tensile value of bars is in excess of stress to be provided for. It is also necessary that the bent bars be properly distributed and since shear is nearly uniform between the supports and the intersection of the beam, the inclined bars should be spaced at points *a*, *b*, *c*.

These points were found by dividing the distance on the center line *A B* into equal parts. They should be laid off on the neutral axis, but since the neutral axis changes for the positive and negative moment, the center line, as lying between the two neutral axes, was selected.

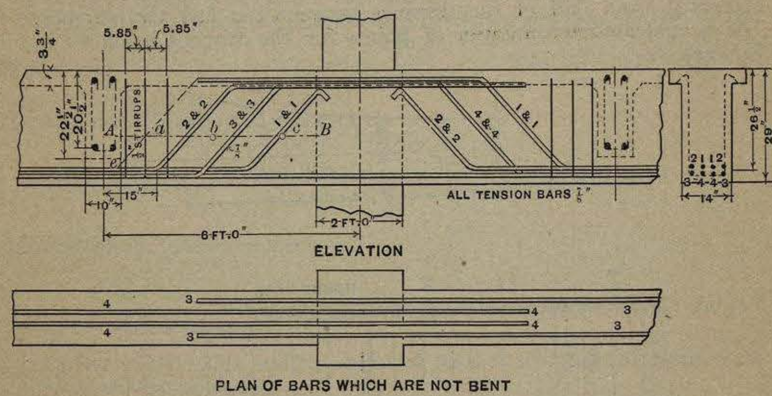

FIG. 149.—Reinforcement for Girder (*See p.* 474).

A study must be made to see whether the tensile stresses in the bottom of the beam will permit this. In this case the girder is loaded by concentrated loads and the moment at the point where the beam intersects the girder is nearly the maximum. Approximate figuring of tensile stresses shows that the first two bars may be bent about 15 inches from the center of the intersection of the beam, while to resist diagonal tension the bar to intersect the center line at *a* should be bent at *e* as shown by the dotted line. To provide for the diagonal tension, between point *a* and the beam stirrups will be introduced. Using ½-inch rods for stirrups, the tensile value of which is $2 \times .196 \times 16\,000 = 6\,270$ pounds, it is necessary to space them $\frac{6270}{1065} = 5.85$ inches apart, as shown in Fig. 149, the shear to be provided for in one inch of length of beam being 1 065 pounds.

EXAMPLE OF BENT BARS AS REINFORCEMENT FOR DIAGONAL TENSION

As indicated in the design for the girder in the example just given it is possible to provide for the diagonal tension by bent bars without stirrups, When the loading is uniformly distributed instead of concentrated, the location of the bends in the different bars as well as the size of the bars to use should be governed by the distribution of the shear. This is illustrated in the example which follows.

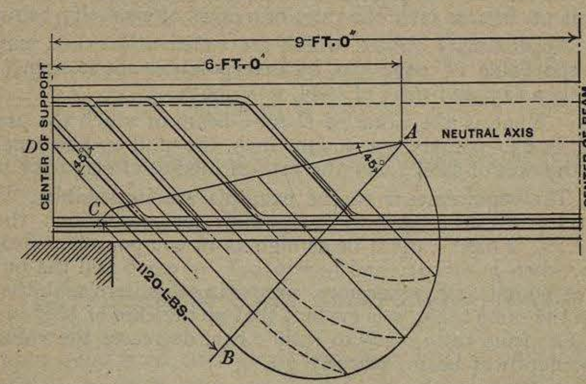

FIG. 150.—Spacing of Bent Bars. (*See p.* 475.)

Example 7—Suppose the 18-foot girder in previous example is loaded uniformly with 4 600 pounds per foot of length, find the locations of the points to bend up the bars to resist diagonal tension.

Solution—The load selected will require a beam of same section and tension reinforcement as the girder in previous example, where breadth of stem, $b' = 14$, depth to steel, $d = 26.5$, and depth from center of compression to tension, $jd = 24.6$. Then $V = 41\,400$ pounds and from page 447, the unit shear $vb' = \frac{V}{jd} = \frac{41\,400}{24.6} = 1\,680$ pounds per inch of length of beam. Two-thirds of this amount or 1 120 pounds per one inch of length of beam has to be provided for by diagonal tension reinforcement. The distance from the support of the limiting point where shear can be taken by concrete itself is $x_1 = 9\,\frac{40 \times 14 \times 24.6}{4\,600} = 6$ feet, formula (38), page 451.

From this point to the right the shear increases from zero to its maximum value of 1 120 pounds at the support, and may be represented by the triangle *A B C*, Fig. 150. This triangle may be drawn in the following manner: From point *A* at the neutral axis draw a line *A B* at 45 degrees, and from point *D* a perpendicular to line *A B* through point of intersection *B*. Lay out the maximum shear *B C*. Now, suppose we intend to bend four bars, all of the same diameter, to take the diagonal tension, then each of them will take an equal part. Divide the area of the triangle into three equal parts, find centers of gravity of each part, and from these centers of gravity draw lines to represent the location of points to bend up the bars in the girder. The method of division of the triangle into an equal number of parts is clearly shown in the drawing where the line *A B* is divided into equal parts and dotted arcs of cricles are drawn with centers at *A*.

MISCELLANEOUS EXAMPLES OF BEAM AND SLAB DESIGN.

Example 8: What is the value of C and the ratio of steel if pressure in concrete is limited to 400 pounds per square inch and pull in steel to 12 000 pounds per square inch, the ratio of moduli of elasticity being 15?

Solution: Approximate values, which are sufficiently exact, may be obtained from the Table 11, page 519, by exterpolation above item (1), from which C equals 0.123, and ratio of steel, p = .0053.

Example 9. What is the value of C for a beam in which the pressure in the concrete is 650 pounds per square inch, the pull in the steel 16 000 pounds, and the area of steel 1.2%, the ratio of moduli of elasticity being 15?

Solution: The requirements in the example are impossible. With the pressure in the concrete limited to 650 pounds per square inch, the pull in the steel, if 1.2% is used, cannot be as high as 16 000 pounds. From Table 11, page 520, when p = 0.012 and f_c = 650, C = 0.090 and the pull in the steel is 12 100 pounds. Furthermore, comparing this item with the line for 0.008 steel in the same table, it is evident that an increase of 50% in the area of the steel, *i.e.*, from ratio 0.008 to ratio 0.012, decreases the value C, and therefore the depth of beam, scarcely 7%.

Example 10: What safe load per square foot can be supported by a slab 5 inches thick and 10-foot span reinforced with ½-inch round bars placed 8 inches apart?

Solution. From slab table, page 514, since the given reinforcement from page 507 is equivalent to 0.196 × 1½ = 0.294 square inches for one foot of width, we find by inspection that for a 5-inch slab the nearest area of steel in column (18) is 0.288. Hence, the total safe load for a 10-foot span is slightly more than 136 pounds, say, 140 pounds per square foot; and deducting the weight per square foot of the slab, column (15), gives 140 − 64 = 76 pounds per square foot safe live load. If slab is square, continuous and reinforced in two directions, the safe load of 140 pounds may be multiplied by 2. Deducting the dead load of 64 pounds, the live load will be 280 − 64 = 216 pounds per square foot.

Example 11: What safe load per square foot can be placed upon an 8-inch slab, 16 foot span, having steel reinforcement of 0.007?

Solution: Since by Rule 3, on page 513, total loads are inversely proportional to the squares of the span, the load for a 16-foot slab is ¼ the load for an 8-foot slab. For the total safe load of an 8-foot slab, we must interpolate between steel ratios of 0.006 and 0.008, thus obtaining

$$\frac{649 + 831}{2}$$

= 740 pounds per square foot. For the 16-foot slab the total safe load is therefore $\frac{740}{4}$ = 185 pounds, and deducting the weight of the slab from column (15) gives a net live load of 185 − 103 = 82 pounds per square foot.

Example 12: Using Table 4 of rectangular beams, page 510, what should be the dimensions and reinforcements for a beam 12 feet span, continuous and loaded uniformly with 1000 pounds per foot of length?

Solution: The assumed stresses are the same as those adopted in the Beam Table. Assuming a width of beam 12 inches, a total load per inch of width of $\frac{1000}{12}$ = 84 pounds per running foot. Referring directly to the Beam Table, we find that the total depth corresponding to a 12-foot beam with this load is about 12 inches. The reinforcement from column (25) is 0.083 × 12 = 1.00 square inch.

Example 13: What total load per foot of length can be carried by a 12-foot simply supported beam 12 inches wide and 25 inches deep?

Solution: There is no value in the Table 4, page 511, for a beam whose total depth is 25 inches, but since, from rule 4, loads are proportional to the square of the depth of the steel, we may calculate the load in this case from the load for a 26-inch beam 12 inches wide. Assuming in both cases that the depth to steel, d, is 2 inches less than the total depth, we have $364 \times \frac{23^2}{24^2} \times 12$ = 4 000 pounds per running foot of beam. Since the table is based on $M = \frac{wl^2}{10}$ for simply supported beams, deduct 20% from the above amount. Hence the safe load is 4000 − 800 = 3200 pounds.

EXPERIMENTS UPON REINFORCED BEAMS

Tests upon reinforced concrete beams have been conducted at various universities in the United States, and by leading scientists in Europe. Valuable data with reference to the location of the neutral axis, the deformation and the ultimate loads with various percentages and classes of steel have been recorded* in the United States by Professors Hatt, Howe, Lanza, Marburg, Talbot, and Turneaure, and in Europe by Messrs. Considère, von Emperger, Feret, Rabut, Ramisch, Ribera and Sanders. An extensive series of tests has been carried on at the United States Government Structural Materials Testing Laboratories at St. Louis, using different materials, different methods of manufacture, and different types of reinforcement.

Special results of many of these tests have been mentioned in the preceding pages.

Tests of Prof. Arthur N. Talbot. At the University of Illinois, Prof. Talbot has made several valuable series of tests to investigate the laws of reinforced concrete, which cover an exceedingly wide range of percentages of steel and types of reinforcement. These are described in detail in various bulletins of the University.†

The fundamental principles of rectangular beams are illustrated in some of the earlier experiments which are summarized in the following table. Although a leaner mixture of concrete was used in these than in his later tests which, therefore, correspond more nearly to practical construction, the principles are not affected. The proportions in these beams were 1 : 3 : 6 based on loose measure of cement, or about 1 : 3½ : 7 based on a unit of 100 pounds cement per cubic foot. The beams were 15 feet 4 inches long, 12 inches wide, 13½ inches deep, with the reinforcement 12 inches below

* See also References, Chapter XXXI.

† Bulletin No. 1, Sept. 1, 1904; Bulletin No. 4, April 15, 1906; Bulletin No. 12, Feb. 1, 1907; Bulletin No. 29, Jan. 4, 1909.

the upper surface. These were tested on a span of 14 feet by two loads which divided the span into three equal parts. The exact proportions of the concrete were 96 pounds Portland cement to $3\frac{3}{8}$ cubic feet sand to $6\frac{3}{4}$ cubic feet broken stone. The sand was well graded in size of grains and weighed 115 pounds per cubic foot loose and dry. The stone was Illinois limestone, with particles smaller than $\frac{1}{4}$ inch and coarser than $1\frac{1}{2}$ inches screened out. The consistency was such that the water flushed to the surface under light ramming. The crushing strength of 6-inch cubes at the age of 60 days averaged 2030 pounds per square inch.

Typical deform tion and deflection curves are given in Fig. 130, page 489.

Prof. Talbot gives the following description of the manner of failure of each beam except those numbered 27, 22, and 28, which crushed at the top at maximum load:

Tests of Reinforced Concrete Beams.

BY ARTHUR N. TALBOT. (*See p.* 479.)

Beam No.	Kind of Steel.	No. of Rods.	Size of Rods. in.	Area of Steel. sq. in.	Ratio of area of steel to beam above steel.	Maximum Load. lb.	Load Considered lb.	Total Elongation of Steel. in.	Ratio of depth of steel to depth of neutral axis k: As Measured.	Calculated by formula. (p. 420)	Talbot's formula. (p. 479)	Estimated Total* Bending Moment. in. lb.	Moment of Resistance calculated from formula (7) or (8), p. 420. in. lb.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	
21	Round	3	$\frac{1}{2}$	0.59	0.0041	9 000	8 000	0.0665	0.34	0.33	0.33	261 000	226 890*b*	2 bars turned up
19	"	3	$\frac{1}{2}$	0.59	0.0041	9 200	9 200	0.0755	0.36	0.33	0.33	294 600	226 890*b*	2 bars turned up
16	Square	3	$\frac{1}{2}$	0.75	0.0052	9 900	9 900	0.065	0.37	0.36	0.35	313 200	284 700*b*	2 bars turned up
17	"	3	$\frac{1}{2}$	0.75	0.0052	10 000	9 500	0.059	0.37	0.36	0.35	302 000	284 700*b*	2 bars turned up
27	"	4	$\frac{3}{4}$	2.25	0.0156	26 900	25 000	0.066	0.53	0.54	0.54	725 500	774 000*a*	2 bars turned up
9	Ransome	3	$\frac{1}{2}$	0.75	0.0052	22 800	18 000	0.142	0.34	0.36	0.35	540 000	474 500*c*	8 stirrups
15	Thacher	3	$\frac{3}{4}$	1.20	0.0083	18 400	15 500	0.0715	0.41	0.43	0.41	466 000	443 300*b*	2 bars turned up
10	"	3	$\frac{3}{4}$	1.20	0.0083	16 600	14 500	0.065	0.43	0.43	0.41	438 000	443 300*b*	2 bars turned up
22	Kahn†	3	$\frac{3}{4}$	2.40	0.0167	24 400	22 000	0.064	0.57	0.55	0.56	641 000	786 200*a*	Bars sheared up
4	"	5	$\frac{1}{2}$	2.00	0.0139	23 000	21 000	0.069	0.47	0.52	0.51	615 000	714 800*b*	Bars sheared up
14	"	4	$\frac{1}{2}$	1.60	0.0111	17 200	17 000	0.062	0.46	0.48	0.46	505 500	580 400*b*	Bars sheared up
5	"	3	$\frac{1}{2}$	1.20	0.0083	15 000	13 000	0.0625	0.42	0.43	0.41	396 000	443 200*b*	Bars sheared up
28	Johnson	6	$\frac{3}{4}$	2.19	0.0152	34 300	31 000	0.101	0.53	0.53	0.53	893 500	768 700*a*	4 bars turned up
13	"	7	$\frac{1}{2}$	1.40	0.0097	29 000	27 500	0.111	0.45	0.46	0.43	800 500	681 400*a*	4 bars turned up
20	"	5	$\frac{1}{2}$	1.00	0.0069	20 900	20 000	0.132	0 44	0.41	0.39	593 500	615 600*a*	3 bars turned up
2	"	5	$\frac{1}{2}$	1.00	0.0069	20 600	19 000	0.119	0.39	0.41	0.39	565 500	615 600*a*	Horizontal bars
7	"	3	$\frac{1}{2}$	0.60	0.0042	14 000	13 000	0.1175	0.33	0.33	0.33	401 000	384 400*c*	Horizontal bars
3	"	3	$\frac{1}{2}$	0.60	0.0042	14 000	12 000	c 1065	0.31	0.33	0.33	373 000	384 400*c*	2 bars turned up
								Average	0.418	0 422	0.411	506 906	507 388	

NOTE: — Columns (6) (11) (12) and (14) have been added by the authors.
*As calculated by Prof. Talbot. Based on "Load Considered" column (8).
a. Based on crushing strength of concrete of 2 030 lb. per square inch because the moment thus obtained is lower than the moment based on yield point of steel.
b. Based on yield point of steel as 36 000 lb. per square inch.
c. Based on yield point of steel as 60,000 lb. per square inch.
†Net areas of steel in Kahn bars at load points are lower than gross areas given, so that moments of beams, 4, 14, and 5, by corrected computation are much higher than shown in col. 13.

A portion of the data resulting from the experiments is tabulated above. Column (10) is taken from a separate table of Prof. Talbot's,* and columns (11), (12) and (14) are added by the authors to compare the actual tests and the theory adopted in this treatise.

Prof. Talbot suggests an empirical straight line formula† for the location of the neutral axis with different percentages of steel, which avoids the more intricate calculations necessary with the usual theoretical formulas involving the modulus of elasticity. Adopting the same notation employed throughout this treatise (see p. 420), let

k = ratio of depth of neutral axis to depth of center of gravity of steel.

p = ratio of area of section of steel to area of section of beam above center of gravity of steel.

Then with a slight change to conform to the use of a ratio of 15‡

$$k = 0.24 + 18\,p \qquad (58)$$

Column (12) gives values of k calculated from this formula, using 0.26 for this concrete instead of 0.24 The formula is adapted to concrete beams with percentages of steel ranging from 0.006 to 0.012.

One of the most important conclusions in the authors' opinion, which, may be drawn from Prof. Talbot's tests, is the fact that computations made by the ordinary theory adopted in this treatise produce values for the neutral axis, and also for the ultimate moment of resistance, which are so near to the experimental results that these theoretical formulas (see p. 420) may be employed with confidence.

Calculating the location of the neutral axis by formula (6), page 420, and employing a ratio of the moduli of elasticity of steel to concrete of 20,— which Prof. Talbot's tests§ of elasticity show to be an average value between loads of 1 000 and 1 700 pounds per square inch (stresses which correspond to the compression in the beam when the neutral axis is as given), the theoretical distances given in column (11) agree almost exactly with the actual measurements in column (10). The moments of resistance calculated in column (14) also agree closely with the total bending moments in column (13).

T-Beam Tests by Prof. Frank P. McKibben. The T-beams tested at the Massachusetts Institute of Technology were made of concrete

* University of Illinois, Bulletin No. 1, September, 1904.
† Prof. Talbot gives the derivation of this formula and a theoretical discussion of his tests in Journal Western Society of Engineers, August, 1904.
‡ The constant in Prof. Talbot's original formula was 0.26.
§ Journal Western Society of Engineers, August, 1904.

mixed in proportion 1 : 2 : 4 by volume based on a unit of 100 pounds cement per cubic foot. The stone used was crushed conglomerate well graded, the range of sizes of particles being from $1\frac{1}{4}$ to $\frac{3}{16}$ inch, while the sand was a mixture of coarse and fine sands in equal parts. The steel reinforcement consisted of plain round bars ranging in size from $\frac{13}{16}$ to 1 inch in diameter. The age of beams when tested was about 30 days. Their dimensions were as follows: span 12 feet, total depth 11 inches, depth to steel 9.5 inches, thickness of flange 3 inches, breadth of stem 8 inches, breadth of flange 2 feet. The percentage of reinforcement varied from 2.22 to 3.12 per cent based on the width of the stem, or from 0.74 to 0.104 per cent based on the width of the flange, using in both cases the depth to steel in computing the area of concrete. The following table gives the results of the tests.

Tests of Reinforced Concrete T-Beams

By Frank P. McKibben. (See p. 479.)

Massachusetts Institute of Technology.

No. of Beams.	No. of Round Rods.	Size of Rods. In.	Area of Steel. Sq. in.	Percentage of Steel. In web width.	Maximum Load. Lb.	Load at Last Measurement. Lb.	Stress in Steel: At First Crack. Lb. per sq. in.	Stress in Steel: At Last Measurement. From Deformations. Lb. per sq. in.	Stress in Steel: At Last Measurement. Computed. Lb. per sq. in.	Ratio of Depth of Neutral Axis to Depth of Steel. Measured.	Ratio of Depth of Neutral Axis to Depth of Steel. Computed.	Bending Moment at Last Measurement. Inch-pounds.	Computed Moment of Resistance. Inch-pounds.	Computed Stress in Concrete. Lb. per sq. in.	Ultimate Strength of Prisms. Lb. per sq. in.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
1	2 1	$\frac{7}{8}$ $\frac{13}{16}$	1.69	2.22	24240	22000	3110	34500	37460	0.37	0.38	528000	515000*a*	1510	2220
2	2 1	$\frac{7}{8}$ $\frac{13}{16}$	1.71	2.25	26415	22000	8230	33300	36900	0.44	0.38	528000	503000*a*	1495	1740
3	2 1	$\frac{7}{8}$ $\frac{15}{16}$	1.90	2.50	24365	22000	6440	38100	33400	0.37	0.39	528000	636000*a*	1410	1700
4	3	$\frac{15}{16}$	2.07	2.73	27215	24000	5400	21600	33500	0.43	0.41	576000	671000*b*	1570	1680
5	2 1	1 $\frac{15}{16}$	2.37	3.12	30940	28000	2040	29600	34400	0.47	0.44	672000	671000*b*	1780	1610

a Based on stress in steel obtained from last measurement.
b Based on crushing strength of concrete, since beam failed by compression.
Note: In figuring the moment of resistance the computed depth of neutral axis for $n = 15$ was used. Percentage of steel in terms of width of flange is $\frac{1}{3}$ of the values in col. (5).

The tests compare well with the results obtained from the formulas given on page 420. The stresses in steel, determined by measurements of stretch, do not vary appreciably from those obtained from the formulas. Beams

No. 4 and 5 failed by compression in the concrete, and the compressive stress in beam near to failure agrees quite closely with the strength of the prisms made of the same mix of concrete. A difference in deflection of the stem and the flange was detected by the tests, which indicates that the compressive stresses are not uniform throughout the whole width of the flange. This, however, in practice is undoubtedly more than balanced by assuming a width of flange smaller than the width of slab that actually assists in taking the compression. First cracks occur, as evident, at very low stresses, but they are very minute and almost invisible and their presence is not dangerous.

Tests of Repetitive Loading of Reinforced Concrete Beams by Prof. H. C. Berry. Fatigue tests of reinforced concrete beams made by Prof.

Fatigue Tests of Reinforced Concrete Beams. Size of Beams: 8″ × 11″. Span: 13 ft. Age: 6 Weeks

By H. C. Berry

University of Pennsylvania. (See p. 481)

Reinforcement	Number of Repetitions	Working Stress in Steel	Working Stress in Concrete	Breaking Load	Maximum Deflection
		lb. per sq. in.	lb. per sq. in.	lb.	in.
4, $\frac{1}{2}$″ round rods . .	1			12 000	0.56
4, $\frac{1}{2}$″ round rods . .	297 000	18 300	785	12 300	0.48
2, $\frac{5}{8}$″ square bars . .	395 000	15 200	628	10 500	0.46
2, $\frac{3}{4}$″ diamond bars	2			13 000	0.62
	718 000	14 300	785		
2, $\frac{3}{4}$″ diamond bars	then				
	422 000	17 100	940	13 600	0.78
3, $\frac{3}{4}$″ corr. bars. . . .	0			20 000	0.66
3, $\frac{3}{4}$″ corr. bars . . .	295 000	10 800	940	17 700	0.55

H. C. Berry* at the University of Pennsylvania in 1908 indicate that as many as one million repetitions of high working stresses do not materially affect the ultimate strength of a reinforced concrete beam, its maximum deflection, or the position of its neutral axis. Duplicate beams were made of concrete mixed in the proportions of 1 part cement, $1\frac{1}{2}$ parts bar sand and $4\frac{1}{2}$ parts $\frac{3}{4}$-inch crushed granite and were reinforced with plain and deformed bars. These beams were tested when 6 weeks old, one being subjected to a repetitive loading sufficient to cause higher stresses than ordinarily allowed in

*Eng. Record, July 25, 1908, p. 90.

good practice, and then tested to failure, while the other was broken in the ordinary manner.

It was evident that the greater part of the set in the deformation in the plane of the steel occurred in the first few thousand applications of the load and that the set in the deformation on the compressive side of the beam was also relatively large for the first few thousand repetitions and increased with the stress applied and the number of repetitions.

The stresses realized and the deflections resulting from the repetitive loadings are shown in the accompanying table on page 481. The breaking strength of the beams sustaining the repetitive loading is substantially the same in every case as the corresponding beam with no appreciable repetitions.

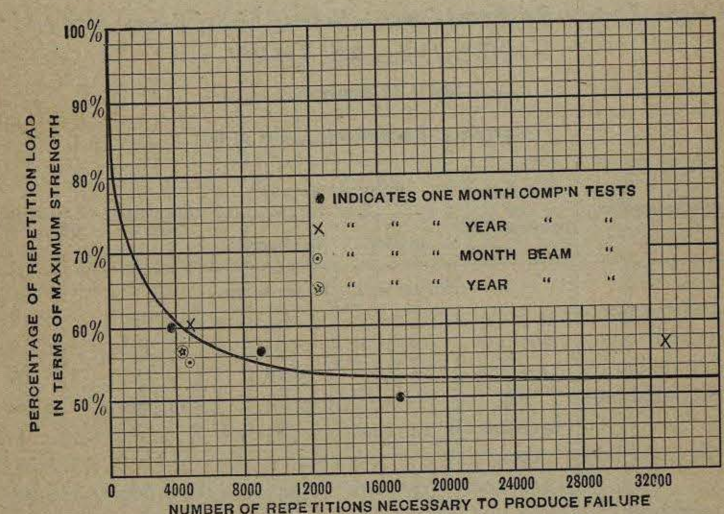

FIG. 151. Fatigue of Reinforced Concrete Beams. (*See p.* 482.)
By Prof. J. L. Van Ornum.

Compression tests by Prof. J. L. Van Ornum* at Washington University made in 1907 agree with the above tests for repetitive loadings under 50 per cent of the maximum strength of the concrete, but for repeated loads greater than this he found that beams will be subject to failure. He concluded that the number of repetitions required to cause this failure depended essentially upon the ratio of the test load to the ultimate strength of the concrete. In these tests, as will be seen from the curve in Fig. 151, which summarizes graphically the results of these experiments, the influence of

* Transactions American Society Civil Engineers, 1907, LVIII, p. 294.

the fatigue of concrete is limited to an intensity of about 50 per cent of the ordinary ultimate strength of the concrete.

Tests at Illinois University, at St. Louis,* and elsewhere confirm the principle illustrated and show that there is a fatigue limit to concrete corresponding in a general way to the elastic limit of metals. This varies with the character of the concrete from ½ to ¾ the ultimate strength. Prof. Talbot finds in columns the deformation to be a measure of this fatigue limit, the latter usually occurring at about ½ the ultimate deformation.

This fatigue limit of concrete, while it does not influence the practice of conservative design, is a warning against the use of too high working stresses.

FLAT SLABS

Besides the usual systems for floors, using a combination of slabs, beams and girders, a floor system of a type of an entirely different design is sometimes employed, which consists of a flat unribbed slab continuous over the whole floor and supported by columns only. The type originally introduced by Mr. C. A. P. Turner of Minneapolis is sometimes termed the Mushroom System.

The reinforcement of the slab consists of bars running in four directions radially from the column, and the head of the column is usually enlarged in order to diminish the bending moment and increase the shearing resistance. The vertical steel in the column reinforcement or a portion of it may be bent and carried into the slab to add to the rigidity of the connection.

The moments and stresses in this system are statically indeterminate, but in order to make an application of the theory of flexure possible, the whole floor is considered as a series of flat circular slabs concentric with the columns and firmly clamped to them, supporting the rest of the floor. Thus the analysis of the whole floor is reduced to that of circular plates clamped to the columns, and flat slabs supported on all edges by these circular plates.

Let Fig. 152 represent a floor of this system, and consider the strip *ab* as separated from the rest of the floor. This strip when loaded will act as a fixed beam. The points of inflexion will be distant approximately one-fifth of the span from the circumference of the enlarged head of the column. The points of inflexion of the floor will thus be located on the dotted curve shown on the drawing. Instead of this curve we may assume the points to be on a circle, represented on the drawing by dash lines, and consider the area within this circle as a round plate, loaded with a uniform load over its area and in addition loaded around its circumference with a load which per

* See page 478.

unit of length is equal to the remaining load of the panel divided by the circumference of the circle.

The part of the slab between the column and the points of inflexion will deflect downwards, while the rest of the slab will deflect as an ordinary supported beam.

The authors have adopted Prof. Eddy's analysis of stresses* in a homogeneous circular plate, and deduced from his general formulas, formulas applying to circular slabs free on their outer edge and clamped round the column. In this analysis the effect of lateral stresses has been taken into account, this being expressed by Poisson's ratio, which is the ratio of the

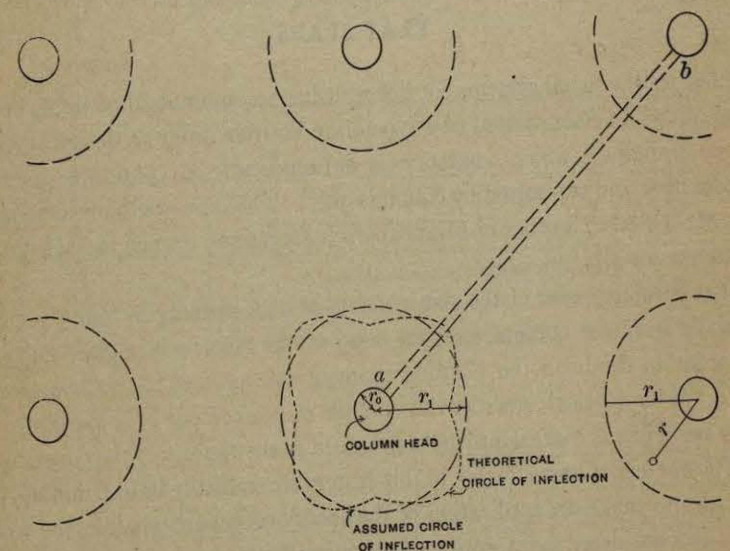

FIG. 152. Flat Slab. (*See p.* 483.)

lateral deformation to that in the direction of stress. Very few tests have been made to determine the value of Poisson's ratio, and the results obtained vary considerably. Many of the earlier tests give as high as 0.2, but since some of the best experiments in our American colleges indicate a value ranging, with concrete of different proportions and strength, from 0.05 to 0.15, the ratio of 0.10 is recommended for use with concrete where the correct value is unknown, as being undoubtedly safe for concrete of 1 : 2 : 4 proportions. It must be noted that the increase of Poisson's ratio tends to diminish the deflection and thus decrease the stress. A high Poisson's ratio therefore means a thinner slab and less steel.

* Engineers' Society, University of Minnesota, 1899.

The meaning of Poisson's ratio as applied to a loaded column is the lateral deformation per unit of width divided by the longitudinal deformation per unit of length. For example, if a certain load causes a 10-inch column to expand laterally 0.0003 inches, while at the same time it shortens 0.03 inches in a gaged length of 100 inches, Poisson's ratio for that loading is $\frac{0.0003 \times 100}{0.03 \times 10} = 0.1$. In a slab supported on columns there is a similar condition of deformations caused by stresses at right angles to each other which are taken into account in the mathematical work involved in the derivation of the formulas.

Let

q = uniform distributed load along the edge of the plate in pounds per foot of length.

w = uniform distributed load on surface of the plate in pounds per square foot.

r_0 = radius of enlarged column in feet.

r_1 = outer radius of assumed plate in feet.

r = any radius in feet.

g = Poisson's ratio.

C_1, C_2, C_3, C_4 = constants to use in formula (52) (54) (55). Table 9, p. 518.

C_a, C_b, C_c, C_d, = constants to use in formula (53) (56) (57). Table 9, p. 518.

M_1 = moment causing circumferential fiber stress
M_2 = moment causing radial fiber stress
} for loading uniformly distributed over the plate.

M_a = moment causing circumferential fiber stress
M_b = moment causing radial fiber stress
} for loading distributed along the edge of the plate.

Then for the maximum moment,* which occurs at the circumference of

* Following formulas may be used for finding of moments at any point of plate.

$$M_1 = wr_0^2 \left\{ 0.1 \left(\frac{r}{r_0}\right)^2 - C_1 \left(\frac{r_0}{r}\right)^2 C_3 \log. \left(\frac{r}{r_0}\right) + C_4 \right\} \quad (54)$$

$$M_2 = wr^2 \left\{ 0.2 \left(\frac{r}{r_0}\right)^2 + C_1 . \left(\frac{r_0}{r}\right)^2 - C_3 \log. \left(\frac{r}{r_0}\right) + C_2 \right\} \quad (55)$$

$$Ma = qr_0 \left\{ - C_a \left(\frac{r_0}{r}\right)^2 - C_c \log \left(\frac{r}{r_0}\right) + C_d \right\} \quad (56)$$

$$M_b = qr_0 \left\{ C_a \left(\frac{r_0}{r}\right)^2 - C_c \log \left(\frac{r}{r_0}\right) + C_b \right\} \quad (57)$$