For under-water work, a larger factor of safety should be employed than for work above ground, the concrete should be slightly richer in carefully selected cement, and the aggregate so proportioned as to give a dense and impervious mixture.

Concrete for the foundations of walls and piers for high office buildings is usually laid in oblong or circular caissons of steel or wood,* after excavating under air pressure. Steel pipes are sometimes sunk with the aid of the water jet, and afterwards filled with concrete.†

*_Engineering News_, Sept. 26, 1901, p. 222.

†Jules Breuchaud, Transactions American Society of Civil Engineers, Vol. XXXVII, p. 31.

CHAPTER XXVI

DAMS AND RETAINING WALLS

For walls to resist the pressure of earth or water, concrete frequently possesses marked advantages over other classes of masonry. With proper management, in most localities its cost may be brought below that of rubble masonry. Its adaptability for thin walls and for certain classes of face work often make it a suitable substitute in complicated designs for first-class masonry, with a consequent large saving in cost. In combination with steel its possibilities for special designs are almost unlimited, and the future will see marvelous advances in its use for ordinary engineering and hydraulic construction.

Water-tightness, often an essential element for this class of structures, has received general treatment in Chapter XIX, page 338. Portland cement concrete may be made water-tight more readily than stone masonry laid in mortar of similar proportions to the cement and sand in the concrete, since large voids or stone pockets in the concrete are more easily prevented than the "rat-holes" so frequently found in the bedding of stones in mortar. Moreover, skill in laying combined with special treatment of the surface or the addition of certain ingredients permits construction in concrete—strengthened with steel reinforcement—of thinner walls for resisting the flow of water than is possible in stone masonry.

Reinforced concrete retaining walls cannot be designed by "rule of thumb," and therefore a careful consideration of the forces acting and of the stresses in the concrete is presented in this chapter. Since the earth pressure is the controlling factor, it has been necessary to introduce a practical discussion of this before taking up the details of the design and examples of the two principal types

RETAINING WALLS

Retaining walls to support the pressure of earth may be designed:

(1) of gravity section with plain concrete or stone masonry;

(2) of thin reinforced concrete section of the inverted T type with spreading base or footing;

(3) of thin section, reinforced and supported by buttresses or counterforts.

Another plan sometimes adapted to cellar wall construction (see p. 619) consists in embedding the base and supporting the top of the wall with timber, steel or reinforced concrete beams, so that the concrete forms a vertical slab supported at top and bottom.

Reinforced concrete retaining walls are almost always more economical than a gravity section of either plain concrete or masonry. In walls of gravity section the materials cannot be fully utilized because the section must be made heavy enough to prevent overturning by its own weight, counterforts or buttresses being of comparatively little advantage because, in stone masonry, the wall is liable to break away from them. In reinforced concrete retaining walls, on the other hand, a part of the sustained material is used to prevent overturning, and the section need be made only strong enough to withstand the moments and shears due to the earth pressure. Since the wall is lighter, exerts smaller pressure on the soil, and may be made if necessary with a very broad base, the special foundations or piling which are often necessary for a gravity wall frequently may be avoided. Reinforced concrete properly designed can be depended upon as absolutely reliable.

The economy of a reinforced concrete wall over one of gravity section for either stone masonry or plain concrete is obvious because of the saving in material. The cost of forms is practically the same for gravity section and reinforced designs.

Whether the T-section of reinforced wall or the wall with counterforts is the more economical depends upon certain conditions. The principal condition is the height of the wall, but the intensity of the earth pressure and the relative cost of concrete and steel and forms also enter into the consideration. The construction of the T-section is simpler and the placing of steel easier, so that it is preferable where skilled labor is scarce. The form construction in the counterforted wall is considerably more expensive. Comparative studies of the two types indicate that the counterfort type is scarcely ever economical when the height is less than 18 feet. Rules for designing walls of gravity section are first given and then, after the discussion of earth pressure, the designs of both a T-type and a counterforted section are treated.

FOUNDATIONS

A firm foundation is essential whatever the type of the design. Piles may be necessary, or to avoid sliding, a stepped base may be required. Unequal settling is more dangerous for a retaining wall than for many other structures, because if it is thrown out of plumb, the earth will move and produce forces much in excess of the calculated ones. Allowable pressures on different soils are referred to on page 640.

The depth of foundation must be sufficient to prevent heaving of the material in front of the wall, and to protect it from frost. A depth of 3 feet may be given as a minimum, while 4 or 5 feet is necessary in temperate or very cold climates.

DESIGN OF RETAINING WALLS OF GRAVITY SECTION

The thickness of base of a retaining wall of gravity section, that is, one in which the earth pressure is resisted by the weight of the masonry, is generally taken without mathematical calculation as a certain ratio of the height of the wall. An easily remembered rule is to make the base ⅜ of the height. The table of empirical values adopted by Mr. Trautwine* for thickness of base of wall to resist earth pressure under average conditions is in accordance with good engineering practice. While he gives no values for concrete, they may safely be assumed equivalent to those for cut stone laid in mortar, which are as given in the following table. The earth is assumed to slope up from the top of the wall till it reaches a level at the height indicated by the ratio in the first column.

Thickness of Retaining Walls of Gravity Section with Earth Surcharge.
BY JOHN C. TRAUTWINE. (*See p.* 661.)

Ratio of Height of Earth to Height of Wall.	Thickness of Base as ratio to Height of Wall.	Ratio of Height of Earth to Height of Wall.	Thickness of Base as ratio to Height of Wall.
1.	0.35	2.	0.58
1.1	0.42	2.5	0.60
1.2	0.46	3.	0.62
1.3	0.49	4.	0.63
1.4	0.51	6.	0.64
1.5	0.52	9.	0.65
1.6	0.54	14.	0.66
1.7	0.55	25.	
1.8	0.56	or more	0.68

The height of the wall is assumed to be measured above the surface of the ground in front of it.

The batter of the face of a retaining wall is customarily limited to 1½ inches to the foot, and the back is usually vertical. This fixes the width on top.

The values in the table may be employed for long walls of concrete with no reinforcement. In many cases, because of the monolithic character of concrete, a ratio of thickness to height from 10% to 20% less may be adopted with safety, if the character of the filling back of the wall precludes

* Trautwine's "Civil Engineer's Pocket-Book", 1902, p. 606.

excessive pressure, and if the base is slightly spread. For more accurate determinations of gravity sections, the principles which follow relating to reinforced designs are applicable.

Angle of Internal Friction. The selection of the angle of internal friction is of much importance as it affects largely the magnitude of the earth pressure. For ordinary cases the values given on page 665 may be used, but for very important structures, where the additional cost is warranted, special experiments may be advisable.

WEIGHT OF EARTH

In the calculation of retaining walls, and many other structures, the weight of earth in place is a prime factor. The weights of dry material, based upon experiments by the authors, are represented in the following table. Most of the figures for weights of earth give the weights per cubic foot after excavation in a loose or a compacted condition. In the authors' experiments the excavation was measured, so that the weights represent the material in place. As fills will eventually assume much the same characteristics as earth in original excavation, the figures may be employed for either natural earth or filled material. The weight of earth containing water varies with the character of the material and with the conditions. Gravel containing ordinary moisture weighs about 2% more than dry gravel and sand may weigh from 3% to 10% more, depending upon its fineness, since fine sands absorb the most water. Wet muck weighs about 75 lb. per cubic foot. These percentages assume that the bank is provided with natural drainage; if the earth is literally filled with water which cannot run off, its weight will be increased by a quantity of water nearly equal in volume to the voids in the material, which vary with the character of the material from 20% to 50% of the bulk of the earth in the bank.

Many of the values appear high, but they are the result of careful tests.

Average Weight of Ordinary Earth before Excavation.

	Pounds per cu. ft.
Sand	105
Gravel	135
Gravelly clay	130
Loam	90
Hard pan	130
Dry muck	40

BACKING

Since the weight of soil saturated with water is much larger than when it is dry, the pressure increasing with the amount of water so that it may even

exceed the hydrostatic pressure, the backing should be provided with adequate drainage. For this, a filling of gravel or crushed stone may be placed directly against the wall with weep holes at suitable distances apart.

EARTH PRESSURE

The principal force governing the dimensions of any retaining wall is the earth pressure. Its magnitude varies largely with the character and wetness of the soil, the inclination of the back of the wall, and the slope of earth above it.

Of the numerous theories, all of which are based on some assumptions not always met with in practice, Rankine's theory seems to be the most reliable yet developed, and although it does not always represent the true conditions, it gives safe results. It is based upon the assumptions that the earth is composed of granular homogeneous particles without cohesion, held only by friction developed between them, and that the mass of earth extends indefinitely. On a vertical plane the resultant pressure always acts parallel to the slope of the earth and at a point one-third of the height from the base, when the surface of the earth is level with the top of the wall or slopes back from it.

The following table of pressures determined by Rankine's formula gives horizontal earth pressures for different heights of wall, based on an angle of repose of earth of 35°—a fair assumption under average conditions—and also average unit pressures for the same assumptions. For other heights of wall, the horizontal unit pressures with the same angle of repose are directly proportional to the heights, and the total pressures are proportional to the squares of the height.

Total Earth Pressure and Average Unit Pressure upon Vertical Walls of Different Heights (See p. 663.)

	Height of Wall in Feet.							
	5	10	15	20	25	30	35	40
Total pressure P, in lb.	350	1400	3150	5600	8750	12600	17150	22400
Average unit pressure in lb. per sq. ft.	70	140	210	280	350	420	490	560

The table assumes (*a*) horizontal surface of earth, (*b*) vertical back of wall, (*c*) weight of earth per cubic foot, 100 pounds, (*d*) angle of repose, 35°. For other weights of earth the values in the table are proportional to the weight per cubic foot.

Passive pressure, that is, the resistance of a mass of earth against moving, is many times as great as the active pressure but because of the shrinkage of filling as ordinarily placed it cannot be counted on for its full value unless the earth is in its natural state.

The general formulas evolved by Mr. Rankine from the assumptions given above and which apply both to gravity walls and to reinforced walls, are presented below.

Wall with Vertical Back. Let

P = resultant earth pressure in pounds on a vertical surface for a length of wall equal to one foot.
H = total height of wall in feet.
H_1 = depth below top of wall of any point in feet.
h = height of surcharge in feet.
w = weight of earth per cubic foot.
δ = angle of inclination of earth behind the wall.
φ = angle of internal friction of the earth.
C_p = constant depending upon δ and φ. (See table on page 665.)

Then*

$$P = \tfrac{1}{2} wH^2 \cos\delta \frac{\cos\delta - \sqrt{\cos^2\delta - \cos^2\varphi}}{\cos\delta + \sqrt{\cos^2\delta - \cos^2\varphi}} \qquad (1)$$

For known values of the angle of inclination and internal friction, the terms embracing them become constant and

$$P = C_p wH^2 \qquad (2)$$

The intensity of pressure at any point the depth of which is H is

$$\text{Unit pressure} = 2\, C_p w H_1 \qquad (3)$$

and its direction is parallel to the direction of the total pressure.*

* For walls with horizontal filling, $\delta = 0$, hence

$$P = \tfrac{1}{2} wH^2 \frac{1 - \sin\varphi}{1 + \sin\varphi} \qquad (4)$$

Unit pressure at any depth, H_1 is $wH_1 \frac{1 - \sin\varphi}{1 + \sin\varphi}$ and acts horizontally.

If angle of slope equals angle of internal friction, i. e., if $\delta = \varphi$,

$$P = \tfrac{1}{2} wH^2 \cos\delta \text{ and Unit pressure is } wH_1 \cos\delta \qquad (5)$$

Formulas (2) and (3), however, apply to these cases by using the proper value of C_p given in the table.

The values of the constant C_p are given in the table below.

Data for Determining the Earth Pressure.

Rule: To find the earth pressure on a vertical wall without surcharge, H ft. high, multiply the proper value of C_p by the square of H in feet and by the weight of the filling per cu. ft. $P = C_p wH^2$ (see p. 664.) For formulas for inclined walls and walls with surcharge, see pp. 665 and 666.

Angle of internal friction φ	Values of constant C_p in Rankine's formula (2), p. 664							
	Slope with horizontal							
	1 to 1	1 to 1½	1 to 2	1 to 2½	1 to 3	1 to 4	Level	
	Corresponding angle of slope δ							
	45°	33° 40′	26° 30′	21° 50′	18° 30′	14° 0′	0	φ
55°	0.09	0.07	0.06	0.06	0.05	0.05	0.05	0.29
50°	0.15	0.09	0.08	0.07	0.07	0.07	0.07	0.32
45°		0.13	0.11	0.10	0.09	0.09	0.09	0.35
40°		0.18	0.14	0.13	0.12	0.12	0.11	0.38
35°		0.29	0.19	0.17	0.16	0.15	0.14	0.41
30°			0.27	0.22	0.20	0.18	0.17	0.43
25°				0.30	0.26	0.23	0.20	0.45
20°					0.36	0.29	0.25	0.47

NOTE: If the angle of internal friction of the earth is unknown, the following average values may be used: Coal, shingle and broken stone, 50°; earth, 35°; clay, 30°; sand dry, 30°; sand moist, 35°; sand wet, 20°.

As stated above, the pressure is assumed to act parallel to the slope of the surface of the earth, and for walls without surcharge acts at one-third of the height of the wall from the base. The maximum unit pressure is at the base, and is equal to twice the average, while the minimum at the top equals zero, so that the variation of the unit pressures may be represented by a triangle.

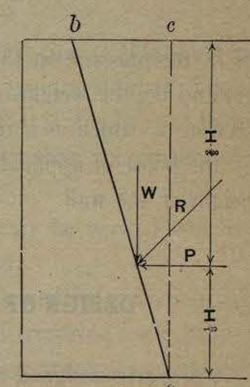

FIG. 213.—Earth Pressure on Inclined Back of a Wall. (*See p.* 665.)

Wall with Inclined Back. The earth pressure, R, on an inclined plane *ab* (Fig. 213) is the resultant of P, the horizontal pressure on the vertical plane *ac*, and W, the weight of the prism of earth *abc*, and acts at one-third the height from the bottom.

Surcharge. When the earth behind the wall is loaded in any way, for example, when a highway or a railway track runs along the wall, or when the embankment is used as a storage for material—then this loading causes additional pressure on the wall, which may be provided for by replacing the load by an equivalent surcharge of earth. The height of this surcharge, h, is the extra load per square foot divided by the weight of a cubic foot of earth. Thus a load of 500 pounds per square foot is equivalent to a surcharge of 5 feet if the earth weighs 100 pounds per cubic foot.

Vertical Back of Wall with Surcharge. The earth pressure on a retaining wall with surcharge equals the pressure on the surface ab less the pressure on bd. Using a constant from the table, page 665,

$$P = wH^2\,C_p - wh^2\,C_p = w\,(H^2 - h^2)\,C_p \qquad (6)$$

and this may be represented by the trapezoid $aced$ (see Fig. 214). The distance of the point of application of this force from below the middle point in the height of the wall,

$$x = \frac{(H - h)^2}{6\,(H + h)} \qquad (7)$$

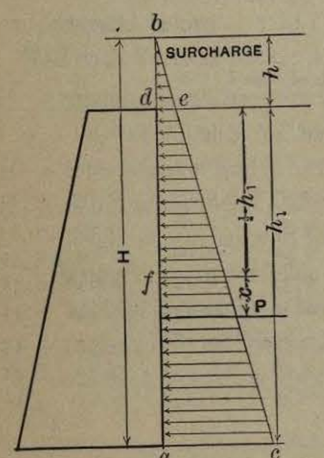

FIG. 214.—Earth Pressure on Vertical Back of Wall with Surcharge. (*See p.* 666.)

Wall with Inclined Back with Surcharge. For an inclined back, the pressure, as in the case of a wall with inclined back without surcharge, is the resultant of P, the pressure on the vertical projection of the wall found by formula (2) and W, the weight of the prism of earth one foot of length, the cross-section of which is a trapezoid. Equation (7) gives the vertical distance of the point of application of the resultant below the middle point in the height of the wall.

DESIGN OF REINFORCED RETAINING WALLS

A properly designed retaining wall, whether of reinforced concrete or of plain masonry, must fulfil the following conditions: It must be stable (1) against overturning, (2) against sliding, (3) against settling, (4) against crushing or overstressing of the material.

To prevent failure by overturning, the moment of downward forces about the outer edge of the base, $M_1 = W_1\,l_1 + W_2\,l_2$, must be greater than that of the overturning moment, $M_2 = Pl_3$ (see Fig. 215). The ratio of those two moments, $\frac{M_1}{M_2}$, is called the factor of safety. For reinforced concrete walls, the factor of 1.5 to 2 may be considered as ample, because the stability of wall is increased by the resistance of earth to shear along the line ab, Fig. 215, and the passive pressure of the filling in front of the wall, which two items are not considered in figuring the factor of safety.

The horizontal component of the resultant pressure on the foundation causes the tendency of the wall to slide. This force is opposed by the resistance to compression of the earth on the plane dc (see Fig. 215) and by the friction F. The friction is equal to the vertical pressure multiplied by the tangent of friction between concrete and earth, or, if

F = total friction,

$W_1 + W_2$ = weight of concrete and earth,

ϕ = angle of friction between earth and concrete

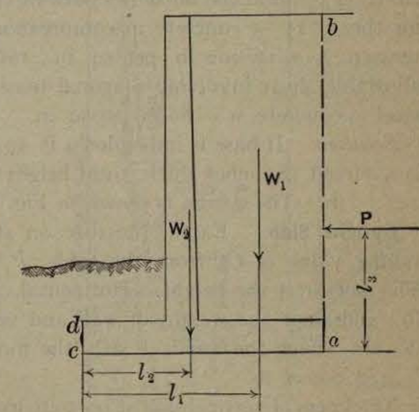

FIG. 215.—Forces Acting upon a Retaining Wall and their Moment Arms. (*See p.* 667.)

Then

$$F = (W_1 + W_2)\ \tan\phi$$

If the wall slides, the cohesion of the earth along the line ab (Fig. 215) must be destroyed, which item increases the stability against sliding. The tangent of the inclination of the resultant pressure, that is, the ratio of its horizontal to vertical component, should not be larger than the tangent of the angle of friction.

Sometimes a vertical projection of the base may be needed, which may be placed in the middle of the base or at either end.

Having determined the earth pressure as explained in preceding pages, the design of a reinforced concrete retaining wall resolves itself primarily into the determination of the thickness and reinforcement of concrete slabs to be obtained by the principles outlined in Chapter XXI on Reinforced Concrete Design. The methods to follow can be illustrated best by practical examples, which are given in full below.