difference is a maximum at *a* and decreases toward *b*. In this case the maximum unit loading is 5566 − 241 = 5325 lb. per sq. ft. The maximum bending moment in this slab, considering it as partly continuous is

$$M = \frac{5325 \times 64 \times 12}{10}$$

= 40 800 in. lb. Depth of steel, $d = 0.29 \times 0.118\sqrt{40800} = 21.75$ in., hence thickness may be taken as 23.25 in. The area of concrete is then 261 sq. in., hence area of steel required is $A_s = 1.31$ sq. in., which is satisfied by ⅞-in. bars spaced 5½ in. on centers. The thickness of this foundation slab may be made uniform, and the spacing of the rods increased as the loading decreases.

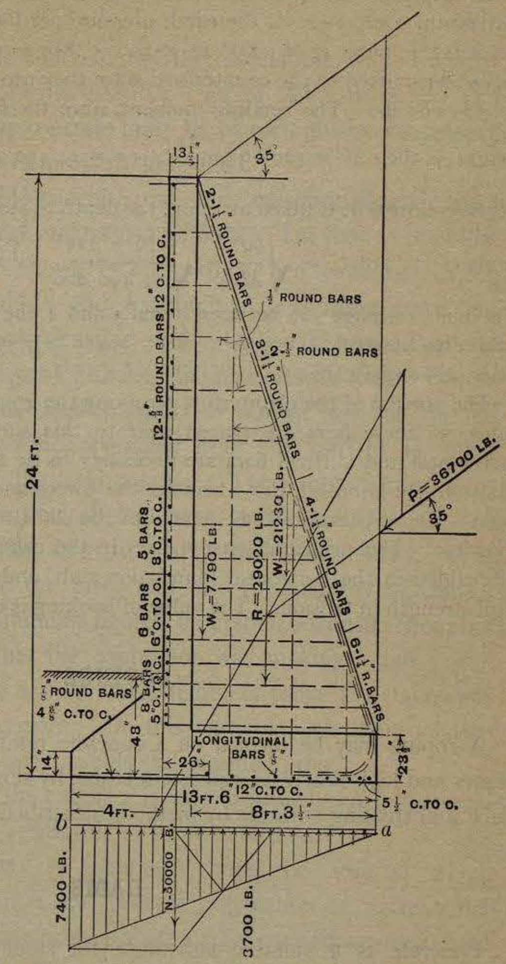

FIG. 217.—Design of Retaining Wall with Counterforts. (*See p.* 671.)

The negative bending moment must be provided for by introducing at the top of the slab, under the counterforts, short rods of equal size and spacing to the bottom ones or else these bottom rods must be bent down at each counterfort. (See p. 428.)

Counterforts. A counterfort is really an upright cantilever beam supported by the horizontal foundation slab and carrying as its load the vertical slab of the wall, which, in turn, takes the earth pressure. The thickness of the counterfort, which must be sufficient to insure rigidity and resist unequal pressures during construction, may be selected by judgment.

To determine the quantity of steel required in the counterfort, we find the horizontal component of the earth pressure per foot of wall to be (from formula (2), p. 664) $.41 \times 22 \times 22 \times 100 \times .819 = 16\,200$ lb.; hence, the total force transmitted to the counterfort, since they are spaced 8 ft., is $8 \times 16\,200 = 129\,600$ lb. The bending moment, since the force acts at one-third the height, is then $M = 129\,600 \times \frac{22}{3} \times 12 = 11\,400\,000$ in. lb. The thickness of the counterfort is taken at 16 in., the depth to steel, $d = 110$ in. From formula (1), p. 418, $C = \sqrt{\frac{bd^2}{M}} = \sqrt{\frac{16 \times 110^2}{11\,400\,000}} = 130$. By interpolation in the Table 11 on page 520 between items 3 and 4, the ratio of steel, $p = 0.00416$ and area of steel $A_s = 110 \times 16 \times .00416 = 7.36$ sq. in. Six 1¼-in. round bars will satisfy this.

The portion of the counterfort receiving the greatest tension is the inclined edge, so these bars are placed near to this surface. Besides these bars, horizontal and vertical bars are necessary to tie the vertical and horizontal slabs to the counterfort, to transfer the forces and provide for diagonal tension. These bars should be bent into the slabs to obtain as good a bond as possible. The principal tension bars in the counterforts also must be well imbedded in the horizontal foundation slab, and bent so as to attain their full strength in tension. The value of hooking is discussed on page 466.

COPINGS

A coping may be formed on a concrete retaining wall, which will shed water and look nearly as well as cut stone, by sloping the top back from the face and treating surfaces by methods described on pages 288 to 293.*

DAMS

Concrete is a suitable substitute for stone masonry (*a*) in gravity dams, where the masonry is laid in large masses, whenever the cost per cubic yard of concrete rubble is cheaper than stone masonry of equal quality, and (*b*) in curtain or arch dams of thin section reinforced with steel.

Concrete of cement, sand, and crushed stone cannot always compete in price with rubble masonry laid in cement mortar, because, although the labor cost of laying concrete is less, more cement is required per cubic yard; but by introducing large stones into the concrete, the percentage of cement per cubic yard may be reduced to the same quantity or even less than in water-tight rubble masonry. Therefore, the concrete rubble is apt to be the cheaper, since the cost of crushing the stone for the concrete is small

* See illustration of form construction in Engineering News, July 9, 1903, p. 37.

compared with the difference in expense of employing skilled masons or unskilled labor.

Methods of laying rubble concrete and the calculation of the quantity of cement per cubic yard are discussed in Chapter XV, pages 300 and 298.† As is there stated, the concrete must be of soft, mushy consistency so that the large stone may be properly imbedded.

The relative cost of rubble concrete and stone masonry depends upon the price of cement at the work and local conditions. The dam at Boonton, N. J., a section of which is shown in Fig. 219, p. 676, contains 240,000 cubic yards of concrete rubble, and was built at a contract price, not including the cement, of $1.98 per cubic yard. Only 0.6 barrels Portland cement were used per cubic yard, although the proportions of the concrete matrix were 1: 2¾: 6¼. This small quantity of cement was due to the large proportion of stones which averaged from one yard to 2½ yards each and occupied 55% of the total volume. The contract price mentioned includes the preparation of the large stones and the crushed stone, and their transportation from a quarry three miles away. It is believed by the authors that the price and also the quantity of cement per cubic yard represent minimum figures in first-class construction, but the force account showed that the contractor was making a fair profit, and inspection of the work and its water-tightness prove that there was no skimping in the use of cement. On this particular job the quotation of the highest bidder was nearly double the accepted price.

With reinforced concrete the engineer is able to branch out into special types whose design may be applicable to local conditions.

Design of Gravity Dams. A foundation must be secured which will resist the pressure upon it and prevent percolation of water under the masonry. The end connections with the adjacent soil or rock must also be carefully considered. The section of the dam must be of such thickness and design as to prevent (1) leakage, (2) overturning, and (3) sliding.

Leakage through a concrete dam of gravity section need only be considered to the extent that no careless work be allowed.

To avoid tension in the foundation it is necessary that the resultant of all the forces of pressure and weight shall pass through the middle third of the base. Dangerous sliding need not usually be feared if the dam is designed to resist overturning. In considering the resistance of friction, Mr. Joseph P. Frizell* states that smooth stone slides on smooth stone

* Frizell's "Water Power", p. 19.

†Tables of Quantities are given on pp. 236, 237.

under a horizontal force of two-thirds its weight, and to slide on gravel or clay, stone requires a force nearly equal to its weight.

The pressure of the water upon any submerged surface is equal to the area of the surface in square feet times the weight of a cubic foot of water times the depth of the center of gravity of the surface below the water level. This pressure tends to overturn the dam, and is resisted by the weight of the dam, and in some cases, where the up-stream face slopes, by the weight of the water upon the dam.

The treatment in Frizell's Water Power of the location of the center of pressure, and the moment produced by it, is especially clear and practical.

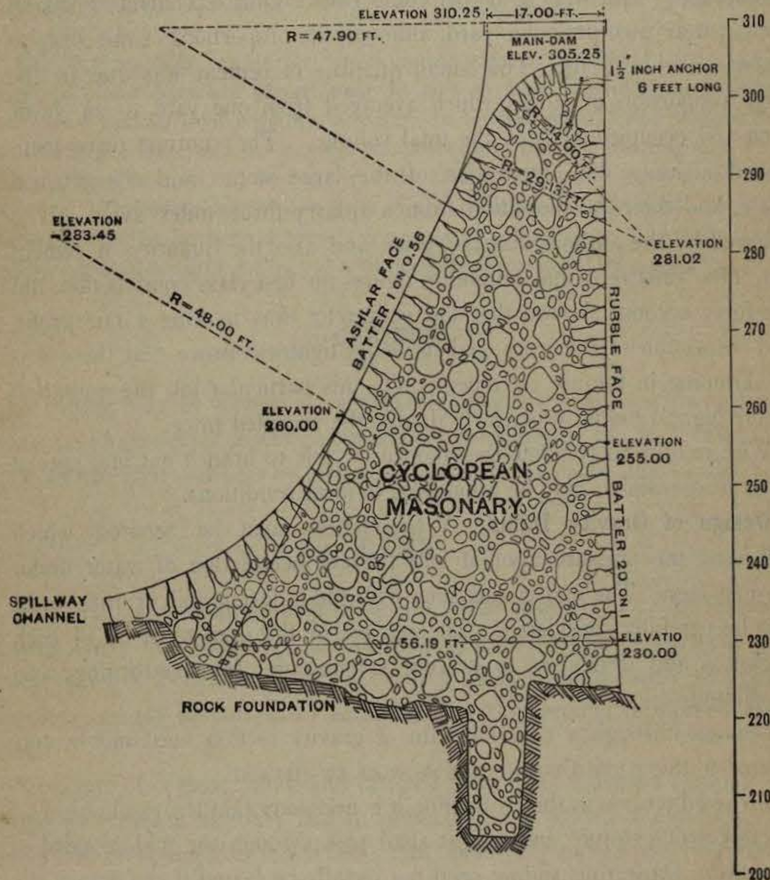

FIG. 219.—Section through Overflow of Boonton, N. J., Dam. (See p. 676.)

Fig. 219 represents a section through the overflow of the concrete dam at Boonton, N. J., the construction of which is described on page 300.

The extreme height of the dam at the highest point above the foundations is 110 feet. An interesting practical test of the water-tightness of concrete occurred when the reservoir was filled. A vertical well was left in the dam in order to provide access to two drainage gates, and although the water in the reservoir is 100 feet deep, and is separated from the well by only 5 feet 6 inches of concrete mixed in the proportions 1 : 2⅗ : 6¼, the well remains entirely dry.

Reinforced Dams. The aim in reinforced dams is to reduce the quantity and cost of materials, and at the same time to permit a much broader base, and a sloping water-tight deck for the up-stream face. The water pressure is thus made to increase instead of oppose stability.

A section of such a dam at Schuylerville, N. Y., 250 feet long and 25 feet high, is shown in Fig. 220. The buttresses are on 10-foot centers, and support a deck tapering from 8 inches to 12 inches thick, while the overfall apron is 8 inches thick. A foot-bridge lighted by electric lights passes through under the crest, giving access from the mill to the railway platform on the other bank.

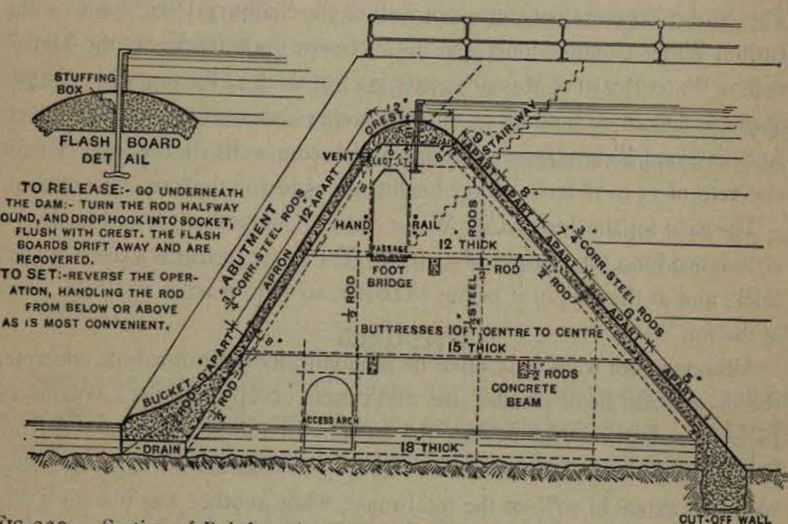

FIG. 220.—Section of Reinforced Concrete Dam at Schuylerville, N. Y. (See p. 677.)

Arched Dams. Curved dams, designed in plan as a single arch, convex up-stream, are considered by foremost authorities to be of doubtful economy, as the extra length requires more material than is saved by the reduced cross-section.

Recently, a type of dams consisting of a series of arches supported by piers or steel lattice work has been suggested, and this idea may receive further development through the introduction of reinforced concrete.

A dam in the form of a buttressed wall with a vertical up-stream surface has been suggested by Mr. George L. Dillman,* the dam in plan consisting of parabolic arches.

The design for a dam at Ogden, Utah,† consists of a number of piers, triangular in vertical section, forming buttresses to support an up-stream sloping face composed of circular concrete arches from 6 to 8 feet thick. The arches are designed to be covered on their upper surface with ¼-inch steel facing. The top of the dam, which is also formed by arches between the piers, carries a roadway.

CORE WALLS

Concrete is largely superseding rubble masonry for core walls in earth dams and dikes. The forms can be roughly made without reference to the appearance of the faces, while a thin wall of concrete may be built water-tight more easily than one of rubble masonry. Unless reinforced, core walls are generally of the same thickness as those of rubble masonry. The Natural cement concrete core wall of the Sudbury Dam, built by the Boston Water Commissioner and his successor upon the work, the Metropolitan Water Board of Massachusetts, is 2 feet thick at the top, with a batter of one in fifteen on both faces, until it reaches a maximum width of 10 feet. At Spot Pond Reservoir, several dikes with core walls of Portland cement concrete, of 15 to 18 feet average height, are 2½ feet in thickness throughout.

The dike for the Jersey City Water Supply Company at Boonton, N. J., is designed for a total height of 54 feet. The lower 30 feet is 4 feet 8 inches thick, and at this height it begins to batter, so as to reach a width of 3 feet at the top.

Although core walls may often be economically built of rubble concrete, the stones must be of smaller size, and cannot occupy so large a volume of the mass as in gravity dams, since the sections are thinner. In the construction of the Boonton Dike, mentioned above, one contractor was placing rubble to the extent of 20% of the total mass, while another was placing 33%. In the former case the stones were loaded on to derrick skips and unloaded by hand; in the latter case, they were hooked by the derrick. This 33% probably represents a maximum for a wall 5 feet thick or less.

Since a thin wall of reinforced concrete may be made equally strong, and more elastic than a thick wall of plain concrete, reinforcement may eventually be employed to reduce the section, and therefore the quantity of material.

*Transactions American Society of Civil Engineers, Vol. XLIX, p. 94.

†Henry Goldmark in Transactions American Society of Civil Engineers, Vol. XXXVIII, p. 290

CHAPTER XXVII

CONDUITS AND TUNNELS

Since the principal stresses in arches are compressive, concrete is peculiarly suitable for all classes of arched structures. Eccentric loading may be provided for by increasing the thickness of the concrete at the points of greatest stress, by steel reinforcement, or by both. The steel may also prevent failure of thin sections of the arch from excessive stresses due to suddenly applied loads or to settlement of the foundation.

Concrete is supplanting cut stone in arch bridges because of its relative cheapness. Although not entirely acceptable from an architectural standpoint because of the difficulty in obtaining a satisfactory surfacing, several methods of treating the face have been used with fair success. (See p. 288.) This objection may also be met by facing the arch with cut stone. Methods of arch design are treated in Chap. XXII.

Concrete arches and conduits are likely to be cheaper than brick even at the same price per cubic yard, because the greater strength of the concrete makes a thinner section possible.

Tunnels (see p. 689) and subways (see p. 692) are now built almost exclusively of concrete, or of combinations of concrete and steel.

CONDUITS

Sewer and water conduits of almost any size or shape may be built of concrete. In the larger sizes, and in conduits under pressure, steel reinforcement occasionally may be advisable from the standpoint of safety and economy.

Concrete was first used in conduits to form in bad ground a foundation for a brick invert. Later it was adopted instead of brick for the entire arch, and finally, in many instances, the brick invert lining has also been replaced by concrete.

While concrete may not be preferable to brick in all localities and under all conditions, its advantages are sufficient to always warrant a very careful investigation of its adaptability to the work in question.

As far back as 1850 sewers and aqueducts of béton or béton-coignet (see p. 1) 8 feet in diameter were constructed in France. The materials consisted of ¼ part heavy Paris cement, one part hydraulic lime, and 5