

DIAGRAM 1. BENDING MOMENTS FOR DIFFERENT SPANS AND LOADS.

$$M = \frac{wl^2}{8}$$

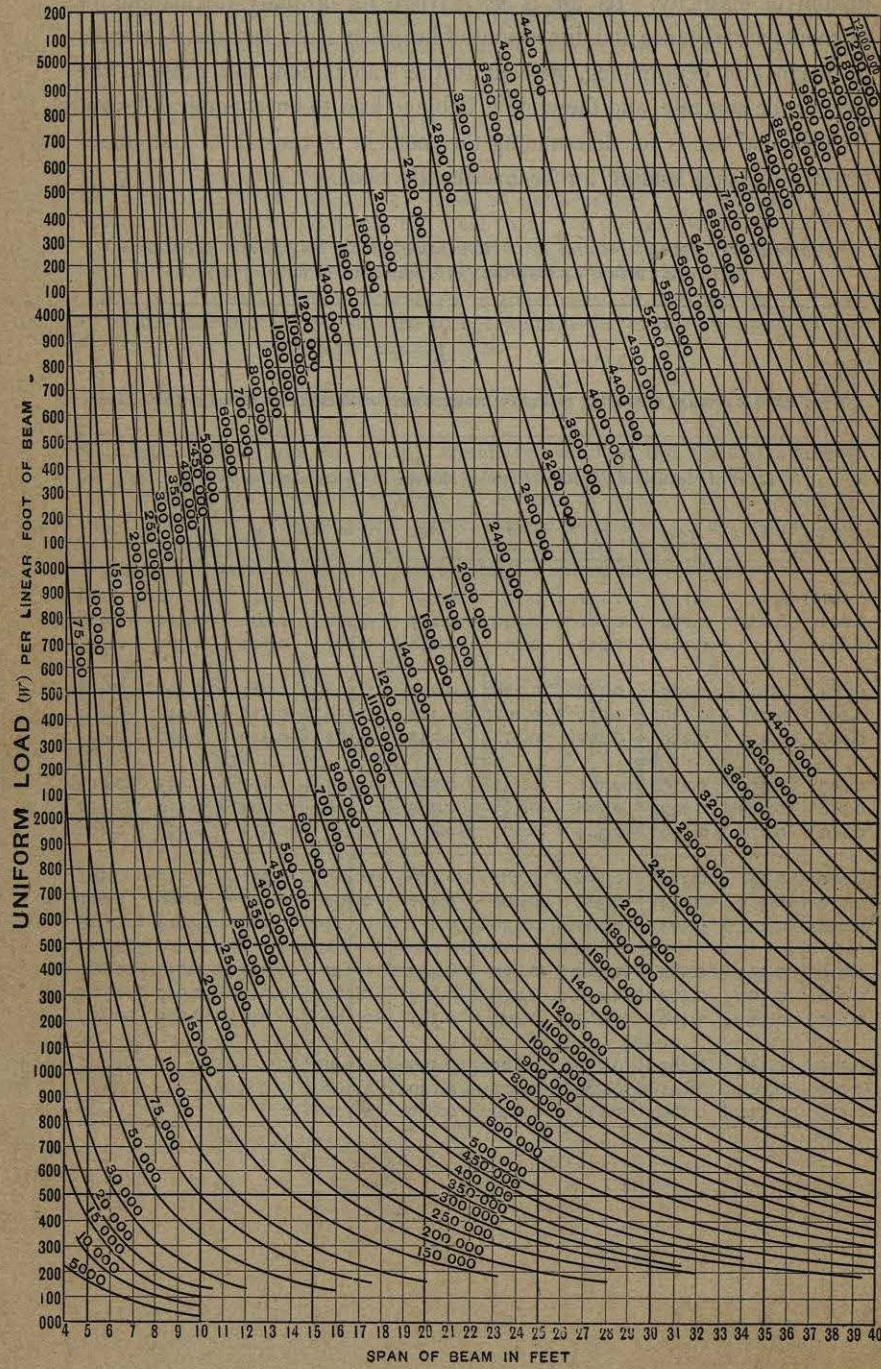


DIAGRAM 2. BENDING MOMENTS FOR DIFFERENT SPANS AND LOADS.

$$M = \frac{wl^2}{10}$$

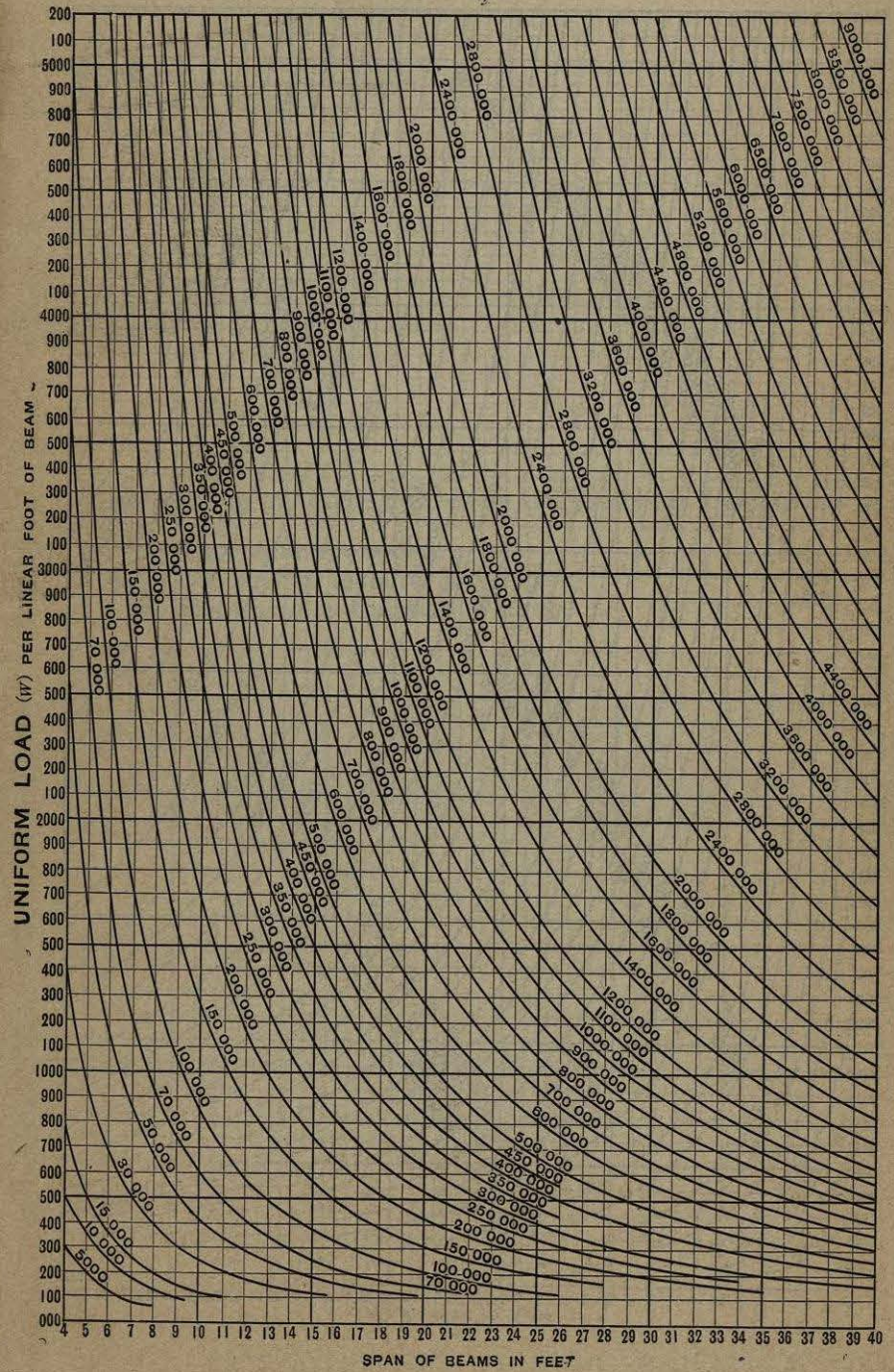


DIAGRAM 3. BENDING MOMENTS FOR DIFFERENT SPANS AND LOADS.

$$M = \frac{wl^2}{12}$$

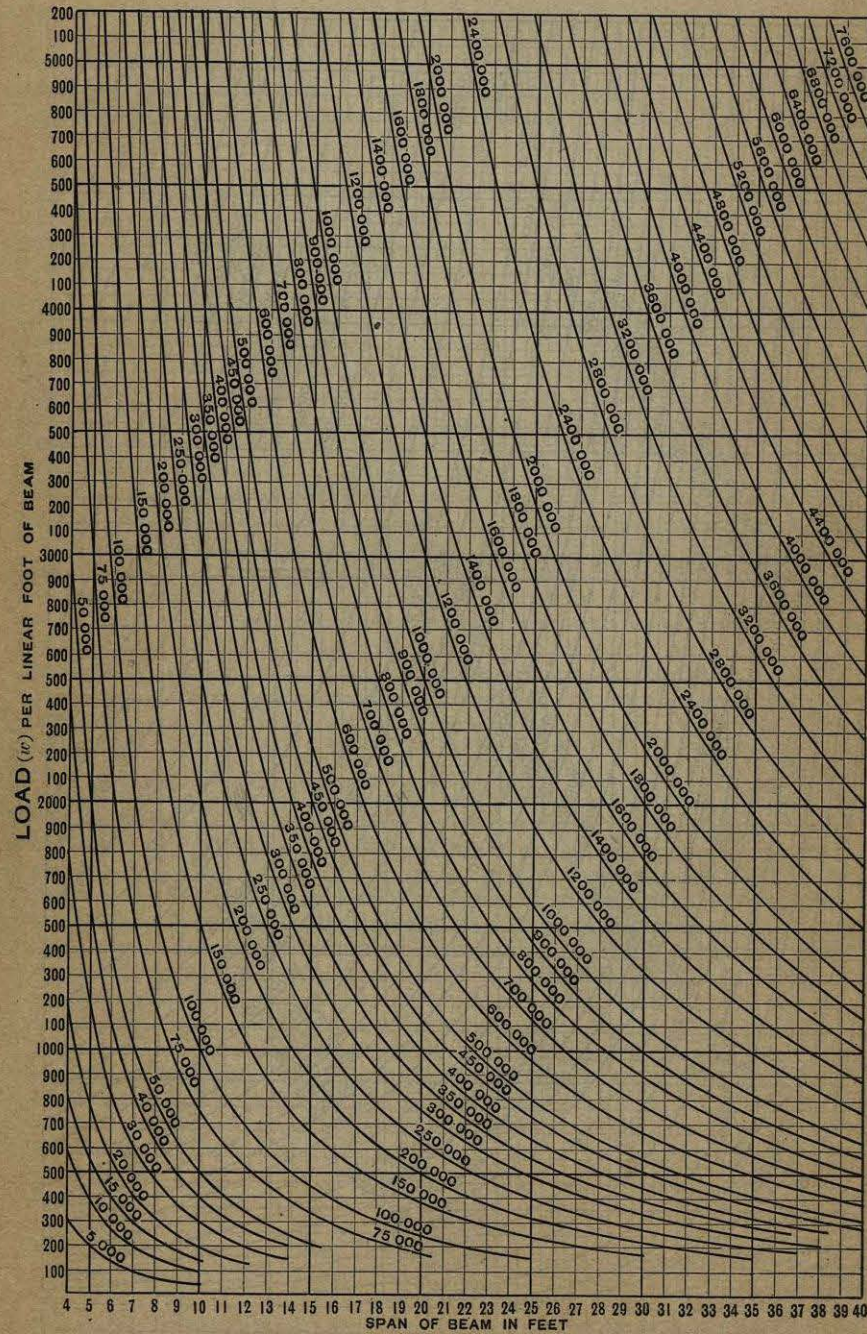
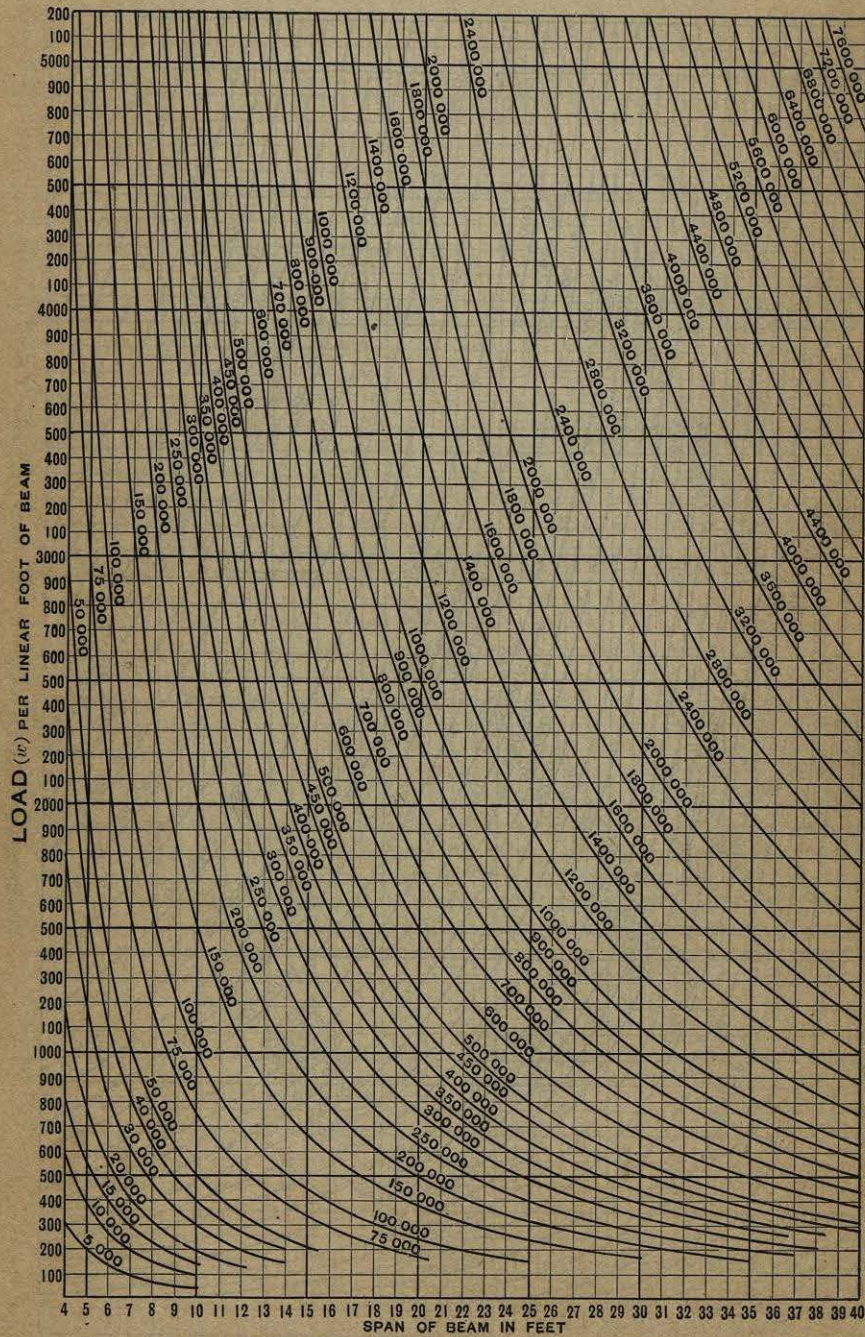


DIAGRAM No. 4. CURVES FOR DESIGN OF T-BEAMS

- To find minimum depth.
- To find area of steel for any depth.
- To find total compression in flange.

DIAGRAM 3. BENDING MOMENTS FOR DIFFERENT SPANS AND LOADS.

$$M = \frac{wl^2}{12}$$



Notation:

$f_c$  = working compression in concrete in lb. per sq. in.

$f_s$  = working tension in steel in lb. per sq. in.

$b$  = breadth of flange in inches.

$t$  = thickness of flange in inches.

$d$  = depth of steel in T-beam in inches.

$jd$  = moment arm, approximately equal to  $d - \frac{t}{2}$ ,

$k$  = ratio of depth of neutral axis to depth of steel.

Curves made from formulas (14) to (19) pages 755 and 756 for  $f_c = 650$ ,  $f_s = 16\ 000$  and  $f_c = 550$ ,  $f_s = 16\ 000$ .

For other stresses the results may be approximated with an accuracy sufficient for practical purposes.

To Determine Minimum Depth of a T-beam Consistent with a Working Compression in Concrete. Enter Diagram 4 at top with value specified for maximum working compression  $f_c$  in concrete times  $bt$ , which is assumed breadth of flange of T-beam, times thickness of slab. Follow this line down vertically until it intercepts the slant line marked  $t$ , the value of which must be previously assumed as some fraction of  $d$ . From point of intersection of these two lines follow horizontal line across the diagram to the left till it intersects a vertical line corresponding to required moment in inch-pounds read from bottom of diagram. This will give **minimum value** of  $jd$ , the distance between center of gravity of steel and center of compression in concrete. The depth from surface of beam to steel is  $d = jd + \frac{1}{2}t$ .

If assumed relation between  $t$  and  $d$  does not correspond to actual, repeat the operation.

To Determine Area of Steel in a T-beam Consistent with a Working Tension in the Steel. Enter diagram at bottom of page with bending moment in inch-pounds and follow this line vertically upwards till it intersects slant line  $jd$ , which is the distance between the center of the steel and center of compression of the concrete and is approximately equal to  $d - \frac{1}{2}t$ . From intersection of these two lines follow horizontal line to left and read off directly the area of steel required in square inches corresponding to the specified working stress in the steel.

To determine the total compression in the flange of a T-beam  $\left( f_c \frac{2kd-t}{kd} bt \right)$

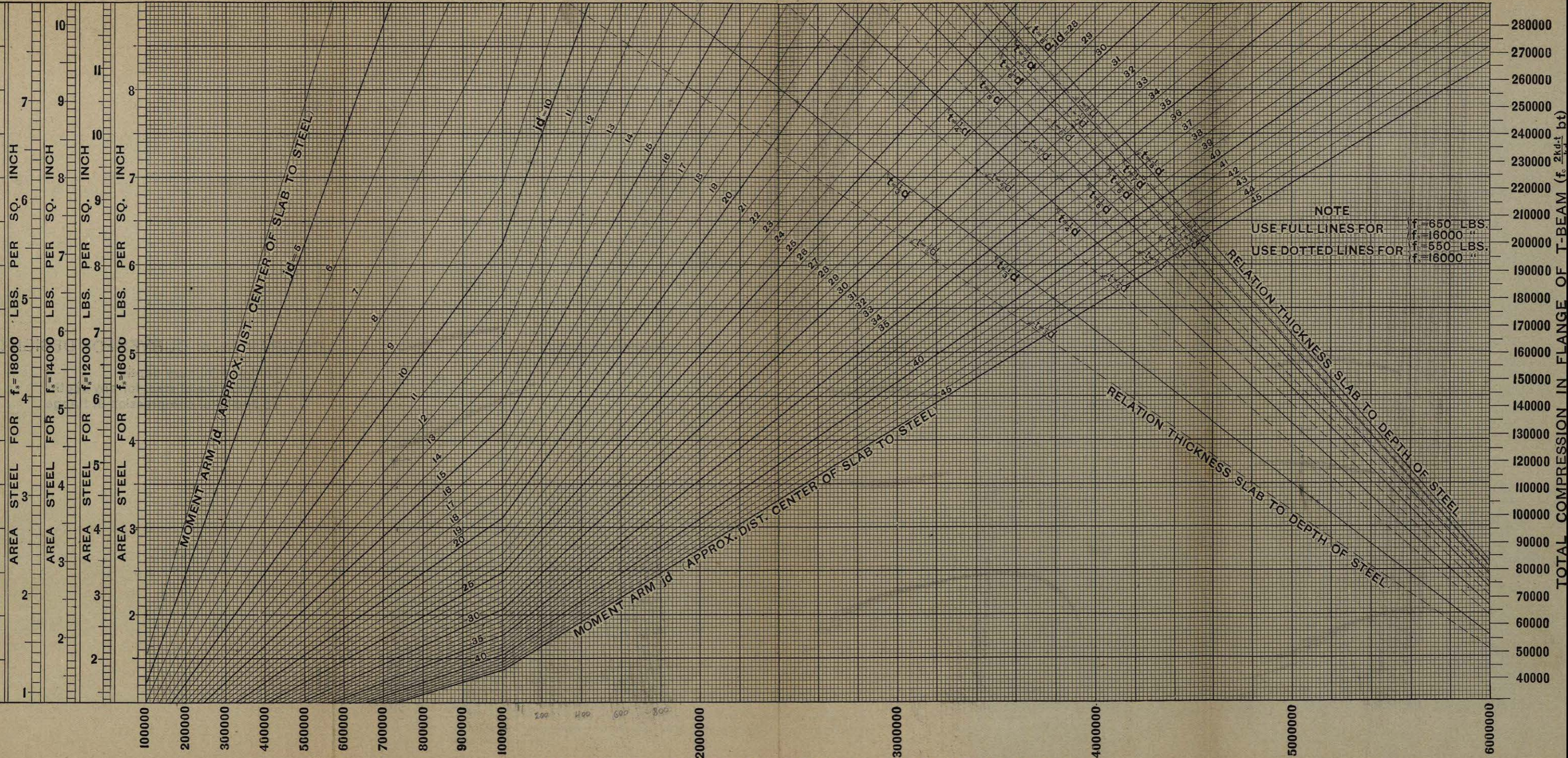
Enter table at bottom with moment in inch-pounds, follow this line up vertically till it intersects slant line  $jd$ . From point of intersection follow the horizontal line to the right hand column of figures, which will give the total compression in the flange of a T-beam which value is  $\left( f_c \frac{2kd-t}{kd} bt \right)$

This value multiplied by the moment arm  $jd$  gives the moment of resistance governed by the concrete. For true compression use *minimum*  $jd$ .

To determine maximum fiber stress  $f_c$ . Determine total compression as above. Equate this value to  $f_c \frac{2kd-t}{kd} bt$ . Assume a value for  $k$  and compute  $f_c$ . Determine value of  $f_s$ . Refer to Table 10, page 519, and see if value of  $k$  corresponds to  $f_c$  and  $f_s$ . If not, select a new  $k$  and recompute.

VALUES OF MAXIMUM WORKING COMPRESSION TIMES AREA OF FLANGE ( $f_c b t$ ) USE FOR FINDING MINIMUM DEPTH

300000 290000 280000 270000 260000 250000 240000 230000 220000 210000 200000 190000 180000 170000 160000 150000 140000 130000 120000 110000 100000 90000 80000 70000 60000 50000



NOTE  
 USE FULL LINES FOR  $f_c = 650$  LBS.  
 $f_c = 16000$   
 USE DOTTED LINES FOR  $f_c = 550$  LBS.  
 $f_c = 16000$

MOMENTS IN INCH POUNDS USE FOR FINDING AREA OF STEEL

**WORKING STRESS IN REINFORCED CONCRETE.**

The Joint Committee on Concrete and Reinforced Concrete, 1909, recommend working stresses as follows:\*

**General Assumptions.** The following working stresses are recommended for static loads. Proper allowances for vibration and impact are to be added to live loads where necessary to produce an equivalent static load before applying the unit stresses in proportioning parts.

In selecting the permissible working stress to be allowed on concrete, we should be guided by the working stresses usually allowed for other materials of construction, so that all structures of the same class but composed of different materials may have approximately the same degree of safety.

The stresses for concrete are proposed for concrete composed of one part Portland cement and six parts aggregate, capable of developing an average compressive strength of 2 000 pounds per square inch at twenty-eight days when tested in cylinders 8 inches in diameter and 16 inches long, under laboratory conditions of manufacture and storage, using the same consistency as is used in the field. In considering the factors recommended with relation to this strength, it is to be borne in mind that the strength at twenty-eight days is by no means the ultimate which will be developed at a longer period, and therefore they do not correspond with the real factor of safety. On concretes in which the material of the aggregate is inferior, all stresses should be proportionally reduced, and similar reduction should be made when leaner mixes are to be employed. On the other hand, if, with the best quality of aggregates, the richness is increased, an increase may be made in all working stresses proportional to the increase in compressive strength at twenty-eight days, but this increase shall not exceed 25 per cent.

**Bearing.**† For compression on surface of concrete larger than loaded area.

32.5 per cent of compressive strength at twenty-eight days, or 650 pounds per square inch on 2 000 pound concrete.

**Columns.** (a) Plain columns or piers whose length does not exceed twelve diameters,

22½ per cent of compressive strength at twenty-eight days, or 450 pounds per square inch on 2000 pound concrete.

(b) Columns with reinforcement of bands or hoops.‡

27 per cent of compressive strength at 28 days, or 540 pounds per square inch on 2000 pound concrete.

Vertical steel reinforcement 8100 pounds per square inch.

\* The form in which these are given corresponds with the 1909 Report of the Reinforced Concrete Committee of the National Association of Cement Users.

† For beams and girders built into pockets in concrete walls, the lower compressive stress of 450 pounds per square inch should not be exceeded.

‡ The amount of band or hoop reinforcement must be at least 1 per cent of the volume of the column enclosed, and clear spacing of the bands or hoops not greater than one-fourth the diameter of the enclosed column.

(c) Columns reinforced with not less than 1% and not more than 4% of longitudinal bars and with bands or hoops spaced not greater than one-fourth the diameter of the enclosed column.

32½% of compressive strength at 28 days, or 650 pounds per square inch on 2000 pound concrete.

Vertical steel reinforcement, 9750 pounds per square inch.

(d) Columns reinforced with structural steel column units which thoroughly encase the core,

32½% of compressive strength at 28 days, or 650 pounds per square inch on 2000 pound concrete.\*

Vertical steel, 9750 pounds per square inch.

**Compression in Extreme Fiber.** For extreme fiber stress of beams calculated for constant modulus of elasticity,

32.5 per cent of the compressive strength at twenty-eight days, or 650 pounds per square inch for 2000 pound concrete.

Adjacent to the support of continuous beams, stresses 15 per cent greater may be allowed.

**Shear.** Pure shearing stresses uncombined with compression or tension, 6 per cent of compressive strength at twenty-eight days, or 120 pounds per square inch for 2000 pound concrete.

**Diagonal Tension.** In beams where diagonal tension is taken by concrete, the vertical shearing stresses should not exceed

2 per cent of compressive strength at twenty-eight days, or 40 pounds per square inch for 2000 pound concrete.

**Bond for Plain Bars.** Bonding stress between concrete and plain reinforcing bars,

4 per cent of compressive strength at twenty-eight days, or 80 pounds per square inch for 2000 pound concrete.

For drawn wire.

2 per cent, or 40 pounds on 2000 pound concrete.

**Bond for Deformed Bars.†** Bonding stress between concrete and deformed bars may be assumed to vary with the character of the bar from

5 per cent to 7½ per cent of the compressive strength of the concrete at 28 days, or from

100 to 150 pounds per square inch for 2000 pound concrete.

\* Lower stresses than these should be used unless the concrete is very carefully proportioned and placed. The authors recommend 500 lb. per sq. in. in general practice.

† No recommendation for deformed bars is given in the report of the Joint Committee; the values given are those suggested by the Reinforced Concrete Committee of the National Association of Cement Users.

**Reinforcement.** The tensile stress in steel should not exceed 16 000 pounds per square inch.\* The compressive stress in reinforcing steel should not exceed 16 000 pounds per square inch, or fifteen times the working compressive stress in the concrete.

**Modulus of Elasticity.** It is recommended that in all computations the modulus of elasticity of concrete be assumed as  $\frac{1}{15}$  that of steel; that is, that a ratio of fifteen be employed.

## STANDARD NOTATION†

$t$	= thickness of slab, <i>i.e.</i> , thickness of T-flange.
$b$	= breadth of beam; in a T-beam, breadth of T-flange.
$b'$	= breadth of stem of T-Beam.
$d$	= depth from surface of beam to center of tension steel.
$p$	= ratio of cross-section of steel in tension to cross-section of beam above this steel.
$p'$	= ratio of cross-section of steel in compression to cross-section of beam above the steel in tension.
$f_c$	= unit compressive stress in outside fiber of concrete.
$f_s$	= unit tensile stress, or pull, in steel.
$f'_s$	= unit compressive stress in steel.
$\bar{f}$	= average unit compression in a column.
$n$	= $\frac{E_s}{E_c}$ = Ratio of modulus of elasticity of steel in tension to modulus of elasticity of concrete in compression.
$k$	= ratio of depth of neutral axis to depth of steel in tension.
$kd$	= distance from outside compressive surface to neutral axis in beam in which the depth to steel in tension is $d$ .
$j$	= ratio of lever arm of resisting couple to depth $d$ .
$jd$	= arm of resisting couple.
$z$	= depth from surface of beam to center of compression.
$M$	= moment of resistance or bending moment in general.
$M_b$	= bending moment.
$M_r$	= resisting moment.
$M_2$	= bending moment in a flat slab causing radial fibre stress for loading distributed along the edge of the fixed plate.
$M_b$	= bending moment in a flat slab causing radial fibre stress for loading uniformly distributed over the plate.
$V$	= total shear.
$v$	= unit shear.
$v'$	= unit working shear.
$u$	= unit bond.
$o$	= circumference of one bar.
$\Sigma o$	= total circumference of all bars in a beam.
$A$	= total area.
$A_s$	= area of steel.
$w$	= unit loading for uniformly distributed load.
$q$	= unit loading for circumferential loading.
$i$	= diameter of bar.
$r$	= ratio unit cost of steel to unit cost of concrete.
$l$	= span of beam.

\*If the steel has a high elastic limit and is of the exceptional quality called for by the specifications on page 38, the authors would frequently permit a stress as high as 20 000 pounds per square inch.

†Substantially as adopted by the Joint Committee on Concrete and Reinforced Concrete and as used in this Treatise.

- $l_l$  = longer span of a rectangular slab.
- $l_s$  = shorter span of a rectangular slab.
- $M_l$  = bending moment of longer beam.
- $M_s$  = bending moment of shorter beam.
- $\alpha$  = denominator in bending moment formula  $M = \frac{w l^2}{\alpha}$
- $m$  = number of bars at the center of beam.
- $m_1$  = number of bars to be bent.
- $C$  = constant from Table 10, p. 519.
- $C_c, C_s, C'_s$  = constant from Table 8, p. 516.
- $C_1, C_2, C_a, C_b$  = constant from Table 9, p. 518.
- $C_s, C_b$  = constant from Table on p. 454.
- $r_0$  = inner radius of flat plates in feet.

COMMON FORMULAS.

Rectangular beam:

	Reference to Page		Reference to Page
Depth of steel, $d = C \sqrt{\frac{M}{b}}$	418	Steel area, $A_s = pbd$	418
Tension in steel, $f_s = \frac{M}{A_s j d}$	420	Compression in concrete, $f_c = \frac{2M}{bd^2 j k}$	420
Unit shear, $v = \frac{V}{b j d}$	447	Unit bond, $u = \frac{V}{j d \Sigma o}$	457
Ratio span to depth not requiring stirrups, $\frac{l}{d} > \frac{\alpha}{1.74 C^2 v}$	456		

T-Beam:

Horizontal steel, $A_s = \frac{M}{f_s \left( d - \frac{t}{2} \right)}$ (approx.)	426	Economical depth, $d - \frac{t}{2} = \sqrt{\frac{r M}{f_s b'}}$	425
Area required by shear, $b' \left( d - \frac{t}{2} \right) > \frac{V}{120}$	424	Length of haunch, $x = \frac{l}{5} \frac{M_b - M_r}{M_b}$ (approx.)	430
Depth of neutral axis, $kd = \frac{2 n d A_s + b t^2}{2 n A_s + 2 b t}$	755	$z = \frac{3 kd - 2 t}{2 kd - t} \frac{t}{3}$	755
Moment arm, $j d = d - z$	755	Compression in concrete, $f_c = \frac{M k d}{b t (kd - \frac{1}{2} t) j d}$	756
Tension in steel, $f_s = \frac{M}{A_j d}$	756		

Beam with Steel in Top and Bottom:

Moment, $M = f_c b d^2 C_c$ } which ever is or $= f_s b d^2 C_s$ } larger	428	Compression in concrete, $f_c = \frac{M}{b d^2 C_c}$	428
Tension in steel, $f_s = \frac{M}{b d^2 C_s}$	428	Compression in steel, $f'_s = \frac{M}{b d^2 C'_s}$	428

Flat Plates

Radial loading, Max. $M_2 = w r_0^2 (0.2 + C_1 + C_2)$	486	Circumferential loading, Max $M_b = q r_0 (C_a + C_b)$	486
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Column:

Ratio steel, $p = \frac{f - f_c}{f_c (n - 1)}$	491	Average unit load, $\frac{P}{A} = f = f_c [1 + (n - 1) p]$	491
Area column, $A = \frac{P}{f_c [1 + (n - 1) p]}$	491		

Stirrups:

Area of stirrups, $A_s = \frac{2 s V}{3 f_s j d}$	449	Spacing of stirrups, $s = \frac{3 A_s f_s j d}{2 V}$	450
Distance from support where no stirrups needed, $x = \frac{l}{2} - \frac{v b j d}{w}$	451	Max. distance from support where rods may be bent, $x_1 = \frac{l}{2} \left( \frac{1}{2} - \sqrt{\frac{8 m_1}{\alpha m}} \right)$	459
Diameter vertical stirrup for square or round bar, $i < C_s d$	454	For other shapes, $\frac{A_s}{o} < \frac{1}{4} C_s d$	454
Diameter inclined stirrup for square or round bar, $i < C_b d$	454	For other shapes, $\frac{A_s}{o} < \frac{1}{4} C_b d$	454

Distribution of slab load to supporting beams:

For longer beam, $M_l = \frac{1}{8} w l_l^2 \left( 1 - \frac{1 (l_s)^2}{3 (l_l)^2} \right)$	431	For shorter beam, $M_s = \frac{3}{8} (w l_s^2)$	431
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