and the thickness of the arch, h, is 30 inches, the value of C_e for 0.8% steel (that is, for $p = 0.008$) is 1.38 and $f_c = \frac{1.38 N.}{b\,h}$

Distribution of Stress When One Surface is in Tension. When the thrust is applied at a distance from the gravity axis with eccentrictiy, e, greater than that given for e_0 by formula (49), page 567, and the concrete is *assumed unable to carry any tension*, the above general formulas are not easily applied and the following method may be used. Here the steel on the side opposite to that on which the thrust acts is designed to carry all the tensile stresses. In this case having a section with a bending moment and thrust, there are three unit stresses to be determined, namely, maxi-

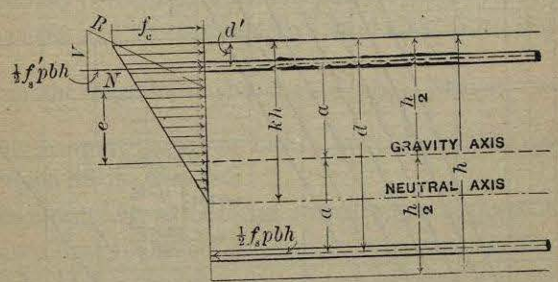

Fig. 178.—Stresses Caused by a Force Producing Compression and Tension upon a Reinforced Section, Tensile Strength of Concrete Neglected. (*See p.* 570.)

mum unit compression in concrete, maximum unit compression in steel, and maximum unit tension in steel. The method of procedure is similar to that used for beams in Appendix II, page 757. Referring to Fig. 178, the unit stress in the upper steel, as shown by inspection, is

$$f'_s = nf_c\left(1 - \frac{d'}{kh}\right) \qquad (53)$$

The unit tension in the lower steel is

$$f_s = nf_c\frac{d - kh}{kh} \qquad (54)$$

Hence, when the compression in concrete, f_c, is known, the stresses in the steel are determined by the above formulas. Since the sum of the stresses

n = ratio elasticity. N = thrust. fc = compression in concrete. fs = tension in steel f'_s = compression in steel. b = breadth. h = height. p = ratio of steel. k = ratio depth neutral axis. d = depth tension steel. d' = depth compression steel. R = resultant of forces.

acting on the section must be equal to the thrust, we have, since each steel area is $\frac{pbh}{2}$

$$N = \frac{f'_s\,pbh}{2} + \frac{f_c\,bkh}{2} - \frac{f_s\,pbh}{2} \qquad (55)$$

Placing values of f'_s and f_s from (53) and (54) in (55),

$$N = \frac{f_c\,bh}{2}\,\frac{k^2 + 2\,npk - np}{k} \qquad (56)$$

The moment of the stresses about the gravity axis, which is obtained by taking the sum of the moments of all the stresses about the gravity axis and eliminating f'_s and f_s by use of equations (53) and (54), is

$$M = f_c\,bh^2\left[\frac{npa^2}{h^2k} + \frac{k}{4} - \frac{k^2}{6}\right] \qquad (57)$$

Designating by C_a the quantity $\left[\frac{npa^2}{h^2k} + \frac{k}{4} - \frac{k^2}{6}\right]$ from equation (57) we may write

$$M = C_a f_c bh^2 \qquad (58)$$

Hence in investigating a given section of an arch, if M, b, h, C_a are known, the unit compression in the concrete is

$$f_c = \frac{M}{C_a bh^2} \qquad (59)$$

To solve this equation easily, values of C_a should be taken from curves. Fig. 180, page 573, gives values of C_a for $n = 15$, $2a = \frac{4}{5}h$ and various values of k.

Evidently before using equations (57) or (59) to find the unit compression in the concrete, the position of the neutral axis must first be determined. To do this we must find the value of k. Since the moment, $M = Ne$, that is, the thrust multiplied by the eccentricity, equation (56) may be multiplied by e and equated to (57). From this process, the following equation containing k is obtained

$$k^3 + 3\left(\frac{e}{h} - \frac{1}{2}\right)k^2 + 6\,npk\frac{e}{h} = 3\,np\frac{e}{h} + \frac{6\,npa^2}{h^2} \qquad (60)$$

M = moment. n = ratio elasticity. N = thrust. fc=compression in concrete. fs=tension in steel. f'_s = compression in steel. b = breadth. h = height. e = eccentricity. p = ratio of steel. k = ratio depth neutral axis. C_a = constant.

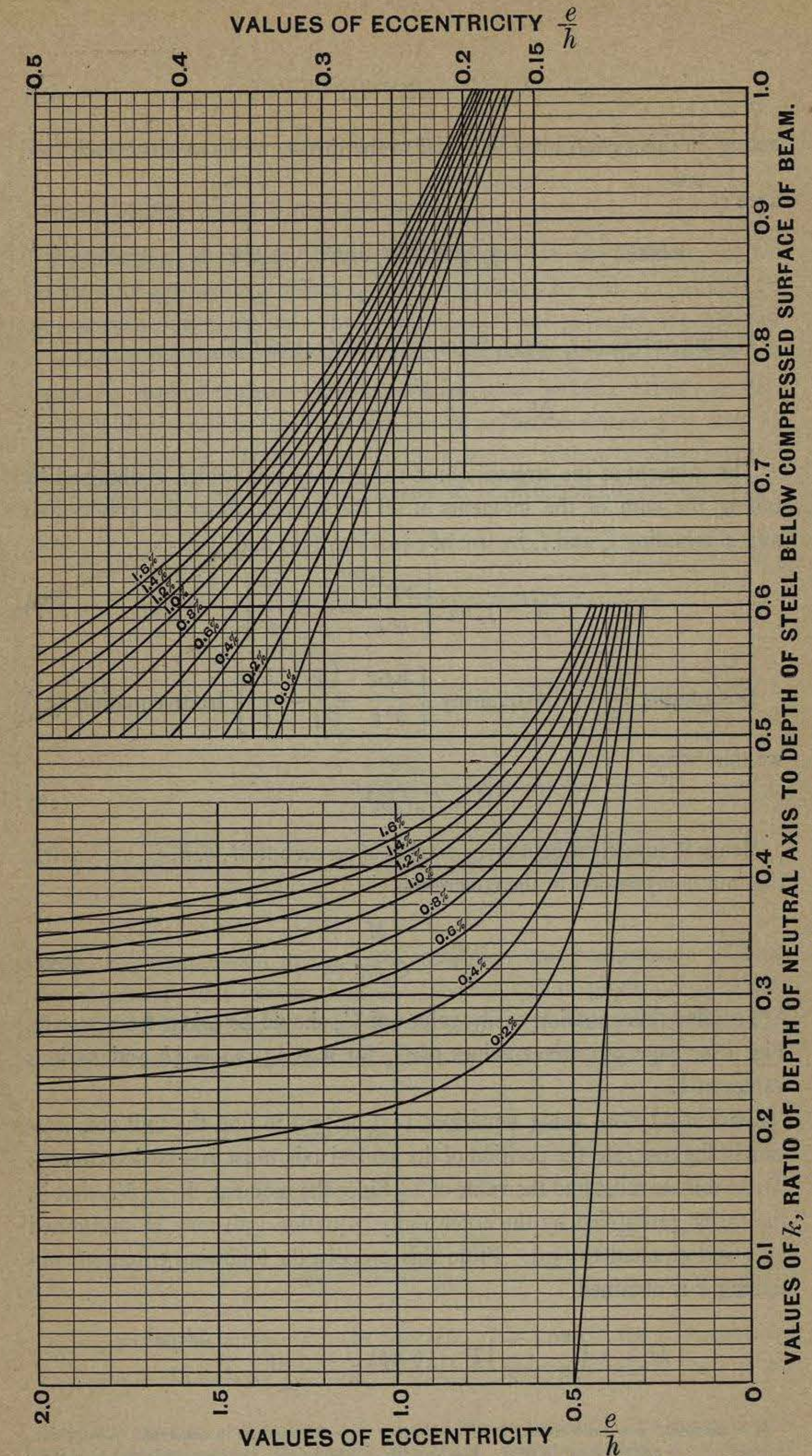

FIG. 179.—Diagram for Determining Depths of Neutral Axis for Different Eccentricities. Based on $n = 15$ and $2a = \frac{4}{5}h$. (See p. 574.)

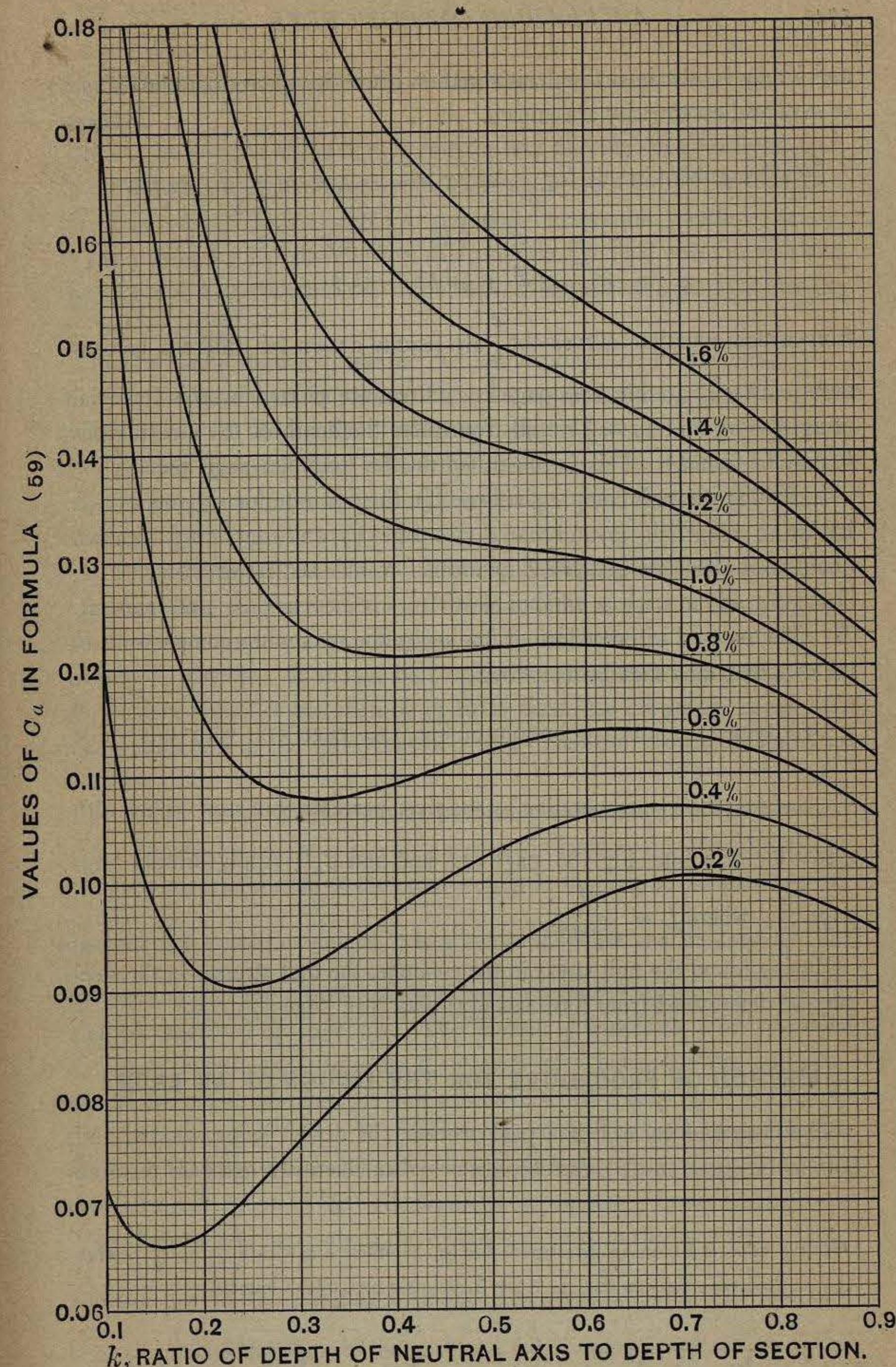

FIG. 180.—Diagram for Determining Constants C_a to be used in Formula (59). Based on $n = 15$ and $2a = \frac{4}{5}h$. (See p. 571.)

and k may be obtained from this equation if the size of section, percentage of steel and eccentricity are known.

By solving this formula for $\frac{e}{h}$, using $n = 15$ and $2a = \frac{4}{5}h$, we have

$$\frac{e}{h} = \frac{-k^3 + \frac{3}{2}k^2 + 14.4p}{3k^2 + 90pk - 45p} \qquad (61)$$

from which equation, curves for $\frac{e}{h}$ are readily drawn for different percentages of steel. In Fig. 179, page 572, curves are plotted by this formula, using $n = 15$ and $2a = \frac{4}{5}h$, and from these the depth of the neutral axis k, may be found.* This is illustrated in the example, page 580.

In finding the unit compressive stress in the concrete for a given section having an eccentricity greater than e_0 (see page 567) and containing a known quant.ty of steel, he following quantities would be known: breadth, b; depth, h; ratio of steel, p; ratio of elasticity, n; eccentricity, e; and moment, M. The method of procedure of finding f_c, the maximum compression in the concrete, may then be as follows.

Determine $\frac{e}{h}$. Enter the bottom of Fig. 179, page 572, with this value of $\frac{e}{h}$ and find the k corresponding for the given percentage of steel. Then with this value of k enter Fig. 180, page 573, and find C_a. Apply formula (59), page 571, where $f_c = \frac{M}{C_a bh^2}$.

Having found the unit stress in the concrete, the unit stresses in the steel may be determined from formulas (53) and (54), page 570.

METHOD OF PROCEDURE FOR THE DESIGN OF AN ARCH.

The design of an arch is a trial process; the design being selected and then investigated to see if the sections are of sufficient strength. If the arch

*If the value of k must be determined directly, substitute $k = z - \left(\frac{e}{h} - \frac{1}{2}\right)$ when equation (60) takes the form $z^3 + pz + q = 0$, and since by Cardan's formula,

$$z = \sqrt[3]{-\tfrac{1}{2}q + \sqrt{(\tfrac{1}{2}q) + \left(\frac{1}{3}p\right)^3}} + \sqrt[3]{-\tfrac{1}{2}q - \sqrt{(\tfrac{1}{2}q)^2 + \left(\frac{1}{3}p\right)^3}}$$

the value of k may be computed. This follows the method suggested by Professor Mörsh in "Der Eisenbetonbau," 1906, p. 111.

h = height. e = eccentricity. p = ratio of steel. k = ratio depth neutral axis.

first chosen is too large or too small it must be revised and the process repeated.

Since the location of the line of pressure and also the stresses are affected by the loading, it is customary either to compute the arch for the dead load plus concentrated loads located at the most unfavorable positions, or else to compute it for the dead load plus a uniform live load covering one-half the arch and also covering the entire arch, to see that the working stresses are not exceeded.

The following steps indicate the method of procedure for the design of a highway bridge shown in folding Fig. 181, opposite page 581. The computations are for the live load over one-half the span. The procedure is similar when the entire span is loaded.

1. Lay out on a drawing the preliminary curve assumed for the intrados. (See p. 540).

2. Assume a crown thickness in accordance with the formula on page 541.

3. Lay out the curve of the extrados and the surface of the roadway. The extrados may be a 3-centered curve, but it is better to use an arc of a circle if possible. It should be so placed as to give a ring thickness at the quarter points of the span of $1\frac{1}{4}$ to $1\frac{1}{3}$ times the crown thickness, and a ring thickness at the springings of 2 or 3 times the crown thickness in this first trial.

4 Draw the arch axis midway between the extrados and the intrados.

5. Divide the arch axis into distances such that the ratio of each distance to the moment of inertia of the cross-section of the ring at the center of the distance is a constant; that is, $\frac{s}{I}$ is a constant. This can be done by trial by beginning at the crown and working towards the springings or by the method described on page 554. The *moment of inertia is of the combined section of concrete and steel about the gravity axis*, hence the size and position of the steel rods must be first assumed, when I may be computed by the formula on page 565. The ratio of area of steel to total area-of section at crown may be arbitrarily taken in the first place from 0.007 to 0.0125, that is from 0.7% to $1\frac{1}{4}$%. The divisions are separated by vertical sections.

In the problem here solved the distance, s, next to the crown is 1.14 ft., and that next to the springing is 7.82 ft. The constant ratio, $\frac{s}{I}$ *for this arch* is 11.4* On folding Fig. 181 the centers of the divisions are shown by circles and are numbered 1, 2, 3, etc. All distances are in feet and all quantities

*Greater accuracy may be obtained by using a larger number of divisions than here chosen, and also by subdividing loads P_1 and P_{20}.

involving distance are in foot units. A section of the arch 1 foot wide transversely is considered.

6. Compute the dead and live loads and enter these loads as indicated by P_1 P_2, etc., at the center of gravity of each division. In the accompanying design, a live load of 100 pounds per square foot covers the right half span, while on the left is the dead load alone of the masonry taken at 150 pounds per cubic foot plus the earth fill taken at 100 pounds per cubic foot.

TABLE 1. *Ordinates and Moments in Computation of Example*

Points	x	y	x^2	y^2	M_L	M_R	$M_L x$	$M_R x$	$M_L y$	$M_R y$
10 and 11	0.56	0.01	0.3	0.00	00	00	00	00	00	00
9 and 12	1.71	0.04	2.9	0.00	391	521	668	891	16	21
8 and 13	2.88	0.11	8.3	0.01	1 205	1 603	3 470	4 616	132	176
7 and 14	4.11	0.23	16.9	0.05	2 520	3 346	10 357	13 752	580	770
6 and 15	5.43	0.39	29.5	0.15	4 471	5 923	24 277	32 162	1 743	2 310
5 and 16	6.89	0.63	47.5	0.40	7 327	9 672	50 483	66 640	4 616	6 093
4 and 17	8.57	0.97	73.5	0.94	11 584	15 216	99 275	130 401	11 237	14 759
3 and 18	10.59	1.50	112.2	2.25	18 242	23 791	193 183	251 947	27 363	35 686
2 and 19	13.17	2.39	173.5	5.71	29 480	380 45	388 252	501 053	70 457	90 928
1 and 20	17.94	5.14	321.8	26.41	58 553	74 192	1 052 235	1 331 004	301 476	381 347
Σ	71.85	11.41	786.4	35.92	133 873	172 309	1 822 200	2 332 466	417 620	532 090

All distances in foot-units; all moments in foot-pounds

Values of H_c, V_c and M_c at crown for Live and Dead Loads.

$$H_c = \frac{10\,(417\,620 + 532\,090) - 11.41\,(133\,873 + 172\,309)}{2\,[10 \times 35.92 - (11.41)^2]} + 13{,}107 \text{ lb.}$$

$$V_c = \frac{1822200 - 2332466}{1573} = -\,324 \text{ lb.}$$

$$M_c = \frac{172\,309 + 133\,873 - 2 \times 13{,}107 \times 11.41}{20} = +\,354 \text{ ft. lb.}$$

Values of H_c and M_c at crown for Rise in Temperature.

$$H_c = \frac{1}{11.4}\,\frac{.0000055 \times 20 \times 41.88 \times 10 \times 2000000 \times 144}{2[10 \times 35.92 - (11.41)^2]} = 2545 \text{ lb.}$$

$$M_c = \frac{-\,2545 \times 11.41}{10} = -\,2900 \text{ ft. lb.}$$

Values of H_c and M_c at crown for Rib Shortening.

$$H_c = -\,\frac{1}{11.4}\,\frac{66 \times 41.88 \times 10 \times 144}{2[10 \times 35.92 - (11.41)^2]} = -\,760 \text{ lb.}$$

$$M_c = -\,\frac{-\,760 \times 11.41}{10} = +\,870 \text{ ft. lb}$$

The horizontal components of the earth pressure are so small that they are neglected, except that, for purposes of illustration, they are shown in the case of the load adjoining each springing, where the horizontal components are computed by formulas for earth pressure on page 666. The point of application of the horizontal and vertical components, as shown for P_1, is taken at the arch axis. In practice, earth pressure is negligible in the design of flat arch rings of the type here selected, and all loads may be taken as vertical. Only where the ratio of rise to span is large need the horizontal components of the earth pressure be considered.

7. Make a table similar to Table 1, page 576. The values of x and y are scaled from the drawing, and are the coordinates of the *center* points of the divisions of the arch axis. The *crown* point of arch axis is here taken as the *origin of coordinates*. The values of M_L and M_R are computed. M_L represents the moment at each of the center points 1 to 10 inclusive of all *loads* lying between the point in question and the crown. Thus M_L for point 10 is 0; for point 9, $M_L = 340 \times 1.15 = 391$ ft. lb.; for point 8, $M_L = 391 + 696 \times 1.17 = 1205$ ft. lb., and so on. The moment at each "center" point being obtained from that at each preceding "center" point. M_R of course represents the moment at each of the center points 11 to 20 inclusive of all *loads* lying between the point in question and the crown. For a symmetrical loading M_L would equal M_R for each pair of center points, such as 1 and 20.

8. Compute H_c, V_c, M_c, that is, the thrust, shear and moment at the crown, as on page 576, by using equations (16), (17), and (18), page 553. If the sign of V_c is plus the line of pressure (equilibrium polygon) at the crown slopes upward towards the left; if minus, as in the present case, upwards toward the right. A plus sign for M_c indicates a positive moment; a minus sign, a negative moment at the crown. For the arch in folding Fig. 181, the crown thrust $H_c = 13107$ pounds, $V_c = -\,324$ pounds and $M_c = +\,354$ ft. pounds.

9. Draw a force polygon as shown in folding Fig. 181 by laying off to scale the loads P_1, P_2, etc., as 0 − 1, 1 − 2, etc. Find the pole by laying off V_c downward (because negative) from the crown point, 10, and then laying off H_c horizontal. The hypothenuse of the triangle having H_c and V_c for sides thus slopes upward to left or upward to right, according as V_c is + or −.

10. Draw the equilibrium polygon as shown on the arch of folding Fig. 181. The resultant pressure acts above the axis at the crown a distance, $\frac{M_c}{H_c} = e$ if M_c is plus, and below by the same amount if M_c is minus. Since here, as is shown later, $e = +0.028$ feet, this distance is laid off vertically above the axis at the crown and through this point the resultant pressure is drawn parallel to the ray O_{10} of the force polygon and so on. It is not really necessary to draw the equilibrium polygon if the moments and eccentricities are computed for the various sections as outlined under item 11, but the polygon, which is the line of pressure, affords a good check on the algebraic work.

11. Determine the moment, thrust, and eccentricity, and if desired the shear at the center points, 1, 2, 3, etc., of the divisions, and enter in a table as shown below. The moment is computed from formulas (19) and (20) on page 554, the values of whose terms have already been found by items 7 and 8. The thrust and shear may be scaled from the force polygon. For example, at section 1 on folding Fig. 181 the thrust line is drawn parallel to the tangent to the axis at 1, and the shear line at right angles to the thrust line. The eccentricities, e, of the sections 1, 2, 3, etc., are computed by dividing the moment on the section (see page 561) by the thrust for that section just scaled. For positive moments and therefore positive values of e, the line of thrust lies above the arch axis.

12. Compute the thrust and moment at the crown due to variation in temperature by formulas (25) and (26), page 556, the moments on the

TABLE 2. *Final Moments and Thrusts*

	LIVE AND DEAD					TEMPERATURE		RIB SHORTENING	
Point	$H_c y$	$V_c x$	Mom.	Thrust	Ecc.	Mom.	Thrust	Mom.	Thrust
1	67370	−5812	+3259	+14360	+0.23	±10180	±1970	−3030	−610
2	31325	−4267	−2068	+14000	−0.15	± 3180	±2310	− 950	−700
3	19660	−3431	−1659	+13920	−0.12	± 910	±2430	− 270	−730
4	12713	−2777	−1293	+13600	−0.10	± 440	±2500	+ 130	−740
7	3014	−1331	− 483	+13240	−0.04	∓ 2320	±2530	+ 690	−760
9	524	− 554	− 67	+13160	−0.005	∓ 2800	±2545	+ 840	−760
12	524	− 554	+ 911	+13120	+0.07	∓ 2800	±2545	+ 840	−760
14	3014	−1331	+1353	+13200	+0.10	∓ 2320	±2530	+ 690	−760
17	12713	−2777	+ 627	+13640	+0.05	± 440	±2500	+ 130	−740
18	19660	−3431	− 346	+14040	−0.03	± 910	±2430	− 270	−730
19	31325	−4267	−2099	+14200	−0.15	± 3180	±2310	− 950	−700
20	67370	−5812	− 656	+14840	−0.04	±10180	±1970	−3030	−610

Thrusts in lb. Moments in ft. lb. Shear in arch design is small and need not be computed.

various sections by formula (27), page 557, and the thrusts and shears by resolving the crown thrust into tangential and radial components, as shown in the small force polygon in the diagram.

A rise in temperature of 20 degrees Fahr., and a fall of the same amount, is sufficient even in the northern part of the United States for arches with filled spandrels.

For the arch shown on folding Fig. 181 the crown thrust H_c, due to temperature, is a tension of 2545 lbs., and a *compression* of equal amount. The crown moment M_c is + 2900 ft. lb. and − 2900 ft. lb.

13. The effect of rib shortening due to the thrust is comparatively slight. Where necessary to compute it, use formula (31) and (32), page 558. (See p. 576.)

For the problem here shown the thrust at crown due to this cause is − 760 lb., and the moment is +870 ft. lb.

14. Having prepared a table similar to Table 2, page 578, showing

thrusts and moments on the various sections 1, 2, 3, etc., due to dead and live loads, temperature, and rib shortening, compute the maximum unit compression in the concrete and maximum unit tension, if any, in the steel by use of formulas on pages 565 to 574.

Table 2 shows thrusts and moments for only a few of the sections of this arch, since it is unnecessary to compute all of them. A selection of the more critical sections may be made by inspection of the equilibrium polygon. The following shows the computation of the maximum unit stresses at the crown for the arch in folding Fig. 181, as outlined in items 11 to 13.

LIVE AND DEAD LOADS AND RIB SHORTENING.

Moment	Thrust	
+ 354	+ 13107	Live and dead
+ 870	− 760	Rib shortening
+ 1224 ft. lb.	+ 12347 lb.	

$$e = \frac{M}{N} = \frac{1224}{12347} = 0.1 \text{ ft.}$$

p = ratio of steel at crown = 0.0092

Consulting lower right hand part of Fig. 177, page 569, it is seen that the value of $\frac{e_0}{h}$ for 0.92% is *greater* than $\frac{e}{h} = 0.1$. Hence there is *compression over the entire section.*

From formula (42), page 565, max. compression in concrete,

$$f_c = \frac{12347}{1 \times 1}\left[\frac{1}{1 + 15\,(.0092)} + \frac{6\,(1)\,0.1}{(1)^2 + 12\,(15)\,.0092\,(\frac{1}{3})^2}\right]$$

= 17100 lb. per sq. ft.

= 119 lb. per sq. in.

Stresses in steel need not be computed.

The above may be more quickly solved by the use of the curves on the left part of Fig. 177, page 569.

LIVE AND DEAD LOADS AND RIB SHORTENING PLUS TEMPERATURE.

Moment	Thrust	
+ 1224	+ 12347	
+ 2900	− 2545	Temp.
+ 4124 ft. lb.	+ 9802 lb.	

$$e = \frac{M}{N} = \frac{4124}{9802} = .42 \text{ ft.}$$

Consulting lower right hand part of Fig. 177, page 569, it is seen that the value of $\frac{e_0}{h}$ for 0.92% of steel is much *smaller* than $\frac{e}{h} = \frac{0.42}{1} = 0.42$. Hence there is *tension over a part of the section.*

From formula (60), page 571, the value of k is found to be 0.6. From formula (59), page 571, the value of the maximum compression = 35 700 pounds per square foot = 248 pounds per square inch. From formula (54), page 570, maximum tension in steel = 1440 pounds per square inch.

The approximated value of above compression in concrete may be more quickly found by the use of curves, Fig 179 and 180, and pages 572 and 573 as shown below.

f_c = compression in concrete. e = eccentricity. M = moment. N = thrust. h = height. k = ratio depth neutral axis. a = distance centre of gravity to steel.

The method of computation for other points in the arch is similar, and stresses should be determined at sections where they appear to be the maximum.

From table 2 it is evident that although at point 20 the moment due to dead and live load is very small, its combination with moments due to temperature and rib shortening makes it one of the critical points. The moment and thrust due to live and dead load and rib shortening is
$M = -656 - 3030 = -3636$ ft. lb. and $N = 14840 - 610 = 14230$ lb.
Hence, $e_0 = \frac{3686}{14230} = 0.26$ ft., for $h = 1.97$, $\frac{e_0}{h} = 0.13$, $p = 0.0037$.

Inspecting the lower part of Fig. 177, page 569, it is seen that the whole section is in compression. From the same diagram for $\frac{e}{h} = 0.13$ and $p = 0.0037$, $C_e = 1.65$. Using formula (52), page 568, $f_c = \frac{14230 \times 1.65}{1.97 \times 12 \times 12} = 83$ lb. per sq. in.

Combine now the moment and thrust due to live and dead load with those due to temperature and obtain $M = -(10180 + 3686) = -13866$ ft. lb., $N = -1970 + 14230 = 12260$ lb., $e = 1.13$ ft. $\frac{e}{h} = 0.57$.

In Fig. 179, page 572, $k = 0.37$ corresponds to $\frac{e}{h} = 0.57$. By locating this value of k in Fig. 180, the constant $C_a = 0.094$ is obtained, which substituted in formula (59), page 571, gives $f_c = \frac{13866}{1.97 \times 1.44 \times 0.094} = 520$ lb. per sq. in. The stress in steel from formula (54) is $f_s = 15 \times 520 \frac{1.67 - 0.37 \times 1.97}{0.37 \times 1.97} = 10000$ lb. per sq. in.

Similar computations should be made for all critical points and when the stresses are either too small or too large, the dimensions or even the shape of the arch must be changed. Small changes may be made without refiguring the whole arch. For larger changes, all computations should be repeated and a new line of pressure determined.

LOADINGS TO USE IN COMPUTATIONS

The usual practice is to make two sets of computations; in the first place, proportion the arch ring for a live load covering the entire span and then for one covering only one-half the span. These two loadings are approximations, more or less exact, to the true loadings which produce the maxi-

FIG 181. EXAMPLE OF ARCH DESIGN
(See pp. 574 to 580)