Solution. A problem of this kind must necessarily be solved by a method of successive trials, since the position of the neutral axis is not known. The location of the neutral axis is determined by the values of f_c, f_s and n, two of which, in this case, are unknown. The method of procedure, therefore, is to assume outright a trial position of the neutral axis, select the constants accordingly, substitute in equations (1) and (2) and solve them for f_s and f_c.

Then see if the position of the neutral axis, as fixed by these values of f_s and f_c and the given n, is the same as the position assumed at the start. If the two positions agree, then f_s and f_c as found are the actual stresses; if not, a new position of the neutral axis must be assumed, new constants selected, and new values of f_s and f_c computed from equations (1) and (2). Thus a series of trials must be made until the location of the neutral axis as assumed is consistent with the computed values of f_c and f_s together with the given n.

In this problem, assuming 30 pounds pressure on the projected area, we have the bending moment due to the wind,

$$M = [8.5 \times 90 \times 30] \times \frac{90}{2} \times 12 = 12\ 393\ 000 \text{ in. lb.}$$

and the total weight of the chimney above the section,

$$W = 3.1416 \times 8 \times 0.5 \times 90 \times 150 = 169\ 646 \text{ lb.}$$

$$A_s = 60 \times .3068 = 18.41 \text{ sq, in.}$$

Now suppose we assume the neutral axis at, say, $k = .400$

For $k = .400$, table 1 gives $C_P = 1.765$, $C_T = 2.224$, $z = .416$

Substituting in equation (1) we have

$$18.41 = \frac{8\ (12\ 393\ 000 - 169\ 646 \times .416 \times 96)}{2.224 \times f_s \times 96}$$

whence $f_s = 11400$

Substituting in equation (2) we have

$$6 = \frac{2 \times 169646 + (2.224 \times 11400 - 1.765 \cdot f_c\ 15)\ \dfrac{18.41}{3.1416}}{1.765 \times f_c \times 96} + \frac{18.41}{3.1416 \times 96}$$

whence $f_c = 416$

Now $f_s = 11\ 400$, $f_c = 416$, and $r = 15$ gives $k = .354$ which does not correspond with our original assumption of $z = .400$. Evidently the true k must lie somewhere between the assumed and determined values, hence if we now assume, say, $k = .375$ and recompute, we obtain $f_s = 11\ 000$ and $f_c = 435$, the values of which together with $n = 15$ gives $k = .371$ which checks fairly well with the assumption of $k = .375$. For all practical purposes we may therefore say that the maximum stress in the steel is 11 000 pounds per square inch, while the maximum stress in the concrete is 435 pounds per square inch. The results indicate that both the thickness of shell and the amount of steel are greater than are necessary for safe stresses.

CHAPTER XXV

FOUNDATIONS AND PIERS

Concrete excels as a material for foundations, and here finds its widest and most important field of usefulness. It is pre-eminently adapted to such construction, because the stresses are chiefly compressive, the forms are easily built, and the surface appearance need not be considered.

Concrete is peculiarly suited to under-water foundations because, although it requires careful handling, it can be placed with great facility. It is now used even in piling. (See p. 650.)

Within recent years concrete has been adopted for foundations above ground, such as bridge piers, and is standing the test of durability even when subjected to excessive wear and impact. (See p. 654.)

Since the design of a foundation or sub-structure is governed almost as much by the character of the underlying rock or soil as by the superstructure, brief reference is made to the standard practice in estimating loads, although the treatment of engineering principles, as such, is not within the province of this treatise.

Reinforced concrete footings are treated in detail (see p. 644).

BEARING POWER OF SOILS AND ROCK

Sound hard ledge will support the weight of any foundation and superstructure, but if the rock is seamy or rotten it may require thorough examination and special treatment. If its surface is weathered, it must be removed. A sloping surface must be stepped or the foundation designed with sufficient toe to prevent sliding.

The sustaining power of earths depends upon their composition, the amount of water which they contain or are likely to receive, and the degree to which they are confined. An approximate idea of the loads which may be safely placed upon uniform strata of considerable thickness is given by Mr. George B. Francis*:

There are several classes of strata that are readily definable, such as ledge rock, hard pan, gravel, clean sand, dry clay, wet clay, and loam, and when these strata are of considerable thickness and uniform for considerable areas, they may be loaded with safety (provided the material

*Journal Association Engineering Societies, June 1903, p. 340.

placed thereon is not of less density than the natural material upon which it is placed, viz., concrete or brick work on ledge rock) as follows

Ledge rock, 36 tons per square foot.
Hard pan, 8 tons per square foot.
Gravel, 5 tons per square foot.
Clean sand, 4 tons per square foot.
Dry clay, 3 tons per square foot.
Wet clay, 2 tons per square foot.
Loam, 1 ton per square foot.

Mr. Francis, however, calls attention to the many kinds and mixtures of materials, and to the consequent impossibility of applying such specific rules as the above to all cases. He also emphasizes the necessity for varied and ample experience when fixing safe allowable pressures.

If the piles are driven to firm strata, such as rock or hard pan, the loading which a pile will stand is determined by the crushing strength of the timber. If supported wholly or in part by friction, it is customary to calculate the safe loading by a formula based upon factors obtained by experiment, or by one based upon the penetration of the pile from the blow of the pile driver.

An engineer experienced in pile driving in a particular locality can often determine by judgment whether the piles have reached a firm bearing, but it is usually safer to formulate exact specifications. Mr. Joseph R. Worcester* advises for piles which meet a hard resistance, a penetration of one inch under a 2 000-lb. hammer falling 10 feet, and for piles which hold by friction, a penetration of 3 inches under a 2 000-lb. hammer falling 15 feet. He prefers heavier hammers if they are available.

A mean of the various formulas† gives for approximate average values, after applying a factor of safety of 3, a safe load of 16 tons for bearing piles and 9 tons for friction piles.* These loads apply to ordinary piles of spruce and Norway pine.

A commonly used formula for determining safe loading on piles with reference to the penetration under blows of the hammer is the *Engineering News* formula, which is as follows:

Let

P = safe load in tons upon a pile.
W = weight of hammer in tons.
h = height of fall in feet.
p = penetration in inches under last blow.

*Journal Association Engineering Societies, June, 1903, p. 285.

†The various pile formulas are tabulated and discussed by Ernest P. Goodrich, in Transactions American Society of Civil Engineers, Vol. XLVIII, p. 180.

Then

$$P=\frac{2Wh}{p+1}$$

Mr. Worcester states with reference to spacing piles:

The minimum distance between centers of piles depends upon two factors: the hardness of the soil and the size of the butts. Ordinary spruce piles may be well driven 24 inches on centers, while large and long piles cannot be driven to advantage closer than 30 inches. Another governing condition must be taken into account, however, and that is the supporting power of the soil as a whole. Where the piles reach a real hard pan, the soil will generally resist all the pressure that the piles can bring on it, unless it consists of a thin crust overlying a soft material; but when the soil is so soft that the piles hold by friction only, and there is enough friction to carry all the soil between the piles down with them, in case they go together, the spacing becomes a question of how much the underlying soil will support per square foot. For example, if the soil can only support 2 tons per square foot, and the piles could each carry 18 tons, it is useless to place them closer than 3 feet on centers.

CONCRETE CAPPING FOR PILES

Although some authorities advocate stone capping for piles, even if the cost is more, it is generally considered good practise to lay the concrete directly upon the head of the pile. The ground is excavated to a depth of one or two feet around the piles, and if very soft, a layer of broken stone or chips may be spread and rammed hard upon it before laying the concrete. The load is distributed by the concrete, and the supporting power of the soil between the piles is utilized.

The thickness of the concrete above the piles must be sufficient to distribute the superimposed weight, and the reactionary load of the pile head acting upwards. If the layer is very thin there may be danger of the pile head shearing through the concrete. The objection sometimes raised to concrete capping is that the upward crushing stress upon the concrete by the head of the pile may be excessive, especially if loaded before the concrete is thoroughly hard. In considering this tendency, it must be borne in mind that under concentrated loading the concrete will sustain a higher stress per unit of area of contact than if the load is distributed. (See p. 249.)

DESIGN OF CONCRETE FOUNDATIONS AND FOOTINGS

The load upon a building foundation need not always be taken as the dead load plus the entire live load for which the superstructure is designed, because in most structures the full live load will never be imposed upon all the floors at the same time. A conservative suggestion for reduction in the live load is given on page 611.

To prevent cracks in a structure, it is not only necessary to select a proper unit pressure on the soil but also to see that this pressure is uniform, so that if there is settlement it will be the same throughout. To satisfy this condition, the center of the loads from the columns or other portions of the structure should coincide with the center of gravity of the base. The area of the footing should be proportional to its load. When such an arrangement is difficult, the foundations under different columns should be separated and the area of base of each be made proportional to the superimposed load.

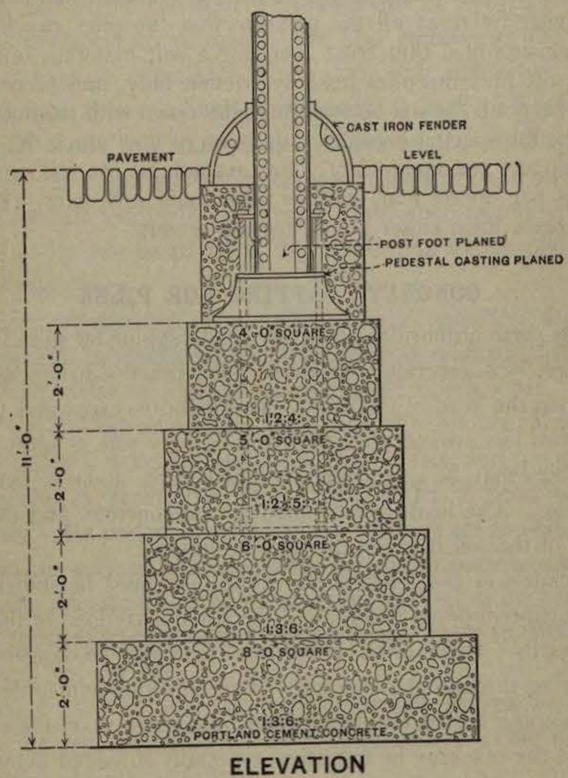

FIG. 202.—Typical Column Foundations of Boston Elevated Railway. (See p. 643.)

Frequently the building line nearly coincides with the property line and the foundation must be placed entirely inside the building. In such cases, to prevent eccentric pressure, either cantilever construction may be used for transmitting the exterior column loads centrally to the footing, or a combined footing designed as explained on page 647.

In structures such as chimneys or narrow buildings which are subject to wind pressure, the foundation should be designed with due consideration of the eccentricity caused by the wind.

With vertical loading upon rock or soil whose sustaining power per square foot is equal to or greater than the unit load, the dimensions of the foundation are fixed by the size of the structure, the safe load which can be sustained by the concrete, or by resistance to overturning. If the load is greater than an equivalent area of soil can sustain, the area of the base of the concrete must be enlarged, and the concrete battered or stepped or reinforced. It is a common engineering practice to make the length of the projections or steps of plain concrete one-half the height of the block, and this usually gives good results in buried foundations where the surrounding earth assists to prevent splitting.

The effect of concentrated loading must be considered when designing a footing. (See p. 367.) The pedestal bases for the Boston Elevated Railway were designed, when covering one-half the area of the concrete, with 25 % higher unit stresses for the concrete in actual contact than when covering the entire area. Fig. 202, page 642, shows a typical foundation for the columns.*

The following figures are suggested as conservative safe loads, when the surface of the concrete is larger than the loaded area. Lower stresses should be used with moving loads or when the area of the foundations is no greater than that of a column which it supports. The figures are based on ordinary concrete with a factor of safety of 4 at one month and a factor of 5½ at six months.

Safe Loads on Foundations.

Proportions of Concrete by volume†	Safe Loading Lb. per sq. in.	Safe Loading Tons per sq. ft.
1:1:3	700	50
1:2:4	650	47
1:2½:5	575	41
1:3:6	500	36

For a vibrating or pounding load these values should be reduced from ¼ to ½, depending upon the nature of the loading.

I-Beam Footings. Formerly, footings were made by imbedding steel I-beams, or in some cases old rails, in concrete for column footings. The concrete serves to distribute the loads and protect the steel. A typical footing, designed by Mr. John S. Branne,‡ is illustrated in Fig. 203, page 644. In this particular case the situation required a cantilever girder connecting this foundation with the next, but the footing shown is itself designed for a total load of 173 tons, of which 120 tons are dead load and 53 tons live load.

* George A. Kimball in Journal Association Engineering Societies, June, 1903, p. 351.
† Based on a barrel of packed cement of 3.8 cu. ft., weighing 376 lb. net.
‡ Journal Association Engineering Societies, Feb., 1901, p. 142.

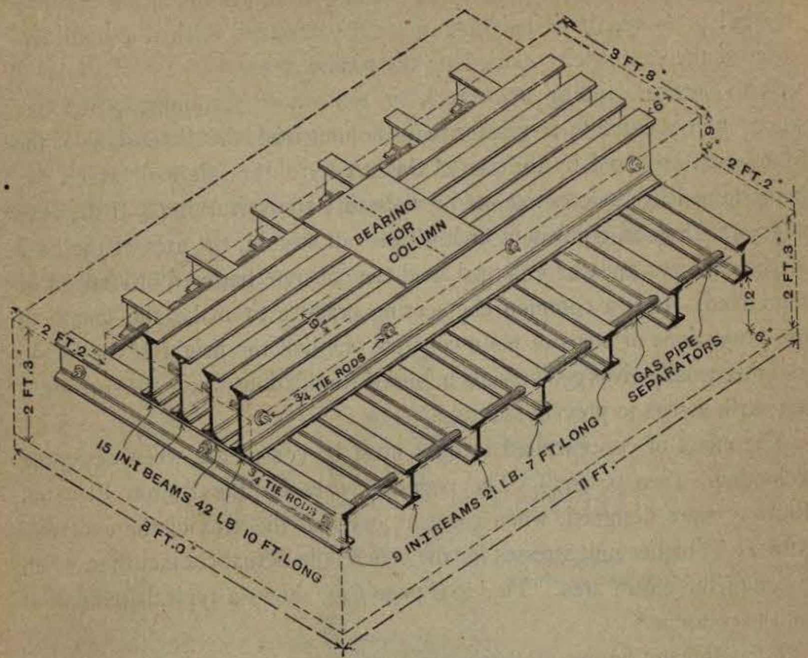

FIG. 203.—Concrete and I-beam Footing. (*See p.* 643.)

REINFORCED CONCRETE FOOTINGS

To distribute the load over a large area of soil without carrying the foundations, by successive steps, to a considerable depth and using a large mass of concrete, a single slab may be employed and reinforced with steel to prevent the projection breaking.

This in almost all cases permits a very great reduction in the quantity of material and reduces the cost. A footing reinforced with rods is designed to utilize the strength of the concrete, and is therefore more economical than the I-beam type of design just referred to, and is always to be preferred to it.

A reinforced concrete footing is really a flat slab and should be designed as such. The theory of the flat plate, explained on pages 483 to 488, therefore applies to it and the formulas on page 485 may be used directly for determining the bending moment. The principal formula for the maximum bending moment is as follows:

Let

M = maximum moment causing radial fiber stress

w = uniform distributed load on surface of the plate in pounds per square foot

r_0 = radius of base of column in feet

r_1 = radius of footing in feet

C_1, C_2 = constants to use in formula

Then

$$M = wr_0^2\,(0.2 + C_1 + C_2)$$

Values of the constants C_1, C_2 are found in the table page 518.

The application of the formula and principles is best illustrated in the example which follows.

Example 1. Find the dimensions of a footing for a column 28 inches square carrying 392000 pounds, when the allowable pressure on the soil is 2 tons per square foot?

Solution. The necessary area of footing is found by dividing the total superimposed load by the allowable unit pressure on the soil, or is $\frac{392\,000}{4000} = 98$ square feet, thus requiring an area 10 feet square. The footing may be considered as a flat slab loaded by the uniformly distributed upward pressure of the soil and fixed rigidly to the column. The formulas given above were deduced for a circular plate, but may be applied without appreciable error to a square footing. The radii to be used in the formulas are the averages of the radii of the circumscribed and inscribed circles.

$$r_1 = \frac{5 + 7.00}{2} = 6.00 \text{ ft.} \qquad r_0 = \frac{1.17 + 1.63}{2} = 1.4 \qquad \frac{r_1}{r_0} = 4.3$$

Using the formula and substituting for the constants values found from the table, page 518, corresponding to $\frac{r_1}{r_0} = 4.3$ and using as Poisson's ratio, $g = 0.1$ we have

$M = 4000 \times 1.4^2\,(0.2 + 6.7 + 3.49) = 81600$ ft. lb. per foot of width, which is equivalent to in. lb. per inch of width. For tension in steel, f_s = 16000; compression in concrete, $f_c = 650$; ratio of elasticity, $n = 15$; ratio of steel, $p. = 0.0077$; the constant from page 519, C is 0.096 and the depth of steel, (*p.* 418), $d = 0.096\sqrt{81600} = 27.2$ in.

The amount of steel will be found in the following manner. Find the area of steel required for the whole circumference of the inner circle of the plate, the radius of which at present is 1.4. Divide this amount by four and place it in two directions, at right angles, distributing it over an area slightly larger than the base of the column. Double the spacing of rods outside of the column, as the bending moment decreases very rapidly as shown in Fig. 204.

Circumference is $2 \times 1.4 \times 12 \times 3.1416 = 105.5$ in. Area of concrete, $A = 105.5 \times 27.2 = 2870$ sq. in. Area of steel, $A_s = 2870 \times 0.0077$ = 22.1 sq. in. Area of steel to be placed in one direction, $A = \frac{22.1}{4} = 5.53$ sq. in. The width of column is 28 in., hence six 1 in. square rods 5 in. on centers may be used. The spacing of the rods on the remaining area of the footing will be made 10 in. Deformed bars are advantageous because of increased bond strength.

Another method of arranging reinforcement is to place the bars in 4 layers, 2 of them diagonally.

The thickness of the footing may be decreased by judgment toward the edges without reducing its effective strength.

Shear Reinforcement. A footing to resist diagonal tension may require shear reinforcement of vertical stirrups or bent bars, placed near the column, where maximum shear occurs and diagonal cracks may be expected.

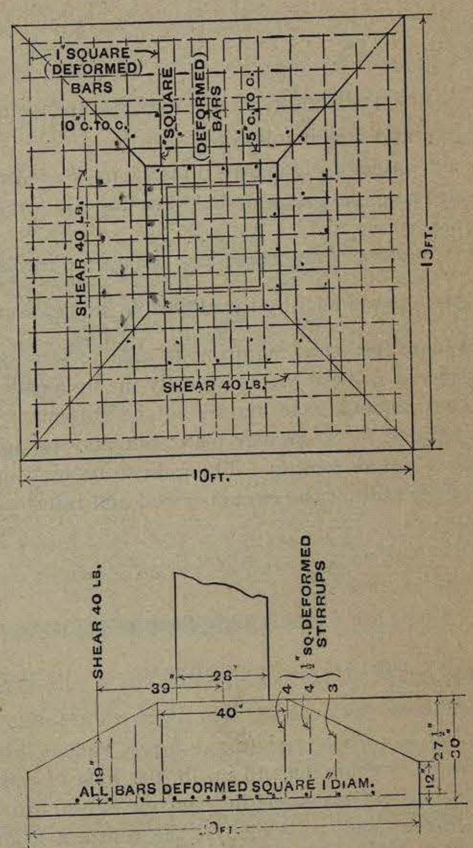

FIG. 204.—Square Footing. (See p. 645.)

While the action of the internal shearing stresses are somewhat complicated, the following plan may be adopted.

Find the unit shear at the edge of the column, dividing the loading tributary to the area of the footing outside of the column by the moment arm jd (which may be taken at 27×0.87) times the circumference 4×40 in. and we have

$$v = \frac{(100 - 5.5)\,4000}{4 \times 40 \times 27 \times 0.87} = 100$$

lb. per sq. in. Assuming that one-third of it is taken by concrete, 66 lb. per sq. in. must be provided for by steel.

Next find by trial the circumference where the unit shear is 40 lb. per sq. in., the amount which may be safely resisted by concrete, so that the shear to be provided for by the steel is zero. In this case this circumference has been found by trial to be distant 39 in. from the center of column, and is shown on the diagram by dot and dash line.

Now multiply the horizontal area enclosed between the circumference of the column and this newly found circumference by half of the previously determined unit shear to be taken by the steel (66 lb.) and obtain the total amount of shear to be taken by the stirrups. Select the diameter of stirrups in accordance with the discussion on page 453, divide the amount of shear by the area of one stirrup and by the unit tensile strength and obtain the number of stirrups. In this case the area requiring stirrups is $78^2 - 28^2 = 5300$ sq. in., and the total amount of shear to be provided for, $\frac{66}{2} \times 5300 = 174\,900$ lb. If $\frac{1}{2}$-inch square stirrups are selected (area 0.25 sq. in.) with a unit tensile strength,

$f_s = 16\,000$ lb. per sq. in., $\frac{174\,900}{16000 \times 0.25} = 44$ stirrups in all, or 11 stirrups each side are necessary. Their spacing is evident from the drawing.

Bond Stress. The bond stresses must also receive attention. It will often be necessary to increase the depth of the footing or the amount of rods to provide for the excessive bond stress. The discussion, page 456, and formulas given there find here a direct application. Reference may be made also to the similar treatment in the design of a retaining wall footing, page 670.

Combined Footings. Sometimes it is necessary to connect the footings of two or more columns. The design of such combined footing differs from that of a single one. When the loads carried by the columns are different, the footing to distribute the loading uniformly should have the shape of a trapezoid. The following example will illustrate method of figuring:

Example. Let P_1 and P_2 be respective loadings of colums I and II, 30 and 24 inches square; $P_1 = 400000$ lb., $P_2 = 580000$ lb. The distance between centers of columns is 15 ft. and the allowable unit pressure on the soil is 4 tons, 8000 lb., per square ft. Find the dimensions of footing. (See Fig. 205.)

Solution. The total superimposed load is 980 000 lb., then the necessary area of footing, $\frac{980\,000}{8000} = 123$ sq. ft. The magnitude of the parallel sides is unknown, and two equations are necessary for the determination. First equation may be obtained from the formula that the area of trapezoid equals the average of the sum of the parallel sides multiplied by its length. The length of the trapezoid is $15 + 1.75 + 1.50 = 18.25$ ft., and the area $= 123 = \frac{a + b}{2} \times 18.25$. Hence $a + b = 13.5$ ft. The second equation may be found from the requirement, that the center of gravity of the trapezoid coincide with the center of gravity of the combined column loading. The distance from A of the center of gravity of column loadings O_1 found by taking moments of loads, is 6.1 ft. and $l = 6.1 + 1.75 = 7.85$. Using the common equation for the center of gravity in a trapezoid gives $l = 7.85 = \frac{18.25}{3}\,\frac{a + 2b}{a+b}$

Solving the two equations for a and b, $a = 9.6$ ft., $b = 3.9$ ft.

To facilitate the finding of bending moments, the length of b, the width of the footing on the center of gravity line, may be computed from the relation $\frac{a - b}{18.25} = \frac{a - b_1}{7.85}$ and the length l_1, from the common formula for the distance of the center of gravity $b_1 = 7.15$ ft. $l_1 = 3.74$ ft. and $l_2 = 7.85 - 3.74 = 4.11$ ft.

Assuming the maximum moment* at center of gravity, $M = 580000 \times 6.1 \times 12 - \left(\frac{9.6 + 7.15}{2} \times 7.85 \times 8\,000 \times 4.11 \times 12\right) = 16\,550\,000$ in. lb. for the width of beam equal to b_1. The moment for one inch of width, $M = \frac{16\,550\,000}{7.15 \times 12} = 193\,000$ in. lb.

*Maximum moment is actually at section of zero shear but the error is inappreciable.

For tension in steel, $f_s = 16\,000$, compression in concrete, $f_c = 650$, ratio of steel, $p = 0.0077$, and $C = 0.096$ (see p. 421) then depth to steel, $d = 0.096\sqrt{193\,000} = 42.2$ in. As $A_s = 42.2 \times 12 \times 0.0077 \times 7.15 = 27.9$ sq. in. for the whole width, 18, 1¼ inch deformed bars will be used.

To prevent bending of the projections of the footing, transverse reinforcement will be introduced. The projections are assumed to act as cantilevers, loaded by half of the column loading multiplied by a ratio of the difference between the width of the footing, a, and the diameter of the column to the

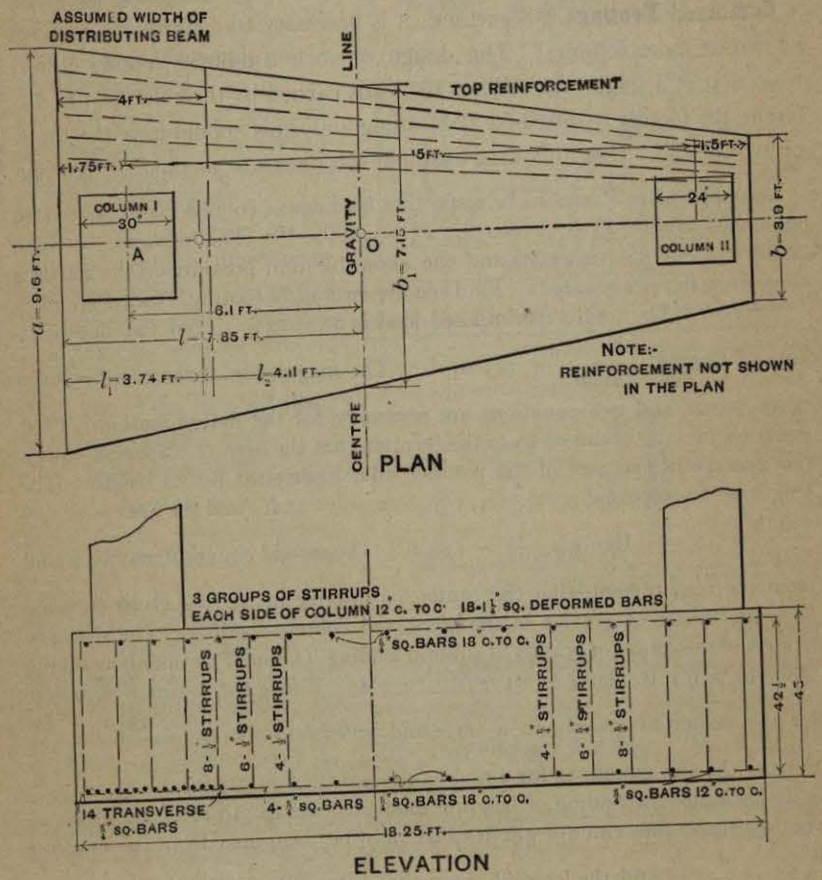

Fig. 205.—Combined footing. (*See p.* 647).

width of footing, or $\frac{9.6 - 2.5}{9.6}$. The moment arm equals half of the length of the projection, and the moment, $M = \frac{580\,000}{2} \times \frac{9.6 - 2.5}{9.6} \times \frac{3.55}{2} \times 12$ $= 4\,570\,000$ in. lb.

Assuming a width of the distributing beam equal to 4 ft., the depth will be

$d = 0.096\sqrt{\frac{4\,570\,000}{4 \times 12}} = 29.6$ in. The depth is smaller than the depth of the whole slab. Since a larger depth is used, the percentage of steel will be found as follows:

$C = \sqrt{\frac{42.2^2 \times 48}{4\,570\,000}} = .137$ from formula (1) (p. 418) and $p = 0.0033$ from table on page 520; $A_s = 42.2 \times 48 \times 0.0038 = 7.70$ sq. in., hence 13⅞ in. square bars will be used.

Note.—The required depth of the distributing beam may be sometimes larger than the depth of the whole slab. In such case the footing may be either thickened under the column or steel introduced at the top and bottom. The latter scheme should be adopted only when additional excavation for the beam cannot be made readily.

In a similar way the distributing reinforcement for column II is found.

$M = \frac{400\,000}{2} \times \frac{3.9 - 2}{3.9} \times \frac{0.95}{2} \times 12 = 555\,000$ in. lb. Assume a width of imaginary beam equal to 3 ft., then $d = 0.096\sqrt{\frac{555\,000}{36}} = 12$ in. As larger depth is used, the percentage of steel will be found as in previous case.

$C = \sqrt{\frac{42.2^2 \times 36}{555000}} = .334$. For this value of C less than 0.1% of steel is needed, and will be taken arbitrarily.

SPREAD FOOTINGS

When the allowable pressure on the soil is very small or when the building is supported by piles sustained by friction, it may be necessary to spread the foundation over the whole area of the building, either using a thick mass of plain concrete or a thinner slab of reinforced concrete design as a flat plate, or a beam and slab system.

Flat Slab Foundations. A flat slab may be designed by the method of flat plates explained on pages 483 to 487. The slab is considered as an inverted flat plate loaded by the reaction of the ground and supported by the columns.

Special provision should be made in the design where there is unequal loading.

Since the distributed pressure acts upward, the bottom of the plate under the columns is in tension and the top of the plate between the columns; hence the steel must be in the bottom of the slab under the columns, and should be bent up to the top of the slab between columns. The column base must be large enough to prevent excess loading or too great moments and shears in the concrete.

Beam and Slab Foundation. For a combination of beams and slabs the principles of floor design are followed except that the distributed load acts upward. The beams or ribs may be built either above or below the slab, the former method permitting a T-beam design, but, on the other hand, requiring an extra fill and separate floor surface in the basement. The formulas and discussion relating to floor design in Chapter XXI apply.