Let

l = length of rod considered in inches.

s_b = distance in the clear between two rods in inches.

i = diameter of rod in inches.

u = adhesion or bond between concrete and steel per square inch of surface of steel.

v = direct shearing strength of concrete in pounds per square inch.

If the beam splits at the rods, it is apt to shear through the concrete between the rods, and break the adhesion between the upper half of the rod and the concrete. When such splitting occurs the shearing strength of the concrete between the rods, on a plane with their centers, is equal to or less than the adhesion of the concrete to the half circumference of one of the rods and the minimum spacing is then*

$$s_b = 1.57 \frac{u}{v} i \qquad (49)$$

If, for example, the working bond stress, u, is assumed as 80 pounds per square inch and the working strength of the concrete in direct shear is taken at 120 pounds per square inch, the formula becomes

$$s_b = 1.05\, D \qquad (50)$$

that is, the minimum net distance in the clear between the rods is approximately equal to the diameter of the rod. Since the concrete is not easily placed between the rods it may have a lower strength there and hence a clear spacing of 1½ diameters (with a minimum of one inch), as suggested above is advisable unless it is determined by computation that the bond stress is much lower than is assumed here. Deformed bars, if stressed to their full bond value should be spaced farther apart than plain bar.

In the middle of a beam the bond stress is low, so that the formulas are most useful in considering the rods in the top of the beam over the support.

DEPTH OF CONCRETE BELOW RODS

The selection of the thickness of the concrete below the rods is governed more by the proper fire and rust protection of the metal than by the stresses in the beam.

* For a short length of rod l, equate the strength in shear of the concrete between the rods to the adhesion between the concrete and the upper half circumference of the rod.

Hence

$$s_b\, l v = \frac{\pi i l u}{2} \qquad s_b = 1.57 \frac{u}{v} i$$

Prof. Charles L. Norton, who has made a careful study of the subject, considers a thickness of 2 inches essential for efficient fire protection. (See p. 333.) Since an excessive thickness adds to the danger of cracking, because the tension in the concrete increases with the depth below the steel, with but slight corresponding gain in strength to the beam, this thickness, measured from the lower surface of the steel, and not from its center of gravity, may be taken as a maximum. Thus, in important members which are liable to severe fire, 2 inches may be considered the standard requirement, while for secondary members and floor slabs, a less thickness, ranging from ½ inch to 2 inches, is probably warranted.

The following thicknesses of concrete below the steel may be employed under ordinary conditions:

Thickness of Concrete below Steel.

Depth of slab or beam, inches	Thickness below lower surface of rods,* inches
1½ to 2	½
2½ to 4	¾
4½ to 8½	1
9 to 12	1¼
13 to 18	1½
19 to 20	1¾
Greater than 20	2

*Values up to depth of 20 inches are from tables of Mr. Edwin Thacher, except that his depths are taken below center of gravity of steel.

The Joint Committee, 1909, recommend slightly greater thickness than given in the above table, and its recommendations should be followed wherever the conditions are especially hazardous. The Committee suggest that "the metal in girders and columns be protected by a minimum of 2 inches of concrete; that the metal in beams be protected by a minimum of 1½ inches of concrete, and that the metal in floor slabs be protected by a minimum of 1 inch of concrete."

BOND OF CONCRETE TO STEEL TO RESIST DIRECT PULL

Tests by different experimenters show that with similar materials the bond is proportional to the area of the surface in contact, and varies with the character of the surface and the nature of the concrete or mortar.

Feret† found that the bond of concrete to iron is nearly proportional to the percentage of cement in a unit volume of concrete, and that there is an

† Thonindustrie-Zeitung 25 (153)2, 213-2, 215, translated in Cement, July 1902, p. 213.

increase of strength of about 50% in concrete two years old over that three months old. The best consistency for concrete he considers to be so plastic as to be almost sloppy. He found that the bond of a very dry concrete was only about one-fourth that of an almost sloppy concrete which gave the maximum density. The bond increased rapidly as the proportion of water increased until the concrete reached its maximum density, when for larger proportions of water there was a slow decrease in bond strength, very wet concrete showing a bond about three-fourths that of the slightly dryer concrete.

In tests made by Prof. M. O. Withey at the University of Wisconsin* in 1906, the results of which are contained in the following table, the bond developed by different specimens averaged about 0.3 of the compressive strength and increased nearly proportionally with it.

*Variation of Bond with the Compressive Strength.**

1 : 2 : 4 Concrete—Age 28 days. (See p. 462.)

By Prof. Morton O. Withey.

Size of rod. In.	Area of Rod. Sq. in.	Depth imbedded. In.	Maximum load. Lb.	Unit stress in rod. Lb. per sq. in.	Elastic limit of steel. Lb. per sq. in.	Bond Lb. per sq. in.	Compressive strength of concrete. Lb. per sq. in.
9/16	0.248	6½	7200	29 000	36 400	627	2155
9/16	0.248	6⅛	5500	22 200	36 400	509	2000
9/16	0.248	6¼	6700	27 000	36 400	607	1850
9/16	0.248	6¼	4625	18 600	36 400	418	1485
9/16	0.248	6¼	4250	17 100	36 400	384	1435
9/16	0.248	6	4100	16 500	36 400	387	1150
½	0.196	6	3600	18 400	38 600	382	1150
½	0.196	5¾	1500	7 600	38 600	166†	795†
½	0.196	6¼	1840	9 400	38 600	187†	584†

As shown in the table, which gives in a condensed form the results of tests made by Prof. Talbot at the University of Illinois‡ in 1905, a 1 : 2 : 4 concrete realized a bond resistance averaging about 13 per cent. higher than a 1 : 3 : 5½ concrete. The effect of the surface of the bar upon bond resistance is also indicated. Plain round mild steel bars developed a bond resistance ranging from 355 to 465 pounds per square inch of contact surface, a bond about 2.7 times that of cold rolled shafting and tool steel and also much greater than that of flat mild steel bars. Deformed bars give a much higher bond resistance. In cases* where the embedment was not so great as to stress the bars beyond their elastic limit, the results indicate a bond strength for deformed bars in ordinary 1 : 2 : 4 concrete of from, say, 400 to 700 pounds per square inch of contact surface.†

* Bulletin, University of Wisconsin, Vol. 4, No. 2, Nov. 1907.

† Made upon concrete in which sand and limestone screenings containing 40% of dirt were used as aggregate.

‡ Bulletin No. 8, University of Illinois, Sept. 1906.

Tests of Bond of Union Between Concrete and Steel.†

Age of Concrete, 60 days. *(See p. 462.)*

By Prof. Arthur N. Talbot.

Type of rod.	Size. Inches.	Proportions.	Encased length. In.	Surface in contact. Sq. in.	Maximum load. Lb.	Bond. Lb. per sq. in.	Frictional Resistance. Total Maximum. Lb.	Frictional Resistance. Unit Lb. per sq. in.	Ratio of Frictional Resistance to bond.
Plain Round	½	1:2.4	6	9.4	3893	412	2135	227	55.2
Plain Round	⅝	1:2:4	6	11.8	5376	465	3485	297	64.0
Plain Round	½	1:2:4	12	18.8	7605	404	4982	266	65.5
Plain Round	⅝	1:2:4	12	23.5	9736	414	5284	223	54.0
	Average	1:2:4			6652	424	3971	253	59.7
Plain Round	½	1:3:5½	6	9.4	3498	372	1983	210	57.0
Plain Round	⅝	1:3:5½	6	11.8	4170	355	2700	227	64.0
Plain Round	½	1:3:5½	12	18.8	7035	373	5066	268	72.0
Plain Round	⅝	1:3:5½	12	23.5	9458	402	5366	228	56.8
	Average	1:3:5½			6040	375	3779	233	62.4
Cold Rolled Shafting	1	1;3:5½	6	18.8	2570	136	1256	67	49.2
Cold Rolled Shafting	½	1:3:5½	6	9.4	1476	157	466	50	31.8
Mild Steel	3/16–1½	1:3:5½	6	20.2	2536	125	1713	84	67.1
Round Tool Steel	¾	1:3:6	6	14.1	2077	147	......	.....	.....

Tests by Mr. Frank A. Bone‡ indicate that the bond of bars in

* Bulletin No. 1, University of Illinois, Sept. 1904, Table 7; Proceedings American Society for Testing Materials, Vol. VII, 1907, p. 467; Engineering Record, Dec. 22, 1906, p. 694.

† Bulletin, University of Wisconsin, Vol. 4, No. 2, Nov. 1907.

‡ Engineering News, May 21, 1908, p. 571.

concrete which is being stressed to a high compression is much greater than in unstressed concrete.

The results of many bond tests are without value because the yield point of the steel was exceeded.

The bond stress of the tension bars in a beam is determined by methods discussed on page 457.

LENGTH OF BAR TO PREVENT SLIPPING

In a reinforced concrete member it is necessary not only to have at a given section the required amount of steel to take the pull or the compression but it is also essential for each bar to have sufficient length of imbedment in the concrete so that the bond (that is, the resistance to slipping) of the bar in the concrete is great enough to develop the necessary direct pull or compression in the body of the bar. Unless a bar is bent up or anchored by some mechanical means (see p. 466) its resistance to slipping is determined by the length of imbedment and the value of the unit bond between the concrete and the steel.

The length of imbedment necessary to develop a required holding power through mere bond between the concrete and steel may be determined thus:

Let

f_s = working tensile or compressive stress per square inch in the body of the bar.

i = diameter of bar in inches.

u = bond in pounds per square inch of surface.

l_1 = necessary length of imbedment of bar in inches.

Then*

$$l_1 = \frac{i f_s}{4 u} \qquad (46)$$

This formula holds for square as well as for round bars. Using the limiting bond stress suggested by the Joint Committee for round bars of mild steel, 80 pounds per square inch, the length of imbedment when the steel is stressed to 16 000 pound per square inch is fifty diameters. Deformed

* If the bar is round the total force to be developed in the body of the bar is $\frac{\pi i^2}{4} f_s$ while the holding power of the bar, or its resistance to slipping is $\pi i u l$.
Equating these and solving for l, we obtain

$$l_1 = \frac{i f_s}{4 u}$$

bars may be given a greater bond stress, while, as indicated in the preceding paragraphs, a smooth steel of the nature of tool steel must be given less. It has been suggested that the bond of deformed bars be taken the same as the bond of plain bars except using for their diameter the diameter of a cylinder based on the longest projections, that is, of a cylinder which would be sheared out by the deformed bar. Ordinarily then, as indicated on page 528, a bond stress for deformed bars varying with the character of the bar from 100 to 150 pounds per square inch may be used, corresponding to an imbedment of 40 to 27 diameters. For smooth metal of the nature of tool steel not over 30 or 40 pounds per square inch, that is 133 to 100 diameters should be permitted. Flat steel or structural steel with flat surfaces should be given a similarly low value per square inch.

The bond in all cases is based on the surface area of the imbedded bar. Later tests by Prof. Withey, which show much lower bond strength when the concrete in the specimens is not affected by the compression, are referred to on page 458. These indicate that a strength in bond for 1 : 2 : 4 concrete cannot be assumed greater than 273 pounds per square inch of surface of round bars, so that the value of 80 pounds per square inch suggested above, is a fair working unit bond stress.

All of these bond values are based on a concrete whose strength at the age of thirty days; when tested in cylinders, is 2 000 pounds per square inch. The length of imbedment also varies with the working stress in the steel in tension or compression. In compression, steel is not apt to be stressed over 8 000 to 10 000 pounds per square inch, so that a shorter length of imbedment is required. Furthermore, the concrete in compression provides a greater grip.

The following table gives the length of imbedment of round or square bars for different unit stresses in the steel:

Length of Imbedment Required for Round and Square Bars.

TENSION IN STEEL f_s.	LENGTH OF BARS TO IMBED IN TERMS OF THE DIAMETER.					
	Allowable bond stress, pounds per square inch.					
Lb. per sq. in.	40	60	**80**	100	120	150
8 000	50	33	**25**	20	17	13
12 000	75	50	**37**	30	25	20
16 000	100	67	**50**	40	33	27
20 000	125	83	**62**	50	41	33

NOTE: The length of imbedment may be obtained by multiplying the value selected from this table by the diameter of the bar.

VALUE OF HOOKED BARS IN BOND

The results of a valuable series of tests on hooked bars, made for the Eastern Concrete Construction Company at the Massachusetts Institute of Technology under the direction of Prof. H. W. Hayward are presented in a table which follows.

In all the tests ¾-inch round bars were imbedded in blocks 12 inches square and 15 inches long, to a depth of 12 inches with an additional bend of different lengths. In one case the straight portion of the bar was greased. Right-angle bends and semi-circular bends on a 3-inch diameter were tested. Seven specimens of each type were tested, the individual results being extremely uniform except in type 1, where the straight bar test gave the usual variations expected in ordinary bond tests.

Tests of Hooked Bars in Bond, Massachusetts Institute of Technology, 1909, *for Eastern Concrete Construction Company*

Proportions of concrete 1 : 2 : 4. Age 28 days. Crushing strength concrete 1130 lb. per sq. in. at 34 days. Round bars ¾-inch diameter.
Yield point of steel 35 900 lb. per sq. in. or 15 870 lb. for ¾-inch bar.
Elastic limit of steel 29 000 lb. per sq. in. or 12 820 lb. for ¾-inch bar.

Type	Length of straight bar	Length and Kind of hook	First slip* lb.	Maximum loa lb.	Bond, lb. per sq. in. of surface	Condition at maximum load
	in.		lb.	lb.		
1	12	0″	7 770		275	Bars pulled out
2	12 (greased)	4″ square bend	16 000	16 000		Concrete crushed at bend finally split block.
3	12	4″ " "	18 700	21 450		Concrete crushed; bars partly straightened.
4	12	2″ " "	not recorded	17 770		Bars straightened out; Block not split.
5	12	6″ " "	17 000‡	19 530		Bars pulled out, or split concrete by kicking back
6	12	4″ " " (no backing of concrete)	9 600	15 590		End of bars raised up and crushed concrete; bars pulled through
7	12	3″ return bend	17 100	22 660		Concrete split
8	12	3″ " " (with 3″ extra length	17 300	23 630		" "

*As shown by drop of beam except in type (6), where the bent ends begin to straighten, the drop of the beam coming either at the same period or at about 1 000 lb. later.

In all the tests except those with straight bars (type 1) and those where there was no concrete back of the bend, the elastic limit of the steel was passed before the bars began to pull out as indicated by the drop of the beam.

The crushing strength and also the bond strength of the concrete was low because the specimens were stored at a low temperature, but this does not affect appreciably the value of the results.

The following conclusions may be drawn from these tests:

(1) A 4-inch right-angle bend in a ¾-inch round bar (5 diameters) is sufficient to stress the steel to its elastic limit. A longer bend than this is not necessary.

(2) A semi-circular bend on a diameter 4 times the diameter of the bar appears to be even more effective than the square bend.

(3) No crushing of the concrete occurs until the elastic limit of the steel is passed.

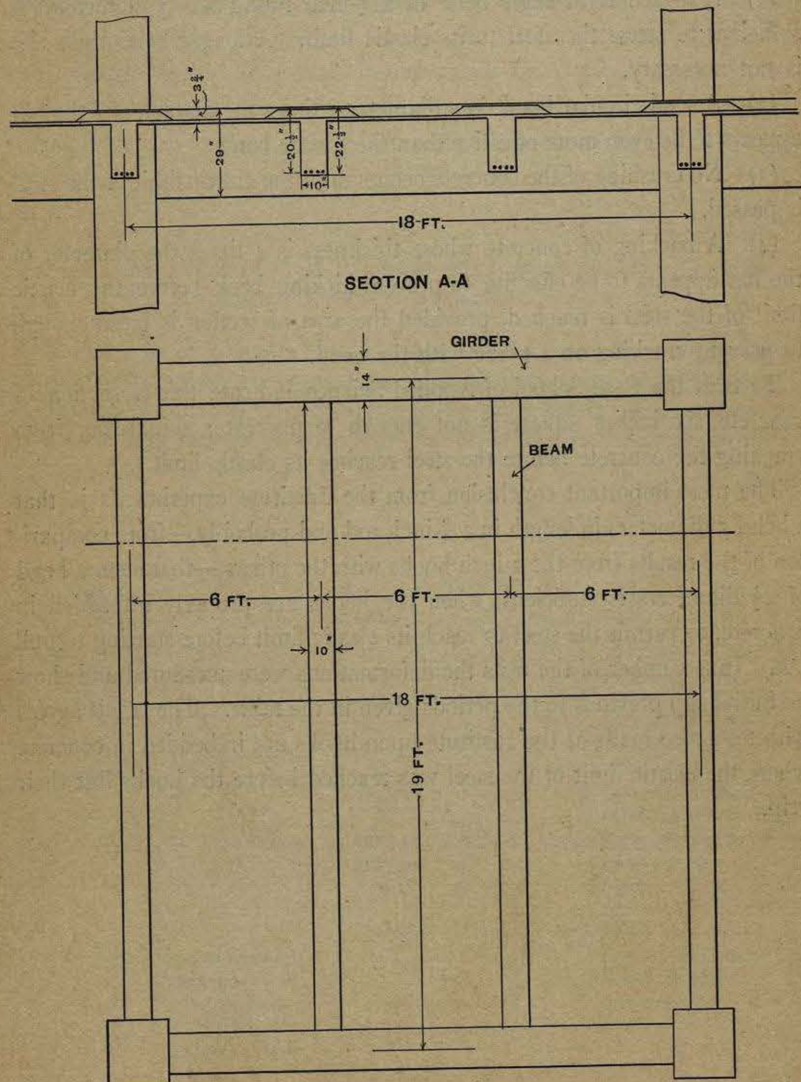
(4) A backing of concrete whose thickness is 4 times the diameter of the bar appears to be effective to prevent kicking back before the elastic limit of the steel is reached, provided the area of section is large enough to prevent cracking on a plane with the bend.

Tests at the Case School of Applied Science indicate that a section of concrete six inches square is not enough to prevent a ¾-inch bar from cracking the concrete before the steel reaches its elastic limit.

The most important conclusion from the Institute experiments is that a bend 5 diameters in length in a ¾-inch rod and probably,—from comparison of the results from the 2-inch hooks with the others,—that even a bend of 2½ diameters is sufficient, when the hooks are properly imbedded in concrete, to permit the steel to reach its elastic limit before starting to pull out. In a number of the tests the deformations were measured and show no initial slip previous to the periods given in the table. The result agrees with tests also made at the Institute upon hooks not imbedded in concrete where the elastic limit of the steel was reached before the hooks lost their grip.

EXAMPLE OF BEAM AND SLAB DESIGN

The use of the formulas given in the preceding pages can be best illustrated by the design of a floor bay consisting of slabs, beams and girders. The design of reinforced concrete structures permits of so many variations by locating steel in different ways that more than one type of design for the same member is almost always possible. The dimensions and reinforcement shown illustrate common methods, and the arrangement of details in the different members is also given as typical. The principles of design follow the recommendations of the Joint Committee on Concrete and Reinforced Concrete, 1909.



PLAN

FIG. 147.—Design of Floor System. *(See page 469.)*

The computations are given with but few comments, but references are entered to the pages upon which each part of the calculation is based.

Example 6: Design a typical slab, beam and girder for a reinforced floor to support a live load of 250 pounds per square foot with columns spaced 18 by 19 feet on centers.

Solution: The girder will be made 18 feet long and the distance between centers of beams 6 feet. The beams are 19 feet long on centers.

	Refer to page
Take allowable fiber stress in concrete, 650 lb. per sq. in.	528
Take allowable tension in steel, 16 000 lb. per sq. in.	529
Take ratio of elasticity of steel to concrete, 15	529
Take direct shear in concrete, 120 lb. per sq. in.	528
Take shear in concrete involving diagonal tension, 40 lb. per sq. in.	528
Take bond between concrete and plain bars, 80 lb. per sq. in.	528
Notation used in Example is Joint Committee standard	529

Slab. Span of slab is 6 ft.
Live load, 250 lb. per sq. ft.
Assumed dead load, 50 lb. per sq. ft.
Total loading, 300 lb. per sq. ft.

Use for moment, $M = \frac{wl^2}{12}$, then $M^* = \frac{300 \times 6^2 \times 12}{12} = 10\,800$ in. lb. 440

Same value may be found directly from curves 524
Since $f_c = 650$, $f_s = 16\,000$ and $n = 15$, then
$C = 0.096$ and $p = 0.0077$, from table 10 519

Hence, depth to steel is, $d = 0.29 \times 0.096 \sqrt{10800} = 2.9$ in. 421

Taking ¾ in. concrete below steel, thickness of slab is 3¾ in. 461
Area steel, A_s, $= 2.9 \times 12 \times 0.0077 = 0.268$ sq. in. 421
Round rods ⅜ inch in diameter spaced 5 inches on centers will give required area. Table 1. 507
The same results may be obtained by using the slab table 5: 513

Since this table is based on $M = \frac{wl^2}{10}$ and we use here $M = \frac{wl^2}{12}$ the total unit weight of 300 pounds per square foot may be reduced ⅙ or to 250 pounds and this value treated in the table, which gives a 3¾ inch slab.

Rods must be bent up to give same steel at top of slab over supports.

Beams. Span 19 feet.
Distance between beams, 6 feet.
Dead and live loads of the slab per foot of length of beam, $6 \times 300 = 1800$ pounds.
Assumed dead load of stem of the beam, 200 pounds per foot of length.
Total unit loading, 2000 pounds.

Use for moment $M = \frac{wl^2}{12}$, then $M = \frac{2000 \times 19^2 \times 12}{12} = 722\,000$ inch pounds.

Reaction at support, which is the maximum shear, is

$$V = \frac{2000 \times 19}{2} = 19\,000 \text{ pounds.}$$

* Only one 12 is inserted in the numerator to change the 6 ft. to inches because the 300 is pounds per foot.

Breadth of Flange. Taking 8 times the thickness of slab plus the breadth of stem of beam (assumed as 10 inches) $b = (8 \times 3\frac{3}{4}) + 10 = 40$ inches.

Minimum Depth. Referring to Table on page 525, since the area of flange times the working strength of concrete, $f_c\,b\,t$, is 97 500 lb. and the assumed ratio of depth of beam to thickness of flange is 3.5 $\left(i.\,e.,\ t = \frac{1}{3.5}\,d\right)$ the minimum distance from center of slab to steel in beam, jd, for a moment of 722 000 is 12 in., or adding $\frac{1}{2}t$, the depth d is 14 in. 426

A larger value of d however will be used for economical reasons as given below, since it reduces both the stress in concrete and the amount of steel. The decrease of depth of beam on the other hand would increase the stresses in concrete above the permissible working strength. 425

Cross-section of Web as Determined by the Shear. 424

$V = 19\,000$ pounds (see above) hence

$$b'\left(d - \frac{t}{2}\right) > \frac{19\,000}{120} \text{ or } 158 \qquad \text{formula (13)}, \ 424$$

Economical Depth. From formula (14), $d - \frac{t}{2} = \sqrt{\frac{r\,M}{f_s \times b'}}$ if the ratio of unit cost of steel to cost of concrete, $r = 70$ 425

$$\text{for } b' = 8,\ d - \frac{t}{2} = 19.85 \text{ inches or } d = 21.7 \text{ inch.}$$

$$b' = 9,\ d - \frac{t}{2} = 18.7 \quad \text{" or } d = 20.6 \quad \text{"}$$

$$b' = 10,\ d - \frac{t}{2} = 17.8 \quad \text{" or } d = 19.7 \quad \text{"}$$

For convenience in placing steel take
$b' = 10$ inches, $d = 20\frac{1}{2}$ inches, $h = 22\frac{1}{2}$ inches 459

Sectional Area of Steel. From formula (15) 426

$$A_s = \frac{722\,000}{18.625 \times 16000} = 2.4 \text{ square inches}$$

4 round bars $\frac{7}{8}$ inches diameter will be sufficient. Two of these may be bent up and lap over the top of the support 429

Steel at Top and Bottom. Negative bending moment at support equals positive M at middle or $-M = 722\,000$ inch pounds. 440

At support the flange of T-beam being in tension is negligible and since four $\frac{7}{8}$-in. round bars are in tensile and two in compressive part of beam, the T-beam changes into a rectangular beam with steel in top and bottom.

The ratios of steel in tension and compression are respectively

$$p = \frac{2.4}{10 \times 20.5} = 0.0117 \text{ and } p' = \frac{p}{2} = 0.0058$$

With these values of p and p' and for $n = 15$, and $a = 0.1$ we obtain from table (p. 516) $C_c = 0.227$ and $C_s = 0.0103$. Maximum pressure in concrete is

$$f_c = \frac{722\,000}{10 \times 20.5^2 \times 0.227} = 760 \text{ lb. per sq. in., formula (18)} \quad 428$$

$$f_s = \frac{722\,000}{10 \times 20.5^2 \times 0.0103} = 16\,700 \text{ lb. per sq. inch, formula (19)} \quad 428$$

Allowable compression in concrete at the support may be 15% larger than that at middle, hence, no haunch necessary. 429

Girder. Span 18 feet, breadth to use for T-beam, 44 in. (assuming breadth of stem as 14 in.) 424

Concentrated loads at $\frac{1}{3}$ points.

Assumed dead load of the stem of the girder, 360 pounds per linear foot.

Load transmitted by the beams is considered as concentrated. 441

Reaction of concentrated loads, $V = 38\,000$ pounds. 434

Maximum moment of concentrated loads with ends of beam simply supported would be, $M = 38\,000 \times 6 \times 12 = 2\,740\,000$ inch pounds. 439

This corresponds to formula $M = \frac{wl^2}{8}$; to correspond to $M = \frac{wl^2}{12}$ it may be reduced by the ratio $\frac{8}{12}$ or 441

$M = 2\,740\,000 \times \frac{8}{12} = 1\,827\,000$ inch pounds.

Moment of dead load, $M = 116\,600$ inch pounds. 440

Total moment, $M = 1\,943\,600$ inch pounds.*

Minimum Depth. From Diagram 4, p. 525, since the area of flange times the working strength of concrete, $f_c bt = 107\,250$ pounds and the assumed ratio of the depth of beam to the thickness of flange equals 6, the minimum depth, $d = 24$ inches. 525

A somewhat greater depth is economical as shown below. 425

Cross-section Determined by Shear. $V = 38\,000 + 3600 = 41\,600$ pounds 424

Using a limit of 120 pounds for total shear

$$b'\left(d - \frac{t}{2}\right) = \frac{41\,600}{120} = 347 \text{ square inches} \quad 424$$

Select by judgment
$b' = 14$ inches, $d = 26.5$ inches, $h = 29$ in. (to allow for 2 layers of steel).

Steel Area. For $M = 1\,956\,000$ from Diagram 4, p. 525, $A_s = 4.90$ square inches, 8 round bars $\frac{7}{8}$ inch diameter will satisfy the moment.

Check of Results by Exact Formulas 14 to 17 755

(This check is unnecessary in practice for an experienced designer.)
$b' = 14$ inches, $b = 30 + 14 = 44$ inches, $t = 3\frac{3}{4}$ inches.
$A_s = 4.90$ square inches.

$$kd = \frac{4.90 \times 2 \times 15 \times 26.5 + 44 \times 3.75^2}{2 \times 4.90 \times 15 + 44 \times 3.75 \times 2} = \frac{3900 + 620}{147 + 330} = \frac{4520}{477} = 9.5 \text{ inches}$$

$$z = \frac{3 \times 9.5 - 7.5}{2 \times 9.5 - 3.75}\ \frac{3.75}{3} = 1.72 \text{ inches.}$$

$$jd = 26.5 - 1.72 = 24.78 \text{ inches.}$$

The value for jd would be a bit lower, when the compression in the stem is also considered. It is evident that the approximate value used in previous figuring, $d - \frac{t}{2} = 24.63$ inches, is practically identical with the more exact moment arm.

Girder at Support. —$M = 1\,943\,600$ inch pounds. 440

Reinforcement at supports consists of $\frac{7}{8}$ inch round bars.

Eight bars are in tensile and four in compressive part of beam, hence

$$\text{ratio tension steel, } p = \frac{4.90}{14 \times 26.5} = 0.0132$$

$$\text{ratio compression steel, } p' = \frac{0.0132}{2} = 0.0066$$

* By method suggested on page 433 the result would be 2 159 000 less 10 per cent or 1 943 000 pounds, a result almost identical with the more exact one.

From Table 8 (p. 516), $C_c = 0.241$ and from formula (18) maximum compression in concrete, 428

$$f_c = \frac{1\,943\,600}{14 \times 26.5^2 \times 0.241} = 820 \text{ pounds per sq. in.}$$

which is excessive. 429

Depth and Length of a Haunch. For depth try $a = 0.1$, $d = 28$ inches 429
For this depth of beam the ratios of steel in tension and compression change to $p = 0.0132 \times \frac{26.5}{28} = 0.0125$, $p' = \frac{0.0125}{2} = 0.0063$ the corresponding values $C_c = 0.236$ and $C_s = 0.0109$ 428
Maximum compression in concrete,

$$f_c = \frac{1\,943\,600}{14 \times 28^2 \times 0.236} = 750 \text{ pounds per sq. inch}$$

and maximum tension in steel 428

$$f_s = \frac{1\,943\,600}{14 \times 28^2 \times 0.0109} = 16\,250 \text{ pounds per sq. inch; formulas (18) and (19)}$$

This stress is allowable and the depth of haunch from top of beam of 28 inches will be accepted.

Length of haunch may be approximated. Moment of resistance of beam without haunch, allowing 15% excess compression or 750 lb. per sq. in., 428
$M_R = 750 \times 14 \times 26.5^2 \times .241 = 1\,780\,000$ inch pounds. Formula (17) 428
$M_B = 1\,943\,600$ inch pounds.
Hence from formula (22) length of haunch 430

$$x = \frac{176\,000}{1\,943\,600} \; \frac{18}{5} \times 12 = 3.9 \text{ inches}$$

Since maximum negative moment occurs in middle of column and necessary length of haunch is only 3.9 inches, no haunch will be introduced outside of the column.

Diagonal Tension Reinforcement of Beam. *Vertical stirrups.*

Take into consideration the beam designed on page 469 for which
$V = 19\,000$ pounds $w = 2000$ pounds.
$b' = 10$ inches $jd = 18.625$ inches and the unit shear

$$v = \frac{19\,000}{10 \times 18.625} = 102 \text{ pounds per sq. in. (formula 30),}$$ 447

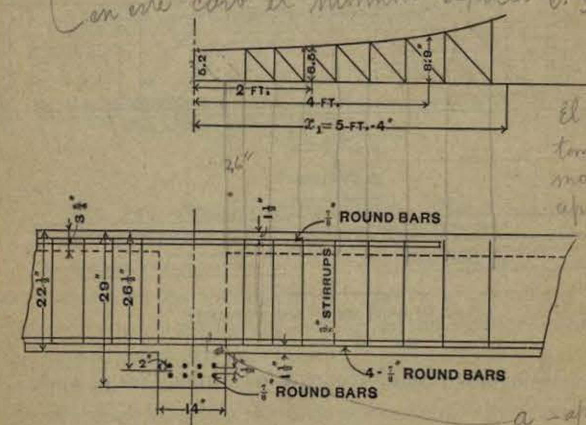
The allowable unit shear in concrete equals 40 pounds, hence stirrups are necessary. 447

Diameter of Stirrups. From formula (40) and Table on page 454 for a bond stress of 80 pounds diameter of a straight-pronged stirrup should not exceed $i = 0.012 \times 20.5$ inches. However, since in the present case the upper ends of stirrups are to be bent, ⅜ inch round bars may be considered as secure against slipping. 467

Location of Stirrups. Stirrups are unnecessary with the unit load w = 2000 pounds, at a distance from support

$$x_1 = \frac{18}{2} - \frac{40 \times 10 \times 18.625}{2000} = 5.3 \text{ feet (formula 38)}$$ 451

Spacing of the ⅜-inch stirrups, the area of both prongs being $A_s = 0.22$ square inches, is obtained by plotting in Fig. 148 values of s from formula (36). At support, $s_1 = \frac{24\,000 \times 18.6 \times 0.22}{19\,000} = 5.2$ inches. 451
For $x = 2$ feet, $s_1 = 6.5$ inches; for $x_2 = 4$ feet, $s_2 = 8.9$ inches. A smooth curve drawn through the points determines the spacing at any part of the beam. The first stirrup is placed half of the minimum spacing from the edge of the support and the last stirrup must not be farther distant from the limiting point, where stirrups are unnecessary, than half of the distance between the last two stirrups. The graphical determination of points for the stirrups is shown in Fig. 148. 452



NOTE – BENT BARS NOT SHOWN.

FIG. 148. Spacing of Vertical Stirrups (*See p.* 452).

Bent-up Bars as Diagonal Tension Reinforcement for Girder. In the girder designed on page 471 four of the eight bars are intended to be bent. If properly placed the bars may be used for taking the diagonal tension. The maximum total shear, $V = 41\,600$. Since the load is concentrated at points at which the beam runs into the girder the shear at the left of that point will be (See Fig. 149, p. 474).
$V = 41\,600 - (6 \times 360) = 39\,440$ pounds and the unit shear, $v = 116$ pounds; at right of the point, V will be $41\,600 - 6 \times 360 - 38\,000 = 1440$ pounds and $v_2 = 4$ pounds. Theoretically, shear reinforcement is needed to the point only where the beam intersects the girder.
The diagonal tension equivalent to the horizontal shear at the support is $\frac{41\,600}{24.6} = 1700$ pounds per one inch of length of beam, at point A is $\frac{39\,440}{24.6} = 1600$ pounds per one inch of length of beam. The total diagonal tension is represented by a trapezoid, the parallel sides of which are 1700 and 1600 and the length 6 feet. Hence total diagonal tension $= \frac{1700 + 1600}{2} \times 6 \times 12 = 118\,800$ pounds. One-third of the shear, or 39 600 pounds, is assumed to be taken by the concrete, hence the tension to be taken by the shear reinforcements is 79 200 pounds. Since six ⅞-inch round rods are to be bent, their area is 3.60 square inches and their tensile value from page 449 is $\frac{A_s \times 16\,000}{0.7}$ = 82 000 pounds. Now comparing the above values it is seen that