An increase and a decrease of 20 degrees Fahr. is probably a sufficient allowance for concrete arches with filled spandrels. For arches with open spandrels the range in temperature of the concrete is somewhat less than that of the surrounding air. For example, in the latter case with a range of temperature of the air from −20 degrees to +100 degrees Fahr., the range for arch computation should be taken at least 40 degrees on each side of the mean temperature.

The methods of combining the temperature moments and thrusts with those due to loads is illustrated in the example, page 579.

EFFECT OF RIB SHORTENING DUE TO THRUST

The thrust acting throughout the arch ring tends to cause a shortening of the span, which, if f is average compression (obtained by averaging values in computation of ring) for unit area, $= \frac{f\,L}{E} = \Delta_L$

Hence

$$H_c = -\frac{I}{s}\,\frac{f\,Lm}{2\,[m\,\Sigma y^2 - (\Sigma y)^2]} \tag{31}$$

and

$$M_c = -\frac{H_c\,\Sigma y}{m} \tag{32}$$

and, as in temperature stresses,

$$M_s = M_c + H_c y \tag{33}$$

The effect of the rib shortening is similar to a fall in temperature.

All the summations above are for one-half the span only. m = number of divisions in one-half of the arch axis.

The effect of rib shortening is slight in many cases but in a flat arch it may be considerable.

DISTRIBUTION OF STRESS OVER CROSS SECTION

The analyses of stress distribution which follow apply not only to an arch but also to any section where there is combined compression and bending.

In an arch, having determined the thrust, shear and bending moment at any given section of the arch ring, the distribution of stress upon the section must be next investigated in order to compute the maximum stresses

M = moment. H_c = crown thrust. m = number divisions of half axis. s = short length of arch axis. I = moment inertia. L = span. y = coördinate of a point. f = compression in concrete.

in the concrete and the steel to see on the one hand that they do not exceed safe working loads, and on the other hand that the design is as economical as possible.

Concrete is strong in resistance to direct shear (Chap. XIX), and hence the shear is generally negligible in concrete and reinforced concrete arches, although it should be considered in stone masonry arches. Since, also, as will be shown, the bending moment is the thrust times its eccentricity, it follows that the determination of the thrust, which is the normal component of all the forces acting, together with the location of its center of pressure, permit the determination of the stresses required in designing any section of an arch or of any section of any member subjected to eccentric stress. Every section of the arch or of a beam or of a column must be of such dimensions or with such reinforcement that the safe working stresses in the concrete shall not be exceeded.

Plain concrete sections and reinforced concrete sections are considered separately, the same notation being used for both.

Notation

Let

R = resultant of all forces acting on any section.
f_c = maximum unit compression in concrete.
f'_c = maximum unit tension in concrete or minimum compression.
N = thrust, a component of the forces normal to the section.
V = shear, the component of the force R parallel to the section.
b = breadth of rectangular cross section.
h = height of rectangular cross section.
e = eccentricity, that is, the distance from gravity axis to the point of application of the thrust which is the intersection of the line of pressure with the plane of the section.
M = bending moment on the section.
y = perpendicular distance from gravity axis to any point in the section.
I = moment of inertia of entire cross section of concrete about the horizontal gravity axis.
I_s = moment of inertia of cross-section of steel about the horizontal gravity axis.
A_c = total area of section of concrete.
A_s = total area of section of steel.
y_1 = perpendicular distance from gravity axis of unsymmetrical section to outside fiber having maximum compression.

y_2 = perpendicular distance from gravity axis of unsymmetrical section to outside fiber having maximum tension or minimum compression.

f'_s = maximum unit compression in the steel.

f_s = maximum unit tension or minimum unit compression in the steel.

p = ratio of steel to total area of section; for rectangular sections p = ratio of steel area to bh.

n $= \dfrac{E_s}{E_c}$ = ratio of moduli of elasticity of steel and concrete.

k = ratio of depth of neutral axis to depth of beam h.

kh = distance from outside compressive surface to neutral axis.

d' = depth of steel in compression.

d = depth of steel in tension.

a = distance from center of gravity of symmetrical section to steel.

e_0 = value of eccentricity which produces zero stress in concrete at outer edge of rectangular section opposite to that on which thrust acts.

C_a, C_e = constants.

DISTRIBUTION OF STRESSES IN PLAIN CONCRETE OR MASONRY ARCH SECTIONS

In designing plain concrete or stone masonry arches, the maximum compressive stresses must be kept within the safe working compressive strength of the material, and the point of application of the thrust must not lie outside of the middle third of the section. When investigating an existing structure, however, it may be found that the thrust acts outside of the middle third, so that a determination of the stresses in such cases must also be considered.

Plain Arches with Rectangular Cross Section. In plain concrete or stone masonry arches of rectangular cross-section there are five special cases depending upon the point of application of the thrust, as follows:

(*a*) Thrust acting at gravity axis of cross-section.

(*b*) Thrust not acting at gravity axis of cross-section, but within the middle third of the section.

(*c*) Thrust acting at edge of middle third of the section.

(*d*) Thrust acting outside of the middle third of the section and material *able* to carry tension.

(*e*) Thrust acting outside of the middle third of the section and material *not able* to carry tension.

Each of these cases will be considered.

(*a*) When the thrust acts at the gravity axis of the cross-section the stress is compression over the entire section and is uniformly distributed as in Fig. 167.

Maximum compression in concrete is

$$f_c = \frac{N}{bh} \qquad (34)$$

(*b*) The thrust acts within the middle third but not at the gravity axis, as shown in Fig. 168. When the thrust acts at any other point than the gravity axis there is combined bending moment and direct stress to be considered. If the bending moment is positive, the thrust acts above the gravity axis; if negative, the thrust acts below. In either case the thrust produces a unit compression of $\frac{N}{bh}$ over the entire section. The moment causes compression on the side of the axis where N acts and tension on the

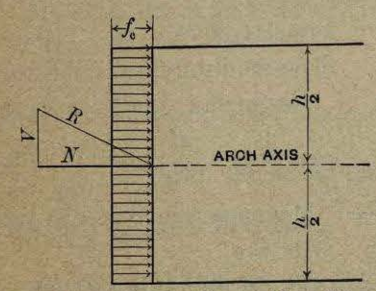

Fig. 167.—Stresses Caused by a Force Acting in the Middle of Plain Concrete Section. (*See p.* 560.)

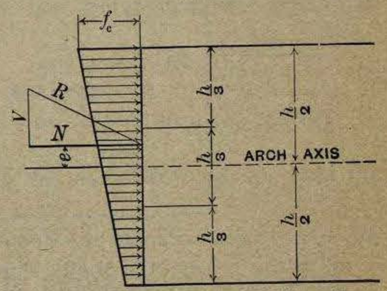

Fig. 168.—Stresses Caused by a Force Within the Middle Third of Plain Concrete Section. (*See p.* 561.)

opposite side. By mechanics, the intensity of stress due to the moment, M, at a distance y from the gravity axis is $\frac{My}{I}$. The actual combined stress at any distance y is then the sum of these two stresses, namely, $\frac{N}{bh} \pm \frac{My}{I}$. The positive sign applies to stresses on the side of the axis where N is applied and the negative sign to stresses on the opposite side.

Since for a rectangular section, $I = \frac{bh^3}{12}$ and $M = Ne$, thrust multiplied by eccentricity, we have:

$$\text{Stress at any point } y \text{ distance from gravity axis} = \frac{N}{bh}\left(1 \pm \frac{12ey}{h^2}\right) \qquad (35)$$

The stress at all points of the section is compression and the maximum

M = moment. f_c = compression in concrete. N = thrust. b = breadth. h = height. y = distance from gravity axis. e = eccentricity. I = moment of inertia.

and minimum values are at the top and bottom of the section, respectively, that is, when $y = \frac{h}{2},$

$$\text{Maximum compression} = f_c = \frac{N}{bh}\left(1 + \frac{6e}{h}\right) \qquad (36)$$

$$\text{Minimum compression} = f'_c = \frac{N}{bh}\left(1 - \frac{6e}{h}\right) \qquad (37)$$

(*c*) When the thrust acts at the edge of the middle third the eccentricity $e = \frac{h}{6}$ and the maximum and minimum values of the stresses are found by placing $\frac{h}{6}$ for e in the last two formulas of case (*b*). Fig. 169 shows the distribution in this case.

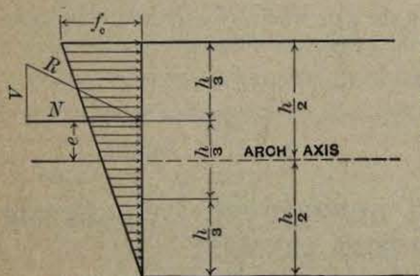

FIG. 169.—Stresses Caused by a Force Acting at the Edge of the Middle Third of Plain Concrete Section. (See p. 562.)

Maximum compression in concrete $= f_c = \frac{2N}{bh}$ (38)

Minimum compression in concrete = 0.

(*d*) If the thrust acts outside of the middle third and the material is capable of carrying some tension, the distribution of stress is as shown in Fig. 170. There will be compression over a large part and tension over the remainder of the section.

$$\text{Maximum compression} = f_c = \frac{N}{bh}\left(1 + \frac{6e}{h}\right) \qquad (39)$$

$$\text{Maximum tension} = f'_c = \frac{N}{bh}\left(1 - \frac{6e}{h}\right) \qquad (40)$$

Evidently in each of the above cases (*a*), (*b*), (*c*), (*d*) the maximum compressive and minimum compressive (or maximum tensile) unit stresses are given by the one general formula $\frac{N}{bh}\left(1 \pm \frac{6e}{h}\right)$. This applies for rectangular sections and will hold so long as the safe tensile strength of the concrete is not exceeded.

In arches of plain concrete or of stone, tension should not be allowed to exist.

(*e*) When the thrust acts outside of the middle third and the material

is not able to carry tension, the stress is distributed as compression over a depth less than the entire depth of the section, and cracks may be expected on the "tension" side. The distribution of stress is shown in Fig. 171 below, where (in addition to notation already presented on page 559).

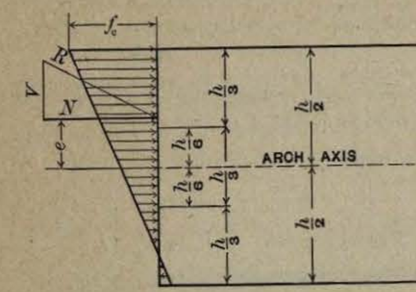

FIG. 170.—Stresses Caused by a Force Acting Outside of the Middle Third of Plain Section. (*See p.* 562.)

g = distance from point of application of thrust to most compressed surface.

Maximum compression =

$$f_c = \frac{2N}{3bg} \qquad (41)$$

Plain Arches with Irregular Cross Section. If the section is not rectangular, the maximum unit compression $= \frac{N}{A_c} + \frac{Ney_1}{I}$, and the minimum unit compression (or maximum unit tension) $= \frac{N}{A_c} - \frac{Ney_2}{I}$. When the second term of this equation is greater than the first, the concrete is in tension.

DISTRIBUTION OF STRESSES IN REINFORCED CONCRETE SECTIONS

Reinforced Concrete Sections of any Shape. The distribution of stress caused by combined thrust and bending moment over a section containing steel reinforcement is shown by the following formulas.

As in column design (page 490) the area of the steel in compression may be replaced by an equal area of concrete by multiplying the steel area by n, the ratio of the modulus of elasticity of steel to the modulus of concrete. Similarly, their moments of inertia may also be compared, and the section treated as if it were of concrete without steel.

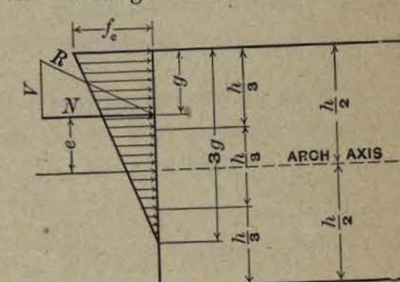

FIG. 171.—Stresses Caused by a Force Acting Outside the Middle Third of Plain Concrete Section. (*See p.* 563.)

The unit stress then in the concrete at any distance, y, from the gravity

y_1 y_2 = distances respectively from gravity axis to maximum compression of, tension.
b = breadth of cross section.

axis is $\dfrac{N}{A_c + nA_s} \pm \dfrac{Ney}{I + nI_s}$. The stress may be compression over the entire section or may be compression over a portion of it and tension over the remainder. The formulas apply in either case so long as the safe tensile stress in the concrete is not exceeded.

Maximum compression in concrete $= f_c = \dfrac{N}{A_c + nA_s} + \dfrac{Ney_1}{I + nI_s}$, where y_1 is the distance from the gravity axis to outermost fiber of concrete on the side of the gravity axis on which the thrust acts.

Maximum compression in steel $= f_s' = n\left[\dfrac{N}{A_c + nA_s} + \dfrac{Ney_3}{I + nI_s}\right]$, where y_3 is the distance from gravity axis to center of gravity of steel on side of gravity axis on which the thrust acts.

Minimum compression in concrete $= f_c' = \dfrac{N}{A_c + nA_s} - \dfrac{Ney_2}{I + nI_s}$, where y_2 is the distance from the gravity axis to the outermost fiber of concrete on

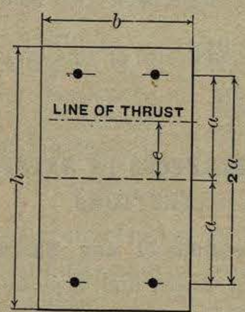

FIG. 172.—Cross Section of an Arch Rib. (*See p.* 564.)

the side of the gravity axis opposite to that on which the thrust acts. This minimum compression is of course tension when the second term in the last equation is greater than the first.

Min mum compression in steel $= f_s = n\left[\dfrac{N}{A_c + nA_s} - \dfrac{Ney_4}{I + nI_s}\right]$, where

y_4 is the distance from the gravity axis to center of gravity of steel on the side of gravity axis opposite to that on which the thrust acts.

Reinforced Concrete Rectangular Sections. For rectangular sections the above general formulas for sections of any shape may be put into slightly simpler forms by substituting the proper terms and assuming equal amounts of steel above and below the center. Special

cases for convenient use in design are also treated below. Since total moment of inertia of combined section, $I + I_s = \dfrac{bh^3}{12} + npbha^2$; area of section, $A_c = bh$ and area of steel in the section, $A_s = pbh$, we have:

Unit stress in concrete at any point at a distance, y, from gravity axis

$$\text{is } \frac{N}{bh}\left[\frac{1}{1 + np} \pm \frac{12\,ye}{h^2 + 12\,npa^2}\right] \qquad (41)$$

Maximum unit compression in concrete

$$f_c = \frac{N}{bh}\left[\frac{1}{1 + np} + \frac{6\,he}{h^2 + 12\,npa^2}\right] \qquad (42)$$

Maximum unit compression in steel

$$f_s' = \frac{nN}{bh}\left[\frac{1}{1 + np} + \frac{12\,ae}{h^2 + 12\,npa^2}\right] \qquad (43)$$

Minimum unit compression (or maximum unit tension) in concrete

$$f_c' = \frac{N}{bh}\left[\frac{1}{1 + np} - \frac{6\,he}{h^2 + 12\,npa^2}\right] \qquad (44)$$

Minimum unit compression (or maximum unit tension) in steel

$$f_s = \frac{nN}{bh}\left[\frac{1}{1 + np} - \frac{12\,ae}{h^2 + 12\,npa^2}\right] \qquad (45)$$

There are four cases which may occur dependi ıg upon value of e.

(1) Thrust applied at gravity axis, as in Fig. 173, where there is no moment on the section and the stress is uniformly distributed. In this case the second term in the brackets of the above formulas becomes zero.

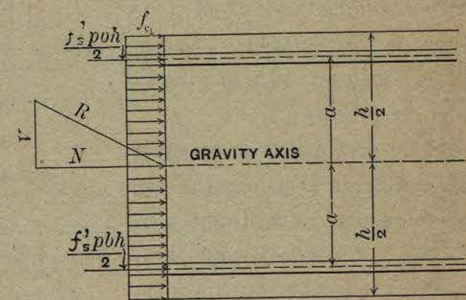

FIG. 173. Stresses Caused by Force Acting at Gravity Axis. (*See p.* 565.)

(2) Thrust applied at such distance from the gravity axis as to cause compression on whole section of the concrete, as in Fig. 174. Here the above formulas are used directly.

n = ratio elasticity. N = thrust. y = distance from gravity axis. f_c = compression in concrete. f_s = tension or minimum compression in steel. b = breadth. h = height. e = eccentricity. p = ratio of steel. a = distance from center of gravity of section to steel.

(3) Thrust applied at such distance from gravity axis that the compression at one surface becomes zero, as in Fig. 175. In this case the formulas may be simplified as follows:

Maximum unit compression in concrete,

$$f_c = \frac{2N}{bh(1 + np)} \tag{46}$$

Maximum unit compression in steel,

$$f'_s = \frac{nN}{bh(1 + np)}\left[1 + \frac{2a}{h}\right] \tag{47}$$

Minimum compression in concrete = 0.

Minimum compression in steel, which is very small,

$$f_s = \frac{nN}{bh(1 + 2p)}\left[1 - \frac{2a}{h}\right] \tag{48}$$

(4) Thrust applied at such distance from the gravity axis that there is tension at one surface, as in Fig. 176.

The most important question for decision is the compression in the concrete, which must not exceed a safe working stress and is readily found from formula (42), page 565, and the determination of whether the opposite surface is in tension or compression. A simple method for determining this is given in the following paragraphs together with a diagram, further simplifying the process.

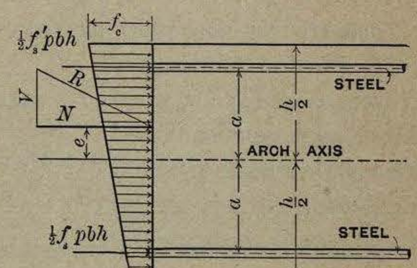

FIG. 174.—Stresses Caused by a Force Producing Compression upon the Whole Reinforced Section. (*See p.* 565.)

If the eccentricity is so great that tensile stress is found in the concrete as determined by methods described in the following paragraphs, a special treatment must be given to determine the stresses, as discussed on page 570.

In case the result from formula (44) is negative, the stress on this surface of the concrete is tension.

Eccentricity. As in plain concrete arches, the location of the center of thrust determines the distribution of the stress. The stress on one side of

n = ratio elasticity. N = thrust. f_c = compression in concrete. f_s = tension or minimum compression in steel. b = breadth. h = height. p = ratio of steel. R = resultant of forces.

the gravity axis is always compression and if the thrust acts at the gravity axis there is uniform compression over the section. As the center of thrust lies farther and farther from the graviy axis, the compression at the opposite surface decreases until finally it becomes zero and then tension.

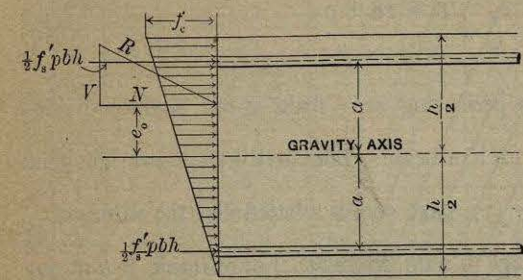

FIG 175.—Stresses Caused by a Force Acting at a Distance e_0 from Center of Gravity of Reinforced Section. (*See p.* 566.)

Equation (44), page 565, gives a means of determining the eccentricity for which there can be neither tension nor compression at the surface opposite to that on which the thrust acts. For a reinforced arch this eccentricity is usually more than $\frac{h}{6}$, that is, the line of pressure must not necessarily lie within the middle third.

When the first term in the brackets of this equation is greater than the second, the minimum stress in the concrete will be in compression; when the two terms are equal, the stress is zero in the outer edge of the concrete on the side opposite to that on which the thrust acts; when the second term is greater than the first, this stress will be tension. By equating the two terms the value of the eccentricity, e, may be found for which the stress at the edge is zero.

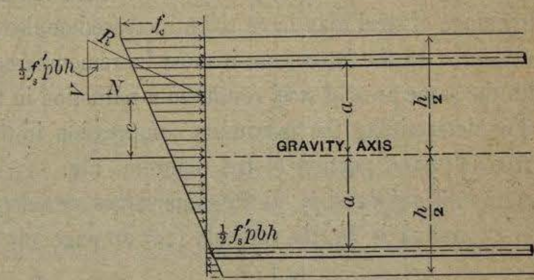

FIG. 176.—Stresses Caused by a Force Acting at a Distance Larger than e_0 from the Axis of Gravity of Reinforced Section. (*See p.* 566)

Using previous notation and also letting e_0 = value of e which makes the stress zero

$$e_0 = \frac{h^2 + 12\,npa^2}{1 + np}\,\frac{1}{6h} \tag{49}$$

If the computed eccentricity, e, is greater than e_0 the concrete is in tension.

Diagram for Determining Compression and Eccentricity. By introducing selected values for some of the letters in formula (49) its solution is simplified. If the ratio of moduli of elasticity of steel to concrete is

e = eccentricity. h = height. n = ratio elasticity. p = ratio of steel.

assumed to be 15, and if the steel is assumed to be imbedded in the concrete $\frac{1}{10}$ of the total depth from each surface so that $2a = \frac{4h}{5}$, which is a close approximation in ordinary design, formula (49) becomes

$$\frac{e_0}{h} = \frac{1 + 28.8\,p}{6 + 90\,p} \qquad (50)$$

where $\frac{e_0}{h}$ is the eccentricity producing zero stress at one surface divided by the total thickness of the arch or beam. The curve in the lower right hand portion of the diagram, Fig. 177, page 569, is plotted and the values of $\frac{e_0}{h}$ can be read for any percentage of reinforcement. For example, if h at any section is 30 inches and the percentage of steel 0.8% (i. e., if $p = 0.008$) $\frac{e_0}{h} = 0.183$ from the diagram, and hence $e_0 = 5.49$. That is, the center of thrust cannot be more than 5.49 inches from the gravity axis without producing tension in the concrete.

If, then, the eccentricity of the thrust on the section in question as previously determined from the line of pressure, or from computation, is greater than the e_0 derived from the curve, there is tension in the concrete, and the percentage of steel may have to be increased or else the depth of section, h, increased. In the latter case it must be remembered that an increase in h with the same area of steel results in a reduction in the percentage of steel.

For determining the maximum compression in the concrete, the curves in the left-hand portion of the diagram, Fig. 177, have been drawn for certain values of n and p. If the same values are selected as are given above. $n = 15$ and $2a = \frac{4}{5}h$, the formula (42) on page 565 becomes

$$f_c = \frac{N}{bh}\left[\frac{1}{1 + 15\,p} + \frac{e}{h}\,\frac{6}{1 + 28.8\,p}\right] \qquad (51)$$

or for definite values of e, h and p

$$f_c = \frac{NC_e}{bh} \qquad (52)$$

In the diagram mentioned, the values of C_e are plotted for values of $\frac{e}{h}$ and different percentages of steel.

To illustrate the use of the diagram, after having found that the eccentricity does not produce tension in the concrete, if the eccentricity is 3 inches

N = thrust. f_c = compression in concrete. b = breadth. h = height. e = eccentricity. p = ratio of steel. C_e = constants.

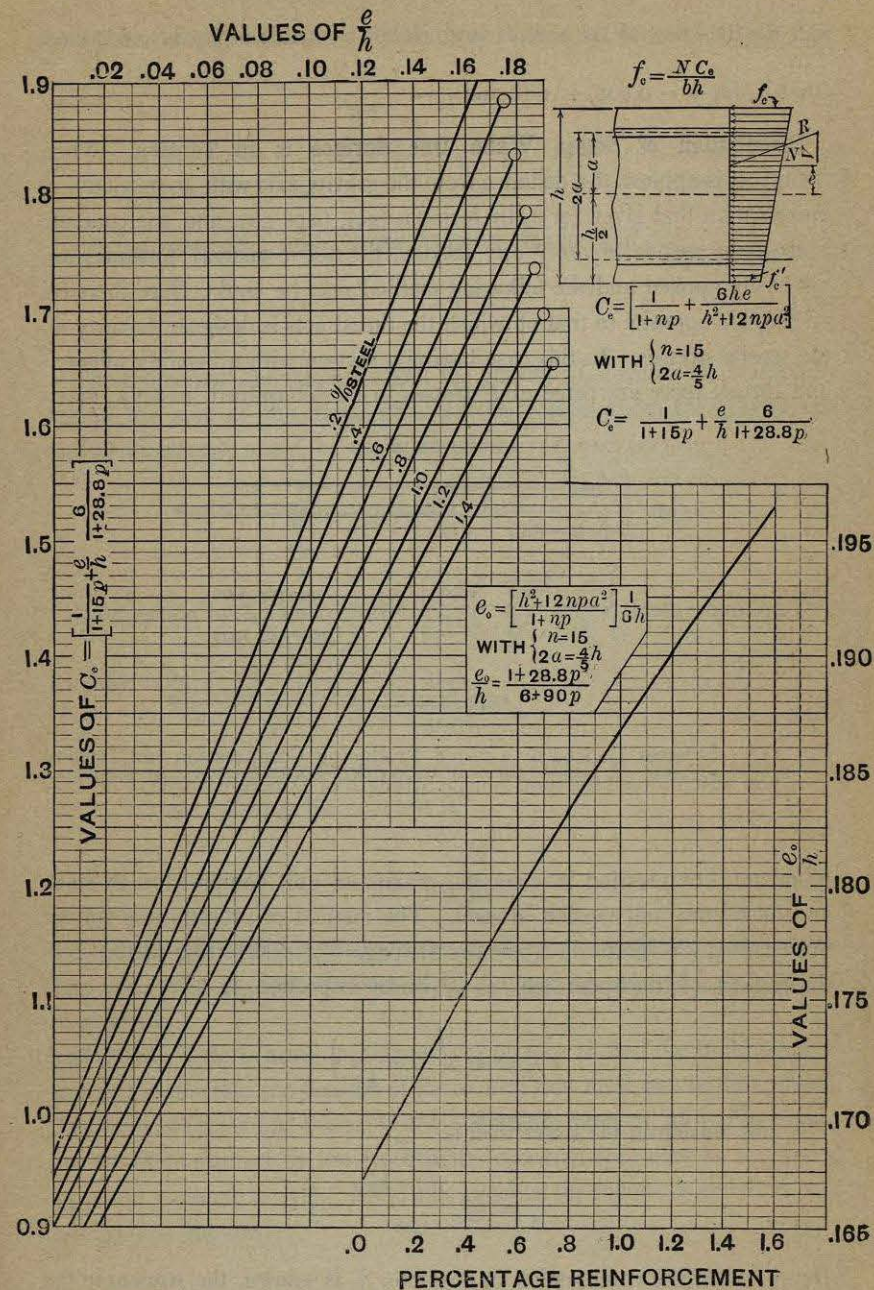

FIG. 177.—Diagram for Determining Compression and Eccentricity. (See p. 567.)