the concrete it may be termed reinforcement. In this case the steel is figured in the same way as vertical bars and the stresses determined from formula ( 60 ), page 49I. If, for example, the allowable stress on the concrete is $45^{\circ}$ pounds per square inch and a ratio of $I_{5}$ is used, the steel can be figured only for a compressive stress of 6750 pounds per square inch.

To utilize the full working strength of the steel, the plan has sometimes been followed of separating the structural steel core from the concrete so that they will work independently, and designing the columns in the lower stories so that the steel will take the entire weight of the upper stories while the concrete surrounding the steel supports the weight of the lower stories.

Structural steel reinforcement is sometimes in the form of a cross in the center of the column, or, as in the case of the McGraw building,* channels counected by riveted latticing are placed and concrete poured within the reinforcement as well as providing a protective layer around it. Stresses for this type are suggested on page 528 .
Tests $\dagger$ of columns reinforced with structural steel shapes frequently show lower ultimate strength than similar columns reinforced with the same quantity of steel in the form of vertical round bars. This probably is due in part to the difficulty in properly placing the concrete around the structural steel.

## COLUMN EXAMPLES.

Example 15: What size of square column reinforced with 2 per cent of longitudinal bars without bands will be required to support a load of 94000
pounds? pounds?

Solution: By paragraph (a), page $5^{2} 7$, the allowable compression on 2000 pounds concrete is limited to 450 pounds per square inch. For this allowable stress, using $2 \%$ of longitudinal reinforcement and a ratio of moduli of elasticity of 15 , the area of column from formula (62), page 49 1 , is

$$
A=\frac{94000}{450(\mathrm{I}+14 \times 0.02)}
$$

$=163$ square inches, corresponding to 12.8 inches square. The denominator of this expression may be obtained directly from table on page 492. Allowing 2 inches for protective covering gives 14.8 inches, or, say, 15 inches square.
Example 16: Find the diameter of a round column reinforced by 1 per cent of hooping only, designed to support a load of 120000 pounds. Assume the allowable pressure on plain concrete as 450 pounds and a ratio of moduli of elasticity, $n=15$

Solution: The allowable unit compression on hooped columns may be increased 20 per cent over that on plain concrete (see paragraph (b), page $5^{27} 7$ ),
*William H. Burr, Transactions American Society of Civil Enginees Vo. LX, 1908, p. 443. $\dagger$ M. O. Withey in Engineerng Record, July 10, 1909, P. 41
hence $f=450+20 \%=540$. The area of section from formula $(63)$ is

$$
A=\frac{120000}{540}
$$

$=222$ square inches, giving an effective diameter of 16.8 inches. Adding 3 inches for protective covering gives a total diameter of 20 inches
Example 17: What sectional area of vertical steel will be required for a square column limited to 36 inches diameter, which has to bear I 000000 pounds with pressure in plain concrete limited to $45^{\circ}$ pounds per square inch?
Solution: By paragraph (c), page 528 , in a column reinforced with vertical bars and $1 \%$ of bands or hoops, the allowable pressure on the concrete may be increased $45 \%$ over that on plain concrete, hence $f_{c}=450+45 \%=652$ rogo square inches as effective, the unit pressure from page 491, will be

$$
f=\frac{1000000}{1000}
$$

$=918$ pounds per square inch
Assume $n=15$, then from formula (6I)

$$
p=\frac{918-652}{14 \times 652}
$$

$=0.029$, and area of steel, $A=1090 \times 0.029=31.6$ square inches. From table on page $50 \%$, it is found that 18 round rods $\mathrm{I}_{\frac{1}{2}}$ inches diameter will give the required area.
Example 18: What should be the area of a column 10 feet high supporting I 000000 pounds, reinforced with $3.5 \%$ of longitudinal reinforcement and rimited to 450 pounds?
Solution: Since the column is reinforced with longitudinal and hooping reinforcement, the unit compression on concrete may be taken as $f_{c}=450$ $+45 \%=652$ pounds per square inch (paragraph c, page 528 ). Then from formula (62), page 491, the column area is

$$
A=\frac{1000000}{65^{2}(1+14 \times 0.035)}
$$

$=1030$ square inches. The denominator of this expression may be ob tained directly from table on page 492.

## REINFORCEMENT FOR TEMPERATURE AND SHRINKAGE STRESSES

All masonry is subject to temperature cracks, but when they are distrit, uted in the many joints between bricks or stones they do not show so plainly as on the smooth surface of concrete
Expansion from a rise in temperature rarely causes trouble except at angles where the lengthening of the surface may produce a buckling or a sliding of one portion of the wall past the end of the other. In a building, the walls and floors are generally so well bonded together and free to move
as a unit, that no provision need be made for expansion. In a structure like a square reservoir, the effect of expansion must be taken into account in the design to prevent failure at the corners.
Contraction is often more serious, although cracks are by no means necessarily dangerous. To prevent cracking due to the shrinkage of the concrete in hardening (see p. 287) or to the lowering of the temperature, reinforcement should be inserted or joints formed to localize the cracks. (See p. 285.)

Reinforcement properly placed distributes the contraction stresses so as to make the cracks very small, practically invisible, but it does not prevent them entirely.

The steel must be sufficient in quantity, and should be of small diameter and placed as close as practicable to the surfaces to distribute the cracks and thus make them very fine. Deformed bars, that is, bars with irregular surfaces which provide a mechanical bond with the concrete, are more effective than smooth bars, and steel of high elastic limit also is advantageous.

In practice, from $\frac{2}{10}$ of $\mathrm{I} \%$ to $\frac{4}{10}$ of $\mathrm{I} \%$ (a ratio of 0.002 to 0.004 ) of steel, based on the cross-section of the concrete, is commonly used as temperature or shrinkage reinforcement.

The tensile strength of concrete is so low that a small change in temperature will crack it. For example, the coefficient of expansion of concrete is 0.0000055 (see p. 287 ) and the modulus of elasticity is generally assumed as 2000000 ; therefore, the stress (see p. 404) per degree Fahrenheit is $0.0000055 \times 2000000=$ II pounds per square inch, and a fall in temperature of $\frac{300}{11}=27^{\circ}$ is sufficient to crack a concrete the tensile strength of which is 300 pounds per square inch.
It is evident, and it has been proved by experience, that there is less cracking in concrete laid in cold than in warm weather.

Longitudinal reinforcement is especially necessary in conduits which must be water-tight.
Shrinkage cracks due to the hardening of the concrete may be prevented by keeping the concrete wet. (See p. 287.)

It has been suggested by Mr. Charles M. Mills that the relation between the tensile strength of the concrete and the bond with the bars is an important factor in governing the size of the cracks, and the following analysis, based on his suggestions, gives a means of estimating the size and distance apart of the cracks so as to form a basis for judgment as to the sizes and percentages of steel to use.

The tensile stress in the steel at a crack tends to pull out the bars from
the concrete, and referring to Fig. 154, the bond stress of the bar in the length $a b$ must equal the tensile stress in the whole cross-section of the concrete at $b$ caused by the contraction of the concrete.

## Let

$x=$ distance apart of cracks.
$D=$ diameter of round bar or side of square bar.
$P=$ ratio of cross-section of steel to cross section of concrete.
Then,* if, as is sufficiently accurate for practical purposes, the strength of concrete in tension is assumed to be equal to the bond between plain steel bars and concrete, the distance apart of cracks is

$$
x=\frac{D}{2 p} \text { for square or round bars. }
$$

The distance apart is inversely proportional to the unit bond*, so that a deformed bar having twice the bond strength would space the cracks one-half as far apart and allow them to be only one-half as wide.


Fig. 154. Reinforcement for Temperature Stresses. (See p. 501.)
It is evident that the distance apart of the cracks is proportional to the diameter of the reinforcing bars, and inversely proportional to the percentage of steel.
From this formula is tabulated the estimated percentage of reinforcement for different spacing of cracks and different sizes of bars, assuming the bonding strength of the steel to the concrete to equal the tensile strength of the concrete.

* In addition to above notation, let
$A_{c}=$ area of section of concrete.
$A_{s}=$ area of section of steel.
o = perimeter of steel bar
$f_{c}^{\prime}=$ tensile stress in concrete.

$$
\text { Then } A_{c} f_{c}^{\prime}=\frac{1}{2} u o x \text {, or } x=\frac{2 A f_{c}^{\prime}}{u_{0}} \text {. If } f_{c}^{\prime}=u, x=\frac{2 \mu_{c}}{0} \text {, and since } p=\frac{A_{s}}{A_{c}}
$$

$$
x=\frac{2 A_{8}}{0 p} \text {. Also, } \frac{A_{s}}{0}=\frac{D}{4} \text { for both round and square bars, hence } x=\frac{1}{2} \frac{D}{p} \text {. }
$$

Estimated Percentage of Reinforcement for Different Spacing of Cracks
Distance apart of cracks with

| plain bars......... deformed bars *. |  | $\begin{aligned} & 12^{\prime \prime} \\ & 8^{\prime \prime} \end{aligned}$ | $\begin{aligned} & 18^{\prime \prime} \\ & 12^{\prime \prime} \end{aligned}$ | $\begin{aligned} & 24^{\prime \prime} \\ & 16^{\prime \prime} \end{aligned}$ | $\begin{aligned} & 36^{\prime \prime} \\ & 24^{\prime \prime} \end{aligned}$ | $\begin{aligned} & 48^{\prime \prime} \\ & 32^{\prime \prime} \end{aligned}$ | $\begin{aligned} & 60^{\prime \prime} \\ & 40^{\prime \prime} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Diameter of round or side of square bar. |  | $\begin{gathered} \% \\ 1.04 \\ 1.56 \\ 2.08 \\ 2.60 \\ 3.12 \\ 3.65 \\ 4.17 \end{gathered}$ | $\begin{gathered} \% \\ 0.70 \\ 1.04 \\ 1.39 \\ 1.74 \\ 2.08 \\ 2.44 \\ 2.78 \end{gathered}$ | $\begin{gathered} \% \\ 0.5^{2} \\ 0.78 \\ 1.04 \\ 1.30 \\ 1.56 \\ 1.82 \\ 2.08 \end{gathered}$ | $\begin{gathered} \% \\ 0.35 \\ 0.52 \\ 0.69 \\ 0.87 \\ 1.04 \\ 1.22 \\ 1.39 \end{gathered}$ | $\begin{gathered} \% \\ 0.26 \\ 0.39 \\ 0.5^{2} \\ 0.65 \\ 0.78 \\ 0.91 \\ 1.04 \end{gathered}$ | $\begin{gathered} \% \\ 0.21 \\ 0.31 \\ 0.41 \\ 0.52 \\ 0.62 \\ 0.73 \\ 0.83 \end{gathered}$ |

Note: To express the steel as the ratio of area of cross-section of steel to cross-section of concrete, divide the percentages by roo; thus 1.04 becomes $p=0.0104$.

* Assuming the bond of deformed bars to be $50 \%$ greater than plain.

The size of the crack is governed by the amount of shrinkage and for cracks due to temperature changes may be estimated as the product of the coefficient of contraction ( 0.0000055 ) by the number of degrees fall in temperature by the distance between cracks.

Estimated Width of Cracks for Different Distances Apart

|  | dibtance apart |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| WIDTH FOR DIEFERENT TEMPERature changes. $\qquad$ | $12^{\prime \prime}$ | 18" | $24^{\prime \prime}$ | $36^{\prime \prime}$ | $48^{\prime \prime}$ | $60^{\prime \prime}$ |
| $30^{\circ} \mathrm{Fahr}$. | 0.0020 | 0.0030 | 0.0040 | 0.0059 | 0.0079 | 0.0099 |
| $50^{\circ}$ " | 0.0033 | 0.0050 | 0.0066 | 0.0099 | 0.0132 | 0.0165 |
| $70^{\circ}$ | 0.0046 | 0.0069 | 0.0092 | 0.0139 | 0.0185 | 0.0232 |

From this, if it can be determined how large a crack will be allowable, the corresponding spacing can be obtained.

To avoid large cracks it may be necessary to use enough steel to prevent its passing its elastic limit. If the bars are continuous for such a length that the ends are practically immovable, as in a long retaining wall, a drop

* $30^{\circ}$ corresponds to a shrinkage of $0.017 \% ; 50^{\circ}$ to $0.028 \% ; 70^{\circ}$ to $0.038 \%$.
in temperature, tending to shorten them, produces a tensile stress which is independent of the distance between the restrained ends. Assuming the coefficient of expansion of steel the same as concrete and the modulus of elasticity of steel as 30000000 , this stress is $30000000 \times 0.0000055$ $=165$ pounds per square inch per degree of temperature, or for $50^{\circ} \mathrm{Fahr}$. is 8250 pounds per square inch. This is well within the elastic limit of the steel and would not, of itself, cause the steel to take a permanent set. However, since the concrete surrounding the steel will be continuous except at certain cracks, the stretch in the steel may be unevenly distributed and largely confined to the immediate vicinity of the cracks. If cracks occur while steel is unstressed, through the concrete shrinking, the steel tends to resist the shrinkage by tension at the crack and compression at the center of the block of concrete, and the tensile stress will be equal to the compressive and each equal to one-half the tensile strength of the concrete. This may be expressed by the following formula, using the foregoing notation:*

$$
f_{s}^{\prime}=\frac{1}{2 p} f_{c}^{\prime}
$$

Since the tensile stress in the concrete is liable to be low at the time shrinkage cracks are formed, it may be assumed, for illustration, as 200 pounds per square inch making

$$
f_{s}^{\prime}=\frac{100}{p}
$$

This represents the stress due to local cracks which is additional to the temperature stresses above described. The total stress is, therefore, for $50^{\circ}$ change of temperature $8250+f_{8}^{\prime}$ or $8250+\frac{100}{p}$. If the elastic limit of the steel is 40000 pounds per square inch, and we must keep below this,

$$
40000=8250+\frac{100}{p} \text { and } p=0.0031
$$

For steel, the elastic limit of which is 50000 pounds per square inch,

$$
50000=8250+\frac{100}{p} \text { and } p=0.0024
$$

These values of $p$ represent the lowest theoretical ratio of area of crosssection of steel to area of cross-section of concrete which can be used without the steel passing its elastic limit at certain of the cracks when the ends are restrained or the length is so great that intermediate parts are practically restrained.

$$
\text { * } \frac{A_{c} f_{c}^{\prime}}{2}=A_{s} f_{s}^{\prime} \text { or } f_{s}^{\prime}=\frac{A_{c}}{2 A_{s}} f_{c}^{\prime} \text { hence } f_{s}^{\prime}=\frac{1}{2 p} \cdot f_{c}^{\prime}
$$

In view of the very slight-stretch required to relieve the stress in the bars when the elastic limit is exceeded, and the probability of its distribution by the restraint to movement by the mass, it is not always essential to consider the elastic limit.

## SYSTEMS OF REINFORCEMENT

One of the earliest recorded examples of the application of reinforced concrete is a boat of concrete and iron, built by Mr. L. J. Lambot in France, and shown at the Paris International Exhibition in 1855.* In 186ı Mr. Coignet began his investigations, and in 1866 Mr . Monier, to whom the invention of reinforced concrete is often attributed, applied the combination of concrete and iron to various structures, and laid the foundation for its future widespread applications.
As long ago as 1877 , Mr. W. E. Ward, $\dagger$ at Port Chester, N. Y., built a house entirely of concrete, reinforced with iron I-beams and round rods.
The rapid development of reinforced concrete has resulted in the introduction of numerous systems, many of them covered by patents, for arranging the metal in the concrete, or for special forms of metal. These systems are fully described in the various French works on reinforced concrete. $\ddagger$
A few of the systems, representing both the arrangement and the form of the metal, are described below, and forms of metal extensively used in the United States are illustrated in Fig. 155.

## Systems of Reinforcement

Bonna. Metal of cruciform cross-section.
Berlini. Girder Frame. Horizontal tension members with vertical sir rups shrunk on to them.
Chaudy and Degon. Cross rods passing under bearing rods, but looped up between them.
Coignet. Round bars in top and bottom of beam connected by diagonal wire lacing.
Columbian. Vertical steel plates with horizontal ribs.
Cottacin. Round rods interlaced in the same manner as in wire netting. Cummings. Bars of different lengths having their ends bent to an incline and formed into a loop to resist internal stresses.
Cup Bar. Special rolled section with longitudinal ribs connected at frequent intervals by cross ribs forming cup depressions.

* Christophe's Betoo Armé, 1902, p. I.

Transactions American Society Mechanical Engineers, Vol. IV, p. 388.
$\ddagger$ See among others Christophe's Beton Armé, 1902, pp. 10-71, and Morel's Ciment Armé, 1902. pp. 88 to 152 .


FIG. 155.-Types of Reinforcing Steel. (See pp. 504 and 506.

De Man. Undulated Bars. (See Fig. 155.)
Diamond Bar. Bars rolled round with parallel ribs passing along and around the bar forming diamond-shaped shoulders on its surface.
Donath. Inverted T-beams or I-beams connected by horizontal diagonals of light, flat metal on edge.
Expanded Metal. Sheet steel, slit and expanded, so as to form a diamond mesh. (See Fig. 155, p. 505.)
Ferroinclave. Sheet steel with inversely tapered corrugations to be covered on both sides with concrete.

Gabriel. Deformed tension members with trussing of hard drawn wire. Habrich and Düsing. Flat metal twisted hot.
Hennebique. A combination of alternate straight bars and bars with ends bent up at an angle, with vertical U-bars, or stirrups, of flat iron passing around the straight bars and reaching nearly to the top of the beam.
Herringbone Frame. Horizontal tension member with special attachments for stirrups.
Holzer. Metal in form of I-beams.
Hyatt. Flat plates or bars set on edge and pierced with holes through which pass small round rods to form the cross reinforcements.
Johnson. Corrugated bars. (See Fig. 155, p. 505.)
Kahn. Horizontal flanged bars_ with flanges sheared up at intervals. (See Fig. 155, p. 505.)

Lock-Woven Steel Fabric. Steel wire mesh, locked at intersections.
Lug Bars. Twisted bars with projecting lugs at intervals in the surface.
Melan. Steel ribs, either I-beam or 4 angles latticed, imbedded in the concrete of the arch.
Monier. Two series of round parallel bars at right angles to each other.
Mushroom. Flat floor slabs supported by columns with enlarged heads.
Mushroom. Flat floor slabs supported bey with bent ends, to place in the sides of a conduit or the
Parmley haunches of an arch to resist tension.
Rabitz. Various combinations employing galvanized wire.
Rabitz. Various combinations employing galvanized wire.
Ransome. Square steel rods twisted cold. (See Fig. 155, p. 505.)
Roebling. Flat steel bars set on edge, clamped to supporting beams, and held in alignment by flat bar separators.
Schülter. Like Monier System except rods are placed diagonally.
Scofield. An oval bar with projecting shoulders.
Thacher. Bulb bars. (See Fig. 155, p. 505.)
Triangle Mesh. Wire mesh reinforcement with transverse metal placed diagonally.
Trussit. Expanded metal or herringbone lath bent to V-shaped section.
Visintini. Beams of concrette, cored out so as to form lattice girders.
Welded Wire Fabric. Wire mesh reinforcement with wires at right angles to each other and welded at intersections.

TABLE I. AREAS, WEIGHTS AND CIRCUMFERENCES OF BARS.
Areas and Weights. of Square and Round Rods and Circumjerences of Round Rods.
One cubic foot weighs 490 lb .

|  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 0 \\ \frac{1}{16} \\ \frac{1}{8} \\ \frac{3}{16} \end{gathered}$ |  |  |  |  |  | 2 | 4.0000 | 3.1416 | 6.2832 | 13.60 | 10.68 |
|  | 0.0039 | 0.0031 | 0.1963 | 0.013 | 0.010 | 15 | 4.2539 | 3.3410 | 6.4795 | 14.46 | II. 36 |
|  | 0.0156 | 0.0123 | 0.3927 | 0.053 | 0.042 |  | 4.5156 | 3.5466 | 6.6759 | 15.35 | 12.06 |
|  | $0.035^{2}$ | 0.0276 | 0.5890 | 0.119 | 0.094 | $\frac{3}{16}$ | 4.7852 | 3.7583 | 6.8722 | 16.27 | 12.78 |
|  | 0.0625 | 0.0491 | 0.7854 | 0.212 | 0.167 |  | 5.0625 | 3.9761 | 7.0686 | 17.22 | 13.52 |
|  | 0.0977 | 0.0767 | 0.9817 | 0.333 | 0.261 | $\frac{\frac{5}{15}}{}$ | 5.3477 | 4.2000 | 7.2649 | 18.19 | 14.28 |
|  | 0.1406 | 0.1104 | I. 1781 | 0.478 | 0.375 |  | 5.6406 | 4.4301 | 7.4613 | 19.18 | 15.07 |
|  | 0.1914 | 0.1503 | 1.3744 | 0.651 | 0.511 | $\frac{7}{16}$ | 5.9414 | 4.6664 | 7.6576 | 20.20 | 15.86 |
| $\begin{aligned} & \frac{9}{16} \\ & \frac{8}{8} \\ & \frac{18}{18} \end{aligned}$ | 0.2500 | 0.1963 | 1.5708 | 0.850 | 0.667 | $\frac{1}{2}$ | 6.2500 | 4.9087 | 7.8540 | 21.25 | 16.69 |
|  | 0.3164 | 0.2485 | 1.7671 | 1.076 | 0.845 | $\frac{9}{16}$ | 6.5664 | 5.1572 | 8.0503 | 22.33 | 17.53 |
|  | 0.3906 | 0.3068 | -1.9635 | 1.328 | 1.043 |  | 6.8006 | 5.4119 | 8.2467 | 23.43 | 18.40 |
|  | 0.4727 | 0.3712 | 2.1598 | 1. 608 | 1.262 | ${ }^{\frac{11}{16}}$ | 7.2227 | 5.6727 | $8.443^{\circ}$ | 24.56 | 19.29 |
| $\begin{array}{r} \frac{3}{4} \\ \frac{3}{4} 8 \\ \frac{7}{8} \\ \frac{1}{8} \\ \frac{18}{18} \\ 1 \\ 1 \\ \frac{1}{16} \\ \frac{1}{8} \\ \frac{1}{16} \end{array}$ | 0.5625 | 0.4418 | 2.3562 | 1.913 | 1. 502 | ${ }_{4}^{4}$ | 7.5625 | 5.9396 | 8.6394 | 25.00 | 20.20 |
|  | 0.6602 | 0.5185 | 2.5525 | 2.245 | 1.763 | $\frac{13}{13}$ | 7.9102 | 6.2126 | 8.8357 | 26.90 | 21.1 |
|  | 0.7656 | 0.6013 | 2.7489 | 2.603 | 2.044 | 7 | 8.2656 | 6.4918 | 9.032 I | 28.10 | 22.07 |
|  | 0.8789 | 0.6903 | $2.945^{2}$ | 2.989 | 2.347 | 3 | 8.6289 | 6.7771 | 9.2284 | 29.34 | 23.04 |
|  | 1.0000 | 0.7854 | 3.1416 | 3.400 | 2.670 | $\frac{15}{15}$ | 9.0000 | 7.0686 | 9.424 | 30.60 | 24.03 |
|  | I.1289 | 0.8866 | 3.3379 | 3.838 | 3.014 | $\frac{1}{15}$ | 9.3789 | 7.3662 | 9.6211 | 31.89 | 25.04 |
|  | I. 2656 | 0.9940 | 3.5343 | 4.303 | 3.379 |  | 9.7656 | 7.6699 | ${ }^{9} .8175$ | 33.20 | 26. |
|  | 1.4102 | 1.1075 | 3.7306 | 4.795 | 3.766 | $\frac{3}{10}$ | -10.160 | 7.9798 | 10.014 | 34.55 | 27.13 |
| $\begin{aligned} & \frac{4}{17} \\ & \frac{4}{8} \\ & \frac{8}{3} \\ & \frac{3}{16} \end{aligned}$ | 1.5625 | 1.2272 | 3.9270 | 5.312 | 4.173 | $\frac{1}{4}$ | 10.563 | 8.2958 | 10. | 35.92 | 28.20 |
|  | 1.7227 | 1.3530 | 4.1233 | 5.857 | 4.600 | $\frac{5}{16}$ | 10.973 | 8.6179 | 10.407 | 37.31 | 29.30 |
|  | 1.8006 2.0664 | I. 4849 <br> I. 6230 | 4.3197 | 6.428 | 5.049 |  | 11.391 | 8.9462 | 10.603 | 38.73 | 30.42 |
|  | 2.0664 | 1.6230 | 4.5160 | 7.026 | 5.518 | $\frac{7}{16}$ | 11.816 | 9.2806 | 10.799 | 40.18 | 31.56 |
| $\begin{aligned} & \frac{9}{18} \\ & \frac{8}{8} \\ & \frac{11}{18} \end{aligned}$ | 2.2500 | 1.7671 | 4.7124 | 7.650 | 6.008 | \% | 12.250 | 9.6211 | 10.996 | 41.65 | 32.71 |
|  | 2.4414 | 1.9175 | 4.9087 | 8.301 | 6.520 | $\frac{9}{16}$ | 12.691 | 9.9678 | II.192 | 43-14 | 33.90 |
|  | 2.6406 | 2.0739 | 5.105x | 8.978 | 7.051 |  | 13.141 | 10.321 | IT. 388 | 44.68 | 35.09 |
|  | 2.8477 | 2.2365 | 5.3014 | 9.682 | 7.604 | $\frac{11}{16}$ | ${ }^{1} 3.598$ | 10.680 | 11.585 | 46.24 | 36.31 |
| $\begin{aligned} & \frac{3}{4} \\ & \frac{13}{16} \\ & \frac{13}{7} \\ & \frac{15}{15} \end{aligned}$ | 3.0625 | 2.4053 | 5.4978 | 10.41 | 8.178 |  | 14.063 | 11.045 | 11.781 | 47.82 | 37.56 |
|  | 3.2852 | 2.5802 | 5.6941 | 11.17 | 8.773 | $\frac{13}{18}$ | 14.535 | 11.416 | 11.977 | 49.42 | 38.81 |
|  | 3.5156 | 2.7612 | 5.8905 | 11.95 | 9.388 | 8 | 15.016 | 11.793 | 12.174 | 51.05 | 40.10 |
|  | 3.7539 | 2.9483 | 6.0868 | 12.76 | 10.02 | $\frac{15}{16}$ | 15.504 | 12.177 | 12.370 | 52.71 | 41.40 |

## BEAM AND SLAB TABLES

Beam Tables. Tables 2,3, and 4, pages 509, 510 and 511 , give the loading and reinforcement for beams based on I inch of width under different conditions. For a beam 10 inches wide, for example, both the safe load per linear foot and the steel area will be ten times the values given in the tables.

The tables are for rectangular beams but may be used for T-beams which have a depth 3 or 4 times the thickness of slab by taking the width of flange as the breadth, , $b$

Table 2 is for a simply supported beam and is based on a working compressive stress in concrete of 500 pounds per square inch and in steel of 14000 pounds per square inch-lower values than are customarily used in construction, but required in many building laws. If the compression in concrete is limited to 500 pounds, while 16000 pounds is permitted in the steel, use the same loading but reduce the steel in the ratio of 16 to 14 .

Tables 3 and 4 are for ordinary design, approved by the authors and corresponding to recommendations of the Joint Committee. All tables are based on a ratio of elasticity of $n=15$. (See p. 408.)

For other working stresses than those given, the loads may be multiplied by ratios of the values of the constant $C$ in Table 10, page 519 , since $C$ is proportional to the load.

The uses of the tables are illustrated in Examples 12 and 13 . As high steel is not recommended for ordinary work on a small scale, no table is presented for safe loads for concrete reinforced with it.

Slab Table. Table 5 is for slab design with different working stresses in the steel and concrete. Ordinarily, the series at the top of the second page of the table is used. Note that the values are based on $\frac{w l^{2}}{10}$. For $\frac{w l^{2}}{12}$,generally used where the slabs are fully continuous over the supports, add $20 \%$ to the toads, leaving the area of steel as given. For square slabs fully reinforced in both directions, the loads may be doubled, or if also fully continuous, they may be doubled and $20 \%$ added also.

Table 6 is more convenient for review of beams already designed. It is computed by using formulas (7) and (8) on page 753, and selecting the lower value of $M$. The most economical ratio of steel for the limiting stresses is $p=0.0077$. For ratios lower than this the safe loads on the slabs are governed by the tensile strength of the steel, while for larger ratios they are limited by the working strength of the concrete in compression.

