Fig. 156. Walnut Lane Bridge, Philadelphia.

## CHAPTER XXII

# ARCHES

### BY FRANK P. McKIBBEN

The treatment of arch design by what is termed the elastic theory, although generally considered a complicated problem, as a matter of fact is easily handled by one who is familiar with elementary mechanics and with the principles of reinforced concrete beam design. The process is necessarily somewhat lengthy, involving extended operations in simple arithmetic, but by following the analysis presented in the following pages it can be readily understood. It is doubtful whether in the whole category of the design of structures there is a prettier application of mechanics and mathematics than the design of a reinforced concrete arch bridge.

While in a volume of this size it is impossible to present all phases of the subject, the underlying principles are treated in sufficient detail and with a discussion thorough enough to permit an engineer to safely design an arch.

Following a brief historical introduction discussing the use of concrete versus steel construction, the different forms of arches are reviewed with suggestions for design; the loading for different conditions is scheduled (p. 541); the outer forces are analyzed, including the effect of temperature (p. 553); the method of procedure to be followed in arch design is taken up in a practical example item by item (p. 574); allowable unit stresses are suggested (p. 583); the design of abutments is outlined (p. 583); and a few illustrations of existing bridges are presented

Girder bridges are not treated specifically in this chapter, but they may be readily designed by applying the principles of reinforced concrete beam and slab construction as treated in Chapter XXI on Reinforced Concrete.

The treatment of conduit or sewer arches which are so deeply imbedded as to require computations for earth pressure is referred to on page 693.

Perhaps the most interesting feature of the present chapter is the complete analysis of a typical arch which is presented on page 574. The steps to be followed are outlined consecutively and the mathematical processes indicated in full.

The formulas for distribution of stress given on page 560 apply not only to arch design but also to column and beam design where there is eccentric

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[*] The authors are indebted to Prof. McKibben for this chapter, which has been especially prepared by him for this treatise.

loading or thrust in place of or in addition to the ordinary loads. To facilitate the understanding of the formulas, a departure is made from the usual notation schedule, which must necessarily be several pages away from the work, by placing in addition, at the bottom of each page, a brief definition of all the symbols used on that page.

## CONCRETE VERSUS STEEL BRIDGES

Reinforced concrete either, as arch or girder spans, is being used not only in preference to steel trusses or steel girders, where the stone arch is too expensive to be considered, but the concrete bridge is frequently replacing the old steel structure. The reasons generally conceded for this widespread growth may be briefly stated as: (1) greater durability; (2) less cost of maintenance; (3) less vibration and less noise; (4) more æsthetic effects.

The relative first cost for concrete and steel depends upon the local conditions. In many places a concrete bridge can be built for less than a first-class steel span, although it cannot so readily compete with the flimsy trussed spans frequently seen. The concrete may be laid with less skilled labor than the steel bridge, but since the concrete structure is built on the spot, while the steel is prepared in an established shop, even more careful supervision and inspection are necessary with the concrete. The foundations for a concrete arch are frequently more expensive than concrete abutments for a steel truss because of the greater area required to take the thrust, while on the other hand, in rock or other hard material, a less quantity of concrete may be required for the arch abutments. This part of the design may often be the determining feature from the economical standpoint.

The most serious objection to steel, especially for highway bridges, lies in the fact that unprotected it cannot resist for a great length of time the oxidation due to air, water and locomotive gases, and unless properly cared for and frequently painted, it rusts badly. The examination by the author of this chapter of approximately 600 highway bridges carrying electric railways proves that frequently these bridges are not properly maintained, many of them receiving little or no attention for years at a time, so that the structures are often badly corroded, and in fact, cases are on record where subordinate members of steel bridges have rusted away completely in less than fifteen years

In a concrete bridge the steel is effectively prevented from rusting by the concrete in which it is imbedded (see p. 327), so that, when properly designed and built, no repairs whatever should be required, and no limit can be placed upon the life of the bridge.

Concrete is strongest in compression, and is therefore eminently suitable for use in arch spans where the stresses are largely compressive. The mass of the concrete and the quantity of earth filling or ballast over the arch so deaden the impact due to traffic that in many cases no impact allowance need be made, while at the same time the noise and vibration which occur in steel spans are avoided.

## USE OF STEEL REINFORCEMENT

The use of steel reinforcement in a concrete arch is desirable but not absolutely necessary, as it is possible to construct a concrete arch like the Walnut Lane Bridge in Philadelphia (see p. 532) with the concrete voussoirs laid in blocks, each block forming a voussoir like the stones in a masonry arch. At the same time under ordinary conditions, while the introduction of steel does not, with the present knowledge of concrete arch design, permit great diminution in section, it does give considerable added strength at comparatively low cost and may prevent the formation of cracks in the concrete and take tension caused by any unforeseen action of the arch, such as settlement of foundations, improper allowance for temperature or shrinkage of the concrete while hardening.

The area of the cross section of the longitudinal steel bars in solid arch rings is to a certain extent arbitrary. Good practice sanctions $\frac{1}{2}\%$ to $1\frac{1}{4}\%$ of the ring at the crown and the exact quantity to use must first be selected by judgment, and then tested by the computation and revised if necessary.

As in column design (see p. 489), it is impossible to stress the steel in compression to an amount ordinarily proper in structural steel work, because in so doing the deformation would be so great as to overstress the concrete. The actual compressive stress in the steel, therefore, can never be greater than the working stress in the concrete multiplied by the ratio of the modulus of elasticity of steel to that of concrete. Under ordinary conditions this limit on the steel may be taken as 7500 pounds per square inch.

Since the beginning of this century there has been a remarkable development in methods of construction and in our knowledge of the principles of reinforced concrete arch bridges, but even yet engineers incline to employ a somewhat excessive quantity of concrete in the solid rings of ordinary highway concrete arches. This is frequently out of proportion to the quantity of material used in a reinforced concrete ribbed arch or a steel arch. Improvements in arch design evidently lie, as is indicated in subsequent pages, in the substitution of comparatively narrow ribs for solid arches and in the use of hollow abutments with earth filling in place of solid concrete abutments. This will considerably reduce the cost of reinforced concrete arches.

## HISTORY OF CONCRETE ARCH BRIDGES

In the development of concrete bridges it is natural that the arch rather than the beam should have been the first type of bridge to be constructed. It was a comparatively short step from the stone voussoir arch to the concrete voussoir or to the monolithic arch. One finds therefore many concrete arch bridges, and, until recently, few beam bridges, although for short spans beam bridges are now being constructed in considerable numbers, both in this country and abroad.

The first plain concrete arch of any importance was built in Europe in 1869 and is known as the Grand Maitre bridge at Fontainebleu Forest. It has a maximum span of 115.8 feet and carries the aqueduct of the Paris waterworks from Vanne. The first plain concrete arch in the United States was constructed in 1871 by John C. Goodridge in Prospect Park, Brooklyn, and has a span of 31 feet. The earliest reinforced concrete arch in Europe of which there is a well defined record was built in Copenhagen Denmark, in 1879, with a span of 71.7 feet. It is probable, however, that Jean Monier of Paris was the inventor of the reinforced concrete arch and that he built some bridges before the dates mentioned. In the United States the first reinforced concrete arch on record was erected in 1889, with a span of 35 feet, by Ernest L. Ransome at Golden Gate Park in San Francisco.

When these structures are compared with the 233 feet span of the Walnut Lane Bridge in Philadelphia, which in 1908 was, with perhaps one exception, the longest plain concrete arch in existence, with the 230 feet, 3-hinge Grünwald Arch at Munich, Bavaria, or still more sharply with the Hudson Memorial design for an arch across the Spuyten Duyvil Creek with a span of 703 feet, a wonderful development is observed.

Although in a very few cases concrete bridges built during this development have failed, every such failure can be traced to a direct disregard of well known principles of design or construction. Moreover, as a matter of fact, accidents to concrete arches have been much fewer than the failures of wrought iron or steel bridges during the corresponding period of metal bridge development.

## CLASSIFICATION OF ARCHES

Arches in general may be classified with reference to the material of which they are made, the arrangement of the spandrels and arch rings, or the

number of hinges. Reinforced concrete arches may be divided as to the arrangement of the reinforcement into three groups: the Monier, Melan and Wünsch types. The Monier arch in its developed form is the type most commonly used in the United States. This system of reinforcement was invented by Jean Monier about the year 1876. As first devised, a wire netting was imbedded in the concrete near the soffit, but later two nettings were used, one near the soffit, and the other imbedded in the concrete near the extradosal surface. Wire netting of small mesh with wires of equal size in both directions obviously is not well suited for use in an arch and considerable improvement was soon effected in this type by making the longitudinal bars of the reinforcement heavier than the transverse.

In the usual design a layer of longitudinal bars is imbedded near the intrados and an equal number near the extrados, the bars of the two layers being connected with small bars or stirrups. Transverse bars, at right angles to the longitudinal, form with them a netting both in the top and bottom of the arch. They serve to prevent cracks in the concrete and distribute the loads laterally. These cross bars also act with the stirrups in holding the longitudinal bars in place during construction.

The principal longitudinal bars are designed to carry tension due to the bending moment and to assist the concrete in compression caused by the thrust and the bending moment.

**Melan Type.** This system was invented by Joseph Melan of Brünn, Austria, in 1892. The reinforcement consists of curved steel ribs imbedded in the concrete and extending from abutment to abutment. For short spans the ribs are simply curved I-beams and for long spans each rib is made of two angles near the extrados latticed to two angles near the intrados. The built-up ribs thus formed are usually deeper at the springings than at the crown of the arch. The principal function of the lattice bars is to hold the angles in position when the latter are stressed, and to make a unit which is easy to handle during erection. By far the most important function of steel reinforcement is to carry bending moment, and the steel in the Melan type can be easily placed and kept in position during erection so as to fix positively its location in the finished structure. The material in the lattice bars of the ribs or in the webs of the I-beams is not economically placed. The first Melan arch in the United States, of 30 feet span, was erected at Rock Rapids, Iowa, in 1894, and many other bridges have since been built of this system.

**Wünsch Type.** Comparatively few bridges have been constructed on this system. The arch, which was invented by Robert Wünsch of Budapest, Hungary, in 1884, has a horizontal extrados and a curved intrados and the reinforcement of the arch ring consists of steel ribs spaced from $1\frac{1}{2}$ to 2 feet apart, with a horizontal upper member placed near the extrados and a curved lower member near the intrados. The two members are connected at each abutment to a vertical member imbedded in the concrete. The bridge at Sarajevo in Bosnia, of 83 feet span, is one of the largest built by the Wünsch system.

## ARRANGEMENT OF SPANDRELS AND RINGS

The spandrel, which is the space between the roadway surface and the top or extrados of the arch ring, may be treated in one of two ways. First, it may be entirely filled with earth or with concrete which carries the roadway; or, second, it may be left more or less open, and the roadway supported upon a deck carried on a series of transverse walls, longitudinal walls, or columns resting upon the arch ring.

**Filled Spandrels.** In this form of construction the earth or concrete filling rests directly upon the arch ring, and is held in place laterally by retaining walls which also rest upon the arch ring. As the depth of these walls, unless they are of reinforced design, increases from the crown to the springing, their thickness, designed to resist the earth pressure, also increases until at the abutments the spandrels may be largely filled with the concrete composing the side walls.

If the side walls simply rest, upon the arch ring a crack is liable to form at the junction of ring and wall due to the deflection of the arch ring from the weight of the earth upon it. On the other hand, if the ring and wall are connected by sufficient steel to prevent the formation of this crack, indeterminate stresses are set up which are undesirable and which may result in transferring the crack to another place. This danger may be obviated by building the spandrel walls as gravity walls, leaving a vertical expansion joint at each junction of spandrel and wing walls and at some intermediate point between this joint and the crown.

Another plan is to build thinner reinforced side walls as vertical slabs tied together, with the lateral pressure resisted by reinforced cross walls. The principal objections to the use of solid fillings are as follows: (1) They increase the weight of the superstructure, and consequently thicker arch rings and larger foundations are required. (2) Unless the earth filling is carefully compacted by rolling, tamping or wetting, it will sink and allow the roadway to settle with it. (3) It is difficult to make the side walls and the arch ring act in unison, and unsightly cracks may be formed. Filled spandrels may be therefore limited properly to bridges with solid arch rings of short span, say not over 80 feet, or to those having a rise of less than $\frac{1}{10}$ the span, where the cost of form construction prohibits an open design.

**Open Spandrels.** The objections just mentioned to the use of filled spandrels are of such importance that during the last few years the use of open spandrels in the larger structures has made rapid progress. In addition to being lighter, the open spandrel construction facilitates inspection and lends itself to more pleasing architectural treatment. It permits indeed a treatment peculiar to concrete, which does not follow the type of design used for so many centuries in stone arch bridges. With open spandrels the roadway may be laid upon small arches or upon I-beams carried by transverse or longitudinal walls which in turn rest upon the arch ring; or it may be laid with reinforced concrete beam and slab construction, making a floor similar to those used in reinforced concrete buildings. The beams in this case are placed longitudinally with the roadway, and rest upon transverse walls.

Upon the adoption of the open spandrel it was soon seen that considerable material was wasted in the transverse walls and in the solid arch rings. The next step, therefore, was to reduce the walls to columns and the ring to a series of longitudinal ribs spaced similarly to the ribs of a steel arch. In some cases these ribs are very wide, in fact, are really two independent arch rings as in the Walnut Lane bridge, Philadelphia,* and in other cases the ribs are narrow as in the Rock Creek bridge on Ross Drive in the District of Columbia.†

## HINGES

The use of hinges in concrete arches is by no means of recent origin. As early as 1873, an arch was constructed near Erlach, Germany, with three asphalt "joints" and many others have been built since then. The chief object of the hinge in the arch rings or ribs is to render the structure more nearly determinate.

Although two or even one hinge can be used, three hinges offer the advantage of definitely fixing the pressure line throughout the ring so that it can be easily and accurately located. Except for the friction of the hinges, the stresses are practically independent of changes of temperature or of any reasonable settlement of the foundations. On the other hand, the hinges are often an expensive detail. It is sometimes claimed also that three-hinged arches are not so rigid as fixed arches, but because of their great weight this criticism does not appear to be well founded.

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*See p. 590.

†See p. 590.

In the design of a hinged structure the moment is usually assumed to be zero at the hinge. This assumption is not strictly correct because as the structure deforms under its load it tends to rotate about its hinges and this produces friction at the hinge due to the thrust acting thereon.

The design of the hinge is a most important feature. One of the most instructive failures in arch construction was that of the Maximilian Bridge at Munich, a three-hinged voussoir masonry arch of two spans, each 144.3 feet, when during construction, both spans of the bridge slipped off the hinges at the springings and dropped about 12 inches. This failure was due to an error in the design of the hinges. The bearing surfaces of the hinges were not given sufficient curvature, and the friction which was relied upon to prevent slipping of the two parts composing each hinge was reduced to a minimum by the use of a lubricant, which gave a low coefficient of friction.

Three-hinged construction is best suited to arches of small rise where the center line of the rib can be made to fit closely the line of pressure resulting in small bending moments. Arches with one or two hinges are more indeterminate than three-hinged arches and have practically all of the disadvantages of both the fixed and the three-hinged types.

## SHAPE OF THE ARCH RING

For hingeless arches the intrados should be either three-centered, five-centered or elliptical, while, if desired, the extrados may be the arc of a circle so placed as to give greater depth to the arch ring at the springings than at the crown. A segmental arch, that is an arch formed by the segment of a single circle cannot often be used to advantage, for it seldom can be made to fit the line of pressure. While many arches are elliptical in form, the three-centered intrados is perhaps the most common and it is pleasing to the eye, easily constructed and gives an economical design. Ribs with three hinges should be deepest at sections nearly midway between the crown and spring hinges, decreasing in depth toward the hinges, since sections near the hinges take only thrust and shear with practically no moment, while the intermediate sections resist a moment in addition to the thrust and shear.

## THICKNESS OF RING AT CROWN

The next step in the design of an arch after deciding on the shape of the intrados is to choose a trial thickness of the ring at the crown and at the springing. The choice may be made by judgment based on experience or with the aid of one of the various empirical formulas in use. Since the crown thickness depends not only on the amount of thrust but also upon the bending moment, which varies greatly in a given arch due to the varying positions of the live load, it is difficult and in fact impossible to devise a rational formula for its determination.

The thickness of the arch ring should vary with the shape of the arch, with the span, rise, amount of filling over the ring, the amount of live load and the material of which the arch is made, and while there is no formula that will apply even approximately in all cases, the formula by Mr. F. F. Weld* gives fairly correct results in ordinary cases. It is as follows:

Let

$h$ = crown thickness in inches.

$L$ = clear span in feet.

$w$ = live load in pounds per square foot, uniformly distributed.

$w'$ = weight of fill at crown in pounds per square foot.

Then

$$h = \sqrt{L} + \frac{L}{10} + \frac{w}{200} + \frac{w'}{400} \qquad (1)$$

Obviously the thickness for a hingeless arch should increase from the crown to the springing. The radial thickness of the ring at any section is frequently made equal to the thickness at the crown multiplied by the secant of the angle which the radial section makes with the vertical. For a 3-centered intrados and an extrados formed by the arc of a circle, these trial curves may be at the quarter points a distance apart of 1¼ to 1⅓ times the crown thickness and at the springings 2 to 3 times the crown thickness.

These empirical rules should be used only in preliminary study and *never for the final design*. The true shape of the ring and the thickness at different sections must be fixed by computation based on the line of pressure as described in the pages which follow.

## LIVE LOADS FOR HIGHWAY BRIDGES

For highway bridges the kind and magnitude of the live load depend upon the location of the structure. Each location should be studied and the live load chosen to fit the requirements. The following classification is sufficient for stone or concrete arches and may also be applied to beam and slab construction.

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***Engineering Record*, Nov. 4, 1905, p. 529.

**City Bridges.** For *floors* of city or other bridges carrying heavy traffic, three types of loads are recommended as follows:

1. A uniform live load of 100 pounds per square foot on sidewalks and roadway.

2. On each street railway track, one 8-wheel electric car having a wheel spacing of 5, 15, 5 feet between centers of wheels along one rail; each wheel carrying 12,500 pounds. The car is assumed to cover an area 9 feet wide by 40 feet long.

3. One wagon weighing 20,000 pounds on each of two axles 12 feet apart.

In applying these loads to find the maximum stress in the floor, either of the loads mentioned, or that combination of any of the above loads which produces the maximum stress, should be used. If the uniform load is used simultaneously with either of the concentrated loads, the former should cover only that part of the roadway not covered by the latter.

For *arch rings* or *ribs* having a span of 100 feet or less, a uniform load of 1800 pounds per linear foot of each railway track together with a uniform load of 100 pounds per square foot of remaining area of roadway and sidewalks.

For spans of 200 feet or more, a uniform load of 1200 pounds per linear foot of each railway track together with a uniform load of 80 pounds per square foot of remaining area of roadway and sidewalks.

The load on each track should be assumed to cover a width of 9 feet, thus giving 200 pounds per square foot under the track for spans of 100 feet or less and 133 pounds per square foot for spans over 200 feet in length.

For spans between 100 and 200 feet, the loads are to be taken proportionally.

**Suburban, Town or Heavy Country Bridges.** For *floors* of suburban town or heavy country bridges, the same uniform load and electric car load as for floors of city bridges but with wagon weighing 10,000 pounds on each of two axles 10 feet apart.

For *arch rings* or *ribs* having a span of 100 feet or less, a uniform load of 1800 pounds per linear foot of each track, together with a uniform load of 80 pounds per square foot of remaining area of roadway and sidewalks.

For spans of 200 feet or more the values corresponding to the above are 1200 pounds per linear foot of each track and 60 pounds per square foot of remaining area.

The load on each track should be assumed to cover a width of 9 feet.

For spans between 100 and 200 feet, the loads are to be taken proportionally between the limits stated.

**Light Country Bridges.** For *floors* of light country bridges, subjected to light highway or electric railway traffic, on each track one 8-wheel electric car carrying 9000 pounds on each wheel, or one wagon weighing 6000 pounds on each of two axles 10 feet apart. These two loads should be assumed to act together where necessary to produce the maximum stress in the floor.

For *arch rings* or *ribs* having a span of 100 feet or less, a uniform load of 1200 pounds per linear foot of each track, together with a uniform load of 80 pounds per square foot of remaining area of roadway.

For spans of 200 feet or more, the values corresponding are 1000 pounds per linear foot of each track, and 50 pounds per square foot of remaining area.

For spans between 100 and 200 feet the loads are proportional between the limits stated.

It is customary to see that the design is sufficient to carry a steam road roller. The heaviest roller usually specified weighs 30,000 pounds, 12,000 pounds on the front roller, which has a width of 4 feet, and 9000 pounds on each of the two rear rollers, each of the latter having a width of 20 inches. The axles are taken as 11 feet apart and the two rear wheels as 5 feet center to center.

## LIVE LOADS FOR RAILROAD BRIDGES

For railroad bridges the loading depends upon the location of the line, and hence the future traffic which may be expected. Two consolidated locomotives, with 25,000 pounds on each driving wheel, followed by 5000 pounds per foot of each track, is a common loading. An alternate plan quite generally followed for the rings of stone or concrete arches where the filling is of sufficient thickness to distribute the concentrated loads over a considerable area of arch ring is to use 5000 pounds per foot of track with no concentrated load. This load of 5000 pounds per foot of track is equivalent to about 625 pounds per square foot of horizontal area. These values are satisfactory for spans, say, over 80 feet in length.

Generally speaking, the shorter the span the greater should be the assumed uniform load, and hence for spans of, say, 80 feet or less, a uniform load of 1000 pounds per square foot is frequently adopted, this being approximately equivalent to the heaviest locomotive loadings.

A concentrated load on top of a fill is generally assumed to be distributed downward at angles of 45°. The top of the distributing slope may be taken from the ends of the ties. Wheel loads may be taken as distributed over 3 feet of length of surface of fill and at 45° angles through the filling.