Tables for determining the dimensions and loading of slabs can be found on pages 512 to 515 and the methods of practical computation and details of design are illustrated in Example 6, page 469.

Cross Reinforcement of Slabs. Cross reinforcement, that is, bars at right angles to the principal bearing rods, is customarily used to prevent shrinkage and temperature cracks, and to give added strength. Although this reinforcement is not absolutely essential, it stiffens the construction floor and often renders the expansion joints unnecessary.

The amount of steel to use for this usually is selected somewhat arbitrarily, a cross-sectional area of bars equivalent to 0.2 per cent to 0.4 per cent (p = 0.002 to 0.004) of the cross-section of the floors being the most usual practice. This reinforcement is also necessary even when the beam is simply a small stiffener.

The top of the slab over a girder or beam which is parallel to the principal reinforcement bars should be reinforced transversely not only for stiffening the T-beam (see p. 443) but also to provide for the negative bending moment produced with the bending of the slab next to the beam or girder. This reinforcement is also necessary even when the beam is simply a small stiffener.

Computing Ratio of Steel. The ratio of steel in a slab is most readily found by dividing the cross section of one bar by the area between two bars, this area being the spacing of the bars times the depth of steel below top of slab. For example, a slab with steel 4 inches below the top and ½ inch round bars spaced 6 inches apart has a ratio, $p = \frac{0.196}{24} = 0.0082$, or 0.82 per cent steel.

Square and Oblong Slabs. Flat plate design by the elastic theory is treated on page 483. A rule for ordinary cases is to require that when the length of the slab exceeds 1½ times its width, the entire load should be carried by transverse reinforcement. For slabs more nearly square the following table represents the proportion of steel which should be run across the slab. These values, while not exact, are on the safe side.

Steel in Oblong Slabs

Ratio of length to breadth of slab.	Ratio of steel across the slab in terms of the total steel.
1	0.50
1.1	0.59
1.2	0.67
1.3	0.75
1.4	0.80
1.5	0.83

Thus if a slab is square, the reinforcement may be placed half in one direction and half in the other. If the bending moment is $\frac{wl^2}{12}$, the reinforcement in each direction must satisfy $\frac{wl^2}{24}$. The total amount of reinforcement thus determined may be reduced 25 per cent by gradually increasing the rod spacing from the one-third point to the edge of the slab.

DESIGN OF T-BEAM

The quantity of concrete in a beam may be reduced when it is built at the same time as the slab so that there is no joint between them, by considering it to be a T-section, that is, computing a portion of the slab as acting with the upper part of the beam in compression. In Appendix II, pages 754 to 756 inclusive, we present analyses of the two cases which may occur, depending upon the location of the neutral axis: Case I, neutral axis below the slab or flange; Case II, neutral axis at the underside of the flange; or within the flange. These analyses are not required for design and therefore only the working formulas are here reproduced.

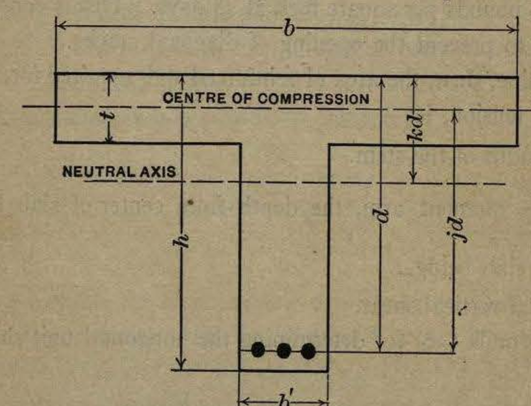

FIG. 132. Section of T-beam. (*See p.* 423.)

The theory of the design is similar to the theory of a rectangular beam, namely, that the total compression in the concrete in the upper part of the beam is equal to the total tension or pull in the steel at the bottom of the beam.

In the design of a T-beam, the thickness of the flange is fixed by the thickness of slab required to support its load, and the width of flange to use is selected in accordance with rules given below. The values to be determined by computation are then the depth of the beam, the width of stem or web, and the amount of reinforcement.

The width of the slab, b, to use for the flange of the T-beam in compression is selected somewhat arbitrarily. In no case of course can it be taken greater than the distance between beams. The Joint Committee has recommended a width not exceeding one-fourth the span length of the beam and also has limited the width to use on either side of the web to four times the thickness of the slab. It is probably safe to use a somewhat greater ratio of width to thickness than this in many cases.

Cross-section of Web as Determined by the Shear. The width of the web of a T-beam is governed by the layout of the tension rods (see p. 459) and by a study of the shearing stresses (see p. 446).

The total vertical unit shear in a beam effectively reinforced with bent bars or stirrups, or both, is limited by the Joint Committee to 120 pounds per square inch for ordinary concrete having a compressive strength (in cylinders) of 2000 pounds per square inch at 28 days. This is conservative but was selected to prevent the opening of diagonal cracks.

To determine, then, the area of reinforced web required for shear involving diagonal tension, let

b' = breadth of the stem.

$d - \frac{t}{2}$ = moment arm, the depth from center of slab to steel, the thickness of slab being t.

V = total vertical shear.

then from formula (30) for determining the horizontal unit shear

$$b'\left(d - \frac{t}{2}\right) \geqq \frac{V}{120} \qquad (13)$$

That is, the area of web at any point in the beam (considering this up to the middle of the slab) must not be less than the total shear divided by the maximum allowable unit shear for the beam with its reinforcement.

The design is illustrated in Example 6, page 470.

Minimum Depth of T-Beam. The minimum depth is the depth at which concrete and steel are stressed simultaneously to their working limits. It is governed by **the compression in the flange which must not exceed the working compressive strength of the concrete.** Greater depth than the minimum is generally used for economical reasons.

The minimum allowable depth may be found from the folding diagram, page 525. If preferred, the rectangular beam formula (1), page 418, may be used where the depth of the beam is not greater than four times the thickness of slab, using in this formula the breadth of the flange, b, for

the breadth of the beam. For ratios of depth of T-beams to thickness of slab larger than four the rectangular beam formula gives unsafe results and the formulas given in Appendix II, page 755, must be used.

The methods are illustrated in Example 6, page 470.

Economical Depth for a T-Beam. Usually a greater depth than the minimum is desirable for economy, because deepening the beam reduces the area of steel proportionally. Professors Turneaure and Maurer* analyze the depth for maximum economy and suggest from this the most economical values.

Using the notation

d = depth of T-beam from compressed surface to center of steel in inches.
t = thickness of flange in inches.
b' = breadth of the stem in inches.
M = bending moment in inch pounds.
f_s = allowable unit tension in steel in pounds per square inch.
r = ratio of unit cost of steel in place to unit cost of concrete in place (using same units for steel and concrete).

$$d - \frac{t}{2} = \sqrt{\frac{r\,M}{f_s b'}} \qquad (14)$$

From this formula the most suitable depth may be selected after two or three trial computations for different widths of stem. The ratio of costs, r, ranges between 38 and 75. For cost of concrete† in place 20 cents per cubic foot, and cost of steel in place 3 cents per pound, the ratio of costs equals 75, while for concrete at 40 cents per cubic foot and cost of steel 3 cents this value will be reduced to 38. In calculations where no unit costs are given, a value of 60 may be selected for r.

The depth of the T-beam should not be made too great in proportion to the breadth of stem. Many designers make the ratio of the depth of a T-beam to its width of web between 2 and 3. For very deep and large beams a ratio of 4 may be accepted, while, on the other hand, if head room is limited, the depth of the beam may be fixed and the width of stem be determined by area required for shear, so that ratio, $\frac{d}{b}$ may be even less than 2.

Another plan sometimes followed in studying designs is to make the depth

* Turneaure and Maurer's "Principles of Reinforced Construction," Second Edition, p. 238.

† The cost of concrete need not include form construction since a variation in depth affects this but slightly.

of T-beam an arbitrary ratio to its span. Comparison of a number of representative designs shows an average ratio of span to depth of beam as about 10 to 12, which suggests the approximate rule to make the depth in inches equal to the span in feet.

Sectional Area of Steel in a T-Beam. The area of cross-section of steel in tension may be obtained very closely by the following formula:

Let

A_s = cross-section of steel in square inches.
M = bending moment in inch-pounds.
f_s = allowable unit tension in steel in pounds per square inch.
d = depth of T-beam in inches.
t = thickness of flange in inches.

then

$$A_s = \frac{M}{f_s\left(d - \frac{t}{2}\right)} \qquad (15)$$

This formula assumes that the center of compression of beam is at the center of the slab. This gives slightly high results for a T-beam with very thin flange in proportion to the dimensions of the web, and too low results for a shallow T-beam with thick flange; ordinarily the error is so slight as to be inappreciable but if d is less than $3\,t$ use formula (4), page 418, taking b as breadth of beam. Formulas for more exact computations or for reviewing T-beams are given in Appendix II, page 749, and quoted below* It is recommended that an inexperienced designer check his results obtained by approximate formulas by the more exact ones.

From the diagram, page 525, the area of steel may be obtained directly and comparisons made between different designs.

Details of Design. The design of a T-beam must also be studied for shear reinforcement (see p. 448), bond of steel to concrete (see p. 456), and especially for the design at the support, which must be adapted to the negative bending moment (see p. 428).

The example on page 470 illustrates the use of the formulas and the principles of design. The selection of bending moments is treated on page 439.

* Let kd = depth of neutral axis; n = ratio of elasticity; b = breadth of flange; f_c = outside fibre compression in concrete. Then,

$$kd = \frac{2\,nd\,A_s + b\,t^2}{2\,n\,A_s + 2\,bt};\ z = \frac{3\,kd - 2\,t}{2\,kd - t}\cdot\frac{t}{3};\ jd = d - z;\ f_s = \frac{M}{A_s\,jd};\ f_c = \frac{M\,kd}{bt\,(kd - \frac{1}{2}t)\,jd}.$$

BEAMS WITH STEEL IN TOP AND BOTTOM

Although concrete is always cheaper than steel to use for compression, it is frequently desirable to place steel in the compression as well as in the tension side of the beam. In a continuous beam, for example, the steel is carried horizontally into the support and may be figured with the concrete to assist it in taking the compression provided its length is sufficient to provide bond.

Analysis of the design with steel located to take compression and tension, with no tension considered in the concrete, is presented in Appendix II, page 757. For convenience, the diagram, Fig. 133, is here reproduced, and the working formulas are given.

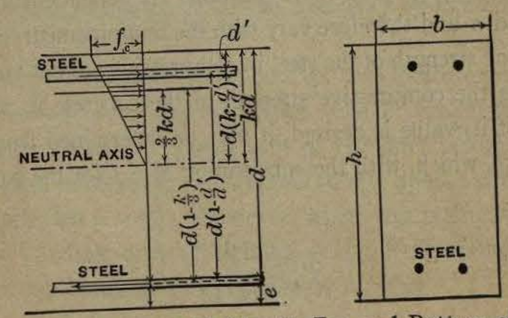

FIG. 133. Resisting Forces with Steel in Top and Bottom of Beam. (*See p. 427 and 757.*)

Let

b = breadth of beam in inches.
d = depth of beam from compressed surface to center of steel in inches.
a = ratio of depth of compressive steel to depth of beam.
p = ratio of cross-section of steel in tension to cross-section of beam bd above this steel.
p' = ratio of cross-section of steel in compression to cross-section of beam above the steel in tension.
f_c = unit compressive stress in outside fiber of concrete in lb. per sq. in.
f_s = unit tensile stress, or pull, in steel in lb. per sq. in.
f_s' = unit compressive stress in steel in lb. per sq. in.
M = moment of resistance or bending moment in general in in. lb.
C_c, C_s, C_s' = constants from Table 8, pages 516, 517.

The location of the neutral axis varies greatly with the location and the area of the steel, so that an approximate formula cannot easily be made.

The allowable stresses must not be exceeded either in the concrete or the steel. The bending moment, therefore, must not exceed the moment of resistance of the concrete and the steel and the beam must satisfy the equations (see p. 757 for derivation):

$$M = f_c bd^2 C_c \quad (17) \quad \text{or} \quad f_c = \frac{M}{bd^2 C_c} \quad (18)$$

and

$$M = f_s bd^2 C_s \quad (19) \quad \text{or} \quad f_s = \frac{M}{bd^2 C_s} \quad (20)$$

These formulas may be solved readily by introducing values of C_c or C_s from the table on page 516. The constants are dependent upon the values of p, p', a and n and therefore vary with the reinforcement of the beam.

The working strength of the steel in compression cannot be reached without exceeding the compressive strength of the concrete in which it is imbedded, but, if its value is desired, it may be determined from the formula (39), page 759, which, with the substitution of C_s' for the square brackets, becomes

$$f'_s = \frac{M}{bd^2 C'_s} \quad (21)$$

The value of C_s' is obtained directly from the table on page 516. The use of the formulas is illustrated in Example 6, page 470.

DESIGN OF A CONTINUOUS BEAM AT THE SUPPORTS

The formulas and table just given for a beam with steel in top and bottom are of the greatest value in designing the ends of a continuous beam.

A number of concrete buildings have been built in the past with beams having insufficient steel through the top of the supports to take the pull and insufficient concrete at the bottom of the ends of the beam to take the compression, and when these have been loaded as designed, cracks, and in many cases serious ones, have occurred at the supports. Just as much care, therefore, is necessary in designing the end of a reinforced beam as the middle.

The tendency to overstress the supports is due to the T-beam design. In the middle of the T-beam the slab takes the compression, but at the support, the compression being in the bottom of the beam because of the negative bending moment, there is only the web of the beam to resist it.

In designing, a slightly higher compression may be allowed in the concrete at the end than at the middle of the beam, using 750 pounds per square inch for 2 000 pounds concrete instead of 650, (see page 528) because the negative moment decreases so rapidly that only a short section is under maximum stress. Besides this, the steel in the lower part of the beam (if sufficiently bonded to the concrete) may be reckoned in compression by the formulas just given. If this is not sufficient to fulfill the requirements, the lower surface of the beam near the support may be dropped so as to form a flat haunch.

By bending up half of the horizontal steel in the beams on each side of the support, and carrying it across over the support, lapping far enough to attain its full strength in bond, the tension in the top of the support will be provided for, since this gives the same tension steel as in the center of the beam. If desired, the stress, f_s, in the steel may be figured from formula (20) above.

Although bond tests with hooked bars (p. 467) indicate that a right angle 5 diameters in length or a semi-circular bend of similar length, properly imbedded, will develop the elastic limit of the steel before giving way, it is the safest plan in ordinary construction to rely upon a straight lap of the required length (see p. 464). However, where this is impossible, as at the wall line in a building, or in a retaining wall, the effectiveness of the hook permits thorough bonding of the members together.

Since the bending moment is a maximum at or near the center of the support, the moment at the edge of the support is slightly less and it is, therefore, frequently worth while to recompute it or estimate it by curves on page 436.

If the allowable compression in the concrete at the bottom of the member exceeds that allowed by formula (18), page 428, the steel in the bottom of the beam or else the concrete or both must be increased in area. The simplest plan in most cases is to make the beam deeper next to the support by forming a flat haunch. When this is not permissible, extra horizontal steel may be inserted instead. While the forms for this haunch are somewhat troublesome to construct, their cost for beams and girders of usual size should not exceed 25¢ to 50¢ each.

The amount of increased depth required may be obtained by trial from formula (18) above, assuming a new depth, and then with the aid of the table on page 516, determining whether the conditions are as specified. This is illustrated in the example on page 472.

Under ordinary conditions the computation need be made only at one point, that is, next to the support, since the point to end the slope can be readily figured from the following formula:

For a uniformly loaded beam, let

M_b = negative bending moment next to the support.
M_r = moment of resistance of the inverted T-beam without the haunch, governed by the concrete.
x = length of haunch.
l = span of beam.

Then

$$z = \frac{l}{5}\,\frac{M_b - M_r}{M_b} \qquad \text{(approximately)*} \qquad (22)$$

An illustration of the use of this formula is given in Example 6, page 472.

EFFECT OF VARYING MOMENT OF INERTIA UPON THE BENDING MOMENT

However the bending moment may be computed, if the beam is built continuously with the next bay, pull or tension is bound to occur over the support with compression at the bottom of the beam. The assumption is sometimes made that if the middle of the beam is designed as freely supported, that is, on a basis of $\frac{wl^2}{8}$, the supports will be relieved and a readjustment will take place. This is only partially true, and usually should not be counted upon in design.

The assumption of reduced bending moment at the support is based on the smaller moment of inertia at the support, but a thorough study by the authors of different conditions shows that a very large difference in the moment of inertia, as great a difference as it is possible to have in any ordinary floor design, causes a reduction in bending moment of less than 10% and under most conditions the reduction is even much less than this. Consequently a beam at the support should be designed, as suggested in the preceding paragraph, for the full negative bending moment as required by the formula $\frac{wl^2}{12}$.

* This formula is based upon the fact that the point of zero moment is at approximately $\frac{1}{5}$ of the span, and from the curves of bending moment on p. 436, it is evident that the variation in the moment between the support and the $\frac{1}{5}$ point is very nearly a straight line. Hence the difference between the bending moment and the moment of resistance is in approximately the same ratio to the bending moment as is the ratio of the distance from the point where the haunch is needed to the point of zero bending moment. When the point of zero moment is not approximately at $\frac{1}{5}$ span the fraction may be altered accordingly.

SPAN OF A CONTINUOUS BEAM OR SLAB

It is customary to consider the span of a continuous beam or slab as the distance between the centers of its supports. In general this is the simplest plan to follow and one which is always on the side of safety. If the support is exceptionally wide, as when a slab runs into a wide beam, or a beam or girder into a large column,* an arbitrary length of span may be taken, if desired, as the net span between supports plus the total depth of the member which is being designed. The maximum negative bending moment may be considered then either at the center of the support or, if the width of the support is greater than the depth of the member, at a point within the support equal to half the depth of the member.

DISTRIBUTION OF SLAB LOAD TO THE SUPPORTING BEAMS

If slabs are reinforced in both directions, the loads carried to the beams supporting them will not be uniformly distributed over the length of the beam, but may be assumed to vary in accordance with the ordinates of a triangle.

Assuming that the slab transmits a load to its nearer support, we have the following formulas for determining the moment to use in computing the long and the short supporting beams.

Let

l_l = the longer span of a rectangular slab in feet.
l_s = the shorter span of the slab in feet.
w = load per linear foot of beam if the slab is considered as supported by longer beams only.
M_l = bending moment in foot pounds of longer beam.
M_s = bending moment in foot pounds of shorter beam.

Then the moments of the two beams, assuming them as freely supported, are found by the application of simple mechanics, to be

$$M_l = \frac{1}{8} w l_l^2 \left(1 - \frac{1}{3}\frac{l_s^2}{l_l^2}\right) \quad (23) \qquad \text{and } M_s = \left(\frac{1}{8} w l_s^2\right)\frac{2}{3} \quad (24)$$

For continuous or fixed beams the fraction $\frac{1}{8}$ may be changed to its proper ratio.

Formula (24) does not apply to girders supporting one or more beams. This case is treated under the heading which follows.

* The deflection and the bending moment of a member are changed as soon as it enters the support because of the change in the moment of inertia.

Example 2: What will be the bending moments in the two continuous beams supporting an oblong panel the whole length of which is twice the breadth and which is reinforced so as to transmit its load both ways?

Solution: Using $\frac{1}{12}\ wl^2$ for the continuous beams instead of $\frac{1}{8}\ wl^2$ and substituting:

$$\text{Moment in longer beam, } M_l = \frac{1}{12} wl^2 \left(1 - \frac{1}{3}\frac{(\frac{1}{2}l_l)^2}{l_l^2}\right) = \frac{11}{144} wl_l^2$$

in terms of the longer span, and

$$\text{Moment in shorter beam, } M_s = \frac{1}{12} wl_s^2 \frac{2}{3} = \frac{1}{18} wl_s^2$$

DISTRIBUTION OF BEAM AND SLAB LOADS TO GIRDERS

When one or more beams run into a girder, the load upon the girder consists of the concentrated live and dead loads from the beams acting at their points of intersection with the girder, the uniformly distributed weight of the girder itself, and the unsymmetrically distributed weight of a small portion of the floor slab, with its live load, which bears directly upon the girder.

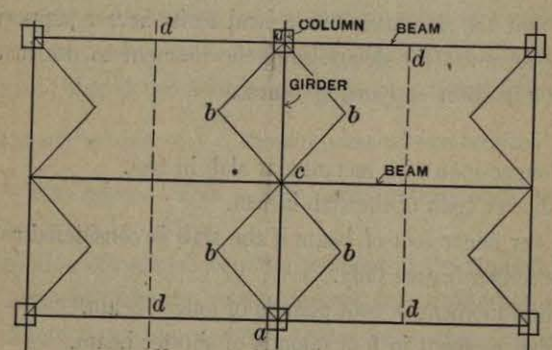

FIG. 134. Distribution of Beam and Slab Loads to Girder. (*See* p. 432.)

To avoid the computation of several moments, a series of studies have been made by the authors for different conditions, and it has been found that the maximum bending moment of a girder may be obtained without appreciable error by considering, as a uniformly distributed load, the weight of the girder plus the weight of slab and its live load, for an area whose length is the length of the girder and whose width is the average length of the beams running from each side into the girder. The sum of these loads divided by the length of the girder gives a uniformly distributed load for which the ordinary formula may be used.

Thus in Fig. 134, instead of computing the moment on the girder as the

sum of the moments produced by loads of the triangles, *a b c*, plus the concentrated loads from the beams at *c*, the entire load *d d d d* may be considered as uniformly distributed over the girder in the length *a a*.

With only one condition is there an appreciable variation from the exact maximum moment, and this is a case where two beams run into a girder at the one-third points. Here the maximum moment obtained by the uniformly distributed method gives slightly too conservative results, and may be reduced by 10%.

Moments in a girder other than the maximum must be computed for individual conditions.

BENDING MOMENTS AND SHEARS

Bending moments and shearing forces have to be computed so frequently in reinforced concrete design that the more common rules and formulas are given here, and besides this elemental matter diagrams are presented for estimating the moments and shears in various kinds of loading, and recommendations are made for the computation of bending moments in design. Shear and diagonal tension in beams are taken up at length.

Rule to Find Reactions at Supports. The reaction at a support must be found in order to determine the bending moment. The sum of the upward forces, which in ordinary beams are the reactions at the supports, is equal to the sum of all the downward vertical forces or loads. In a simple beam supported or fixed at the two ends, the reaction at either end is found by taking moments of all forces about the other support and solving for the reaction desired.

Expressed as a formula, if

R = desired reaction.
P = any vertical load.
l = span.
x = distance of load from the support at which the reaction is desired.
Σ = sum, using $-$ for downward and $+$ for upward forces, then

$$R = \frac{\Sigma P\,(l - x)}{l} \qquad (25)$$

Example 3: In Fig. 135, where there is a uniform load over the entire span and also concentrated loads $P_1 = 200$ and $P_2 = 350$ at the $\frac{1}{3}$ points, what is the left reaction?

Solution: $R = \frac{(200 \times 8) + (350 \times 4) + (100 \times 12)6}{12} = 850$ pounds.