0.0001 to 0.0002 of its length, that is, 0.01 per cent to 0.02 per cent, before showing minute cracks or "water-marks." Cracks become noticeable at a stretching varying in different specimens from 0.0003 to 0.0010 of their length. At this stretch, the steel imbedded in the concrete will have a stress of 9 000 to 30 000 pounds per square inch. Even then, however, the cracks are still so small and are so well distributed by steel properly placed that they are not apt to be noticed in a reinforced structure until the steel has nearly reached its elastic limit.

The concrete in a reinforced beam stretches similarly to the concrete in a plain beam except that the plain concrete beam breaks when the limit of stretch is reached, while if reinforced, the pull is borne partly by the steel and partly by the concrete, and they both stretch together up to the point where cracks, so minute at first as to be almost invisible, occur in the concrete.

The action of the reinforced concrete is shown in the deflection curve in Fig. 130. The inclination of this curve changes at about the same load that is required to break a similar beam of plain concrete.

The diagram shows a typical result of Prof. Talbot's tests of the deformation of the concrete and the deformation of the steel, the deflection of the beam, and the various measured positions of the neutral axis during flexure. Among other conclusions, Prof. Talbot draws the following:

1. The composite structure acts as a true combination of steel and concrete in flexure during the first or preliminary stage, and this stage lasts until the steel is stressed to, say, 3 000 pounds per square inch, and the lower surface of the concrete is elongated about $\frac{1}{10\,000}$ of its length.

2. During the second or readjustment stage there is a marked change in distribution of stresses, the neutral axis rises, the concrete loses part of its tensional value, and tensile stresses formerly taken by the concrete are transferred to the steel. During this stage minute cracks probably exist, quite well distributed, and not easily detected.

3. In the third or straight-line stage the neutral axis remains nearly stationary in position and the concrete gradually loses more of its tensional value. Visible cracks appear and gradually grow larger, though no change in the character of the load-deformation diagram results. It would seem probable that at these cracks the stress in the steel is more than is indicated by the average deformation for the full gage length.

Professor Talbot states that at the load when the curve changes character,—which in the beam shown in the diagram is about 8 000 pounds total load,—there are probably invisible cracks in the lower portion of the beam This change in direction of the curve, indicating a suddenly increased load upon the steel, is strong proof of the loss in tensional resistance of the concrete. Professor Turneaure, moreover, in his experiments, at loads somewhat beyond the point of change in direction, actually discovered these minute cracks. He tested his beams upside down, that is, the load was applied upward, and the minute cracks or water-marks were shown by hair lines on the wet surface of the concrete. Professor Turneaure* says:

> It has been found that by testing the beams when somewhat moist, a crack is made visible when exceedingly small, it appearing first as a narrow, wet streak perhaps $\frac{1}{8}$ inch wide and a little later as a dark hair-like crack. It was not necessary to search for the lines with a microscope as under these conditions they were readily found.
>
> That the wet streak, called a "watermark" hereafter, shows the presence of an actual crack was demonstrated last year by sawing out a strip of the concrete containing such a watermark; the strip fell apart at the watermark.
>
> In the plain concrete no watermarks or cracks were observed before rupture. Comparing the observed and calculated elongations of the reinforced concrete with those for the plain concrete at rupture, it will be seen that the initial cracking in the former occurs at an elongation practically the same as in the latter.
>
> The significance of these minute cracks is an open question. It has been supposed that concrete reinforced by steel will elongate about ten times as much before rupture as will plain concrete. These experiments show very clearly that rupture begins at about the same elongation in both cases. In the plain concrete total failure ensues at once; in the reinforced concrete rupture occurs gradually, and many small cracks may develop so that the total elongation at final rupture will be greater than in the plain concrete. In other words, the steel develops the full extensibility of a non-homogeneous material that otherwise would have an extension corresponding to the weakest section.

These results are somewhat at variance with the conclusions reached by Mr. Considère† in France. He was not able to locate these fine cracks and therefore concluded that while the stretch of plain concrete was about 0.0001 of its length or about 0.01%, in combination with steel it could actually attain a stretch twenty times this, or 0.2%. Because of this apparent action of the concrete, Mr. Considère in his formula for beams assumes the concrete to resist a certain amount of tension.

The stretch, or deformation, in the concrete of a reinforced beam may be estimated approximately from the pull, or stress, upon the steel and the modulus of elasticity of the latter, since

$$\text{elastic deformation} = \frac{\text{stress}}{\text{modulus of elasticity}}$$

* Proceedings American Society for Testing Materials, 1904.

† Considère's Reinforced Concrete, p. 35.

For example, if the steel is pulled to 16 000 pounds per square inch, the stretch per unit of length (disregarding initial tension) is

$$\frac{16\ 000}{30\ 000\ 000} = 0.00053$$

Knowing the stretch in the concrete (and therefore the stretch in the steel imbedded in it) the stress in the steel is readily computed from the same formula.

Tensile Resistance in the Concrete. Professors Talbot and Turneaure both concluded from their tests in 1904 that the tensile strength of concrete may be disregarded in the consideration of the ultimate load carried by a beam. This has since been adopted as current practice in design and is in accordance with the recommendations of 1908 and 1909 in America and Europe. The tensile resistance of the concrete affects the deformation and deflection of the beam under the smaller loads, but if, as is customary, the working strength is taken as a definite fraction of the resistance at the elastic limit of the steel, **the tensile resistance of the concrete need not be considered in the design of reinforced beams.**

Prof. Turneaure says:

The presence of the cracks of course seriously affects the tensile strength of the concrete, and, as they appear at an elongation corresponding to a stress in the steel of 5 000 pounds per square inch or less, it would seem that no allowance should be made for the tensile resistance of the concrete. Furthermore, if such cracks are present the calculation of the tensile resistance of reinforced concrete by the method used by Considère leads to no useful result. In his tests Considère determines the stress in the steel from measurements of its elongation and then assumes the concrete to carry the remainder of the load. Assuming the value of E to be uninfluenced by the concrete, this would be correct so long as the stress in the steel and in the concrete is uniform between points of measurement. As stated by Considère himself, such results are only average values. But the concrete may be cracked entirely through and yet possess a very considerable average tensile strength over a length of several inches. Obviously in that case an average is of no value; the strength of the concrete is really zero.

In practical design the most important question which arises is how far a concrete beam may be cracked without exposing the steel to corrosive influences. In this respect it seems to the writer that the minute cracks which appear in the early stages of the tests can have very little influence. However, the entire question of the effect of cracks and pores in the concrete on the corrosion of the steel needs careful investigation.*

*For later information on this point, see p. 328.

QUALITY OF REINFORCING STEEL

It is generally recognized in reinforced beam design that the yield point of the steel should be considered as the point of failure of this material. Tests show that when the metal reaches its yield point, the beam sags, and this deflection, due to the stretch of the steel and in some cases to the slipping of the steel because of its reduced cross-section, is likely to produce crushing in the concrete.

The yield point of ordinary mild steel purchased in the open market, as determined by the drop of the beam in testing (the true elastic limit is several thousand pounds lower) cannot safely be fixed at a higher value than 30 000 pounds per square inch, although frequently, and in fact in the majority of cases, a value of at least 36 000 pounds, and in many cases 40 000 pounds will be found.

High steel, that is, steel containing a high percentage of carbon, has a much higher yield point than mild steel. If of first-class quality,* a minimum yield point may be placed at 50 000 or 55 000 pounds per square inch and much of it will reach 60 000 pounds. The ultimate strength should be not less than 85 000 pounds per square inch. Thus, if it can be safely employed in reinforced concrete, it is adapted to carry much higher stress than mild steel, and, conversely, a smaller percentage of it is required for the same moment of resistance. Many engineers do not approve of the use of high steel because of its brittleness when of poor quality, and the danger of sudden accident, and because of the fact that it is prohibited in ordinary structural steel work.

Brittleness in steel, however, is less dangerous in reinforced concrete than in many classes of structural steel work because the concrete protects it from shock, and also because smaller sections of steel are used in concrete beams than in steel beams, and the large and irregular shapes of the latter render them much more sensitive to irregular cooling during the process of their manufacture.

Mild steel, that is, ordinary market steel, is manufactured and sold under such standard conditions that for unimportant structures it often may be used without other test than the bending test given on page 415. High steel, on the other hand, must be very thoroughly tested. When tested, however, as per our specifications, page 38, it is entirely safe and to be preferred to mild steel. The objection to it for reinforced concrete is based largely upon the use of a poor quality of material and the extra cost. Another objection which has been raised is that before the elastic

* See Specifications for First-class Steel, p. 38.

limit is reached, the stretch in the high steel may produce excessive cracking in the concrete in the lower portion of the beam, and thus expose the steel to corrosion. The mere fact that cracks are visible does not prove that they are dangerous, because the steel is always designed to take the whole of the tension. Mr. Considère's and Professors Talbot's and Turneaure's tests indicate that there is no dangerous cracking even with high steel until the yield point of the steel is reached.

Tests made in Europe in 1907 (see p. 336) prove quite conclusively that the cement protects the steel from ordinary and even extraordinary corrosive action until the elastic limit of the steel is nearly reached. In cases where very minute cracking of the concrete may cause anxiety (even although not dangerous), the steel, whatever its quality, should not be stressed beyond the ordinary limits of, say, 16 000 pounds per square inch.

A yield point in steel of 30 000 pounds per square inch corresponds to a stretch of 0.0010 of its length and a yield point of 50 000 to a stretch of 0.00167. (See p. 411.)

If steel could be made with a high modulus of elasticity it would be particularly serviceable for reinforced concrete, because the higher the modulus of elasticity of a material, the less is the deformation under any given loading. Unfortunately, however, all steel, whether high or low in carbon, has substantially the same modulus of elasticity (30 000 000 lb. per sq. in.).

It may be stated, then, that high carbon steel, say, 0.56% to 0.60% carbon, of the quality used in the United States for making locomotive tires, is better than mild steel for reinforced concrete provided the steel is well melted and rolled, and is comparatively free from impurities, such as phosphorus. However, a high carbon steel, unless limited by chemical analysis, and made under careful inspection, is in danger of being more brittle than low carbon steel. Its use, therefore, should be limited strictly to work important enough to warrant the ordering of a special steel and the taking of sufficient trouble on the part of the purchaser to insure strict adherence to the specifications. Since manufacturers cannot always be depended upon to exactly follow specifications of this nature, it is necessary that an inspector be sent to the works either by the dealer or the purchaser.

The specifications for first-class steel on page 38 are sufficiently explicit so that steel which comes up to them can be safely used and a working stress of 20 000 lb. per sq. in. will not be excessive. A steel which can be employed with safety for all the locomotive and car wheels of the country certainly cannot be discarded as unsafe for concrete, provided similar precautions are taken in its purchase; on the other hand the extra cost may prohibit its use except under special conditions.

Bending Test for Steel. The most important test in the specifications is the bending test and **no steel which fails to pass this bending test should be used under any circumstances.** The bending test is as follows: Test specimens for bending shall be bent cold to the following angles without fracture on the outside of the bent portion:

Around twice their diameter.	Around their own diameter.
Specimens 1 inch thick, 80°.	Specimens $\frac{1}{4}$ inch thick, 130°.
Specimens $\frac{3}{4}$ inch thick, 90°.	Specimens $\frac{3}{16}$ inch thick, 140°.
Specimens $\frac{1}{2}$ inch thick, 110°.	Specimens $\frac{1}{8}$ inch thick, 180°.

Steel with high elastic limit, whether due to high carbon or to manipulation in manufacture, should be purchased with these reservations even if the working stress is to be no higher than is used with mild steel, say, 16 000 pounds per square inch, because it is liable to be brittle. In case a lot of steel has been delivered without previous test by the purchaser, one bar selected at random in every 100 should be subjected to this test and if it fails to pass, the portion from which it is taken should be rejected.

THE STRAIGHT LINE THEORY

For reasons discussed in the preceding paragraphs, the authors have selected the straight line theory of distribution of stress with the concrete taking no tension.

This theory assumes the following hypothesis as a basis for practical design:

(1) A plane section before bending remains plane after bending.

(2) Tension is borne entirely by the steel.

(3) Initial tension or compression is absent in the steel.

(4) Adhesion of concrete to steel is perfect within working limits.

(5) Modulus of elasticity of concrete within the usual limits of stress is a constant.

Our reasons for selecting this theory may be briefly recapitulated as follows:

(a) Beams designed by it and properly built will be unquestionably safe.

(b) Fine cracks are formed in the tension portion of the beam at an early stage in the loading which actually destroy nearly all of the tensile resistance of the concrete.

(c) The modulus of elasticity in many tests has been shown to be approximately a constant within working loads.

(d) This theory is the simplest, and the most easily understood.

(e) It has been adopted by the highest authorities in America and Europe.

(f) The results from it may be readily compared with other theories.

These assumptions lead to the formulas given in this chapter.

During the period termed by Prof. Talbot the first stage (see p. 410) it is necessary in scientific computations involving the deflection of the beam to take the tension of the concrete into account by the methods given on page 760.

LOCATION OF NEUTRAL AXIS

The location of the neutral axis after the load has been transferred to the steel, is given in formula (6) on page 420 and numerical values for different moduli of elasticity and different percentages of steel on page 521. As is evident from the formula, it varies with the strength and elasticity of both the concrete and steel. Because of the peculiar action of the deformations, as illustrated in Fig. 130, page 409, the location of the neutral axis changes as the load is applied. The question is much simplified in practice by assuming a constant ratio of moduli of 15.

An empirical formula suggested by Prof. Talbot for the location of the neutral axis under normal loading is given on page 479.

Tests show that the neutral axis for small loadings is just below the center line of the beam. For greater loadings it moves gradually nearer to the compression side. As the first cracks develop, the change in position of the neutral axis is more sudden, and the distance of the neutral axis from the compression side soon reaches its minimum, which for usual percentages of steel is 3/10 to 4/10 of the depth from the top, after which the change for additional loading is inappreciable. At failure the change is sudden again.

DESIGN OF A RECTANGULAR BEAM

In a simple rectangular beam we may represent the stresses by the diagram shown in Fig. 131, page 417. At any vertical section through the beam the concrete in the upper portion resists the forces which tend to compress it, and the steel in the lower part of the beam resists the forces which tend to stretch and break it in tension. The compressive resistance acts in one direction and the tensile resistance in another direction, as designated by the large arrows in the diagram. The center of tension in the steel is at the center of the rod, or, if there is more then one tier of rods, through the center of gravity of the set of rods. The center of pressure of

the concrete passes through the center of gravity of the triangle which represents the compressive stresses. The reason for the assumption of the uniformly increasing pressure from the neutral axis to the outside fiber is discussed above.

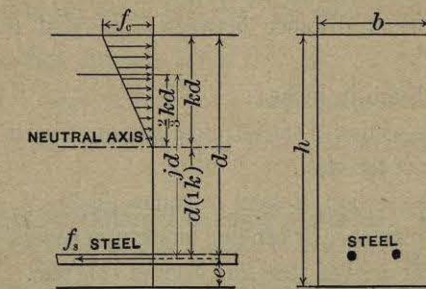

Fig. 131. Resisting Forces in a Reinforced Concrete Beam. (*See p.* 417 *and* 420.)

On page 420 are given simple formulas to review a beam already designed and the various letters in Fig. 131 are there defined.

A complete analysis of rectangular beams is presented in Appendix II, and the reader is referred to the discussion of the theory there given and the derivation of the formulas. (See p. 751.)

The theory of beams requires that the total internal pressure be equal to the total tension or pull in the steel. The safe resisting moment of the beam, which of course must be equal to or greater than the bending moment, is the product of the moment arm (that is, the distance between centers of tension and compression) times either the total safe pressure or the total safe pull. In case the design is unbalanced so that the beam is stronger in compression than in pull, the strength of the beam is limited by the safe moment of resistance determined from the allowable tension or pull in the steel. If, on the other hand, the tensile resistance is greater than the compressive resistance, the concrete governs the strength of the beam.

A beam, then, must have breadth and depth sufficient to prevent excessive compression in the concrete in the top of the beam and enough steel to take all the pull without exceeding the working stress of the steel. Rules for this are given in the simple formulas which follow. The steel must also have sufficient bond (see p. 456) and in many cases inclined or vertical reinforcement is required as treated in connection with diagonal tension, pages 448 to 459. Continuous beams also require reinforcement over the supports, as described in pages 427 to 431.

Having computed the maximum bending moment due to the loads (see

p. 439) the breadth of the beam, b, is assumed and the depth of the steel is found from the following formula·

Let

d = depth of beam from compressed surface to center of steel in inches. (See Fig. 131, p. 417)

jd = moment arm or distance between centers of tension and compression.

b = breadth of beam in inches.

p = ratio of cross-section of steel to cross-section of beam above the center of gravity of the steel.

A_s = area of cross section of steel in square inches.

M = moment of resistance or bending moment in general in inch pounds.

C = a constant for a given steel and a given concrete.

$$d = C\sqrt{\frac{M}{b}} \qquad (1)$$

$$\text{and} \quad A_s = pbd \qquad (2)$$

The constants C and p to be substituted in the above formulas may be taken from the table on page 519 corresponding to the allowable working stresses in steel and concrete and to the ratio of their moduli of elasticity.

Selected working stresses for tension in steel, f_s, and compression in concrete, f_c, require a definite percentage of steel, and the percentage cannot be altered without changing the ratio of these working stresses.*

For a working compression in the concrete of 650† pounds per square inch, a working pull in the steel of 16 000† pounds per square inch, and a ratio of modulus of steel to concrete of 15†, for concrete having a compressive stress in cylinder form of 2 000 pounds per square inch at 28 days.

$$d\ddagger = \frac{1}{10}\sqrt{\frac{M}{b}} \qquad (3)$$

$$A = .0077\, bd \qquad (4)$$

Example 1: What depth of beam having a span of 18 feet and what area of steel are required, using above unit stresses, for a freely supported beam with a load of 600 pounds per running foot?

* See page 752, formula (5).

† Recommended by the Joint Committee, 1909

‡ More exactly, $d = 0.096\sqrt{\frac{M}{b}}$

Solution: Bending moment, M, for $\frac{wl^2}{8}$ is $\frac{600 \times 18 \times 18 \times 12}{8} = 291600$ inch pounds, and using formula (3)

$$d = \frac{1}{10}\sqrt{\frac{291600}{8}} = 19 \text{ inches}$$

With 2 inches of concrete below the steel, the total depth of beam is thus 21 inches.

The area of steel from formula (4) is

$$A = .0077 \times 8 \times 19 = 1.17 \text{ square inches}$$

thus (from Table 1, page 507) requiring four $\frac{5}{8}$ inch round bars, or their equivalent.

Depths and Loads for Different Bending Moments. The depth may be obtained in terms of the unit load, if desired, by substituting for M in formula (1) its value in terms of the load and the span. This may be readily transposed also to give the load, w, which a given beam will carry.

The following table is used for determining on the one hand the depths of a beam to be designed, and on the other hand the safe loads of a beam already designed, for uniformly distributed loads. In the formulas in the table:

d = depth of beam from compressive surface to center of steel in inches.

b = breadth of beam in inches.

w = load in pounds per running foot of beam (including weight per linear foot of beam).

l = span in feet,

C = a constant from Table 10 on page 519.

Formulas for Depth and Loading of Rectangular Beams for Different Bending Moments. (See p. 519 for values of C.)

DEPTH, d, AND LOAD PER FOOT, w.	$\frac{wl^2}{12}$	$\frac{wl^2}{10}$	$\frac{wl^2}{8}$	$\frac{wl^2}{4}$	$\frac{wl^2}{2}$
d	$Cl\sqrt{\frac{w}{b}}$	$Cl\sqrt{\frac{1.2w}{b}}$	$Cl\sqrt{\frac{1.5w}{b}}$	$Cl\sqrt{\frac{3w}{b}}$	$Cl\sqrt{\frac{6w}{b}}$
w	$\frac{d^2b}{C^2l^2}$	$\frac{d^2b}{1.2C^2l^2}$	$\frac{d^2b}{1.5C^2l^2}$	$\frac{d^2b}{3C^2l^2}$	$\frac{d^2b}{6C^2l^2}$

Formulas to Review a Beam already Designed. To review a beam already designed, the following formulas may be used, the derivation of which is given in Appendix II, page 751.

f_c = unit compressive stress in concrete per square inch.
f_s = unit tensile stress in steel per square inch.
d = depth of beam from compressive surface to center of steel in inches.
j = ratio of distance between the centers of compression and tension to depth of beam.
$jd = d\left(1 - \frac{k}{3}\right)$ = distance between the centers of compression and tension.
b = breadth of beam in inches.
A_s = area of cross-section of steel in square inches.
p = ratio of cross-section of steel to cross-section of beam above center of gravity of steel.
k = ratio of depth of neutral axis to depth of beam d.
M = bending moment in inch pounds.
n = ratio of elasticity of steel to concrete.

Then

$$p = \frac{A_s}{bd} \quad (5) \qquad k = \sqrt{2\,pn + (pn)^2} - pn \quad (6)$$

$$f_s = \frac{M}{A_s jd} \quad (7) \qquad f_c = \frac{2\,M}{bd^2 jk} \quad (8)$$

The values of j and k are dependent upon the percentage of steel and the ratio of moduli of elasticity of steel and concrete and may be taken from table, pages 520 and 521.

For a beam with about 0.8 per cent of horizontal steel (in which case, the tension in steel f_s is about 16 000 pounds per square inch and the compression in concrete f_c about 650 pounds per square inch) the distance between the centers of compression and tension jd, is about $\frac{7}{8}\,d$ and the above formulas may be expressed with scarcely appreciable error as

$$f_s = \frac{M}{0.87\,A_s d} \quad (7a) \qquad f_c = \frac{6\,M}{bd^2} \quad (8a)$$

Neither the allowable tension in steel nor the allowable compression in concrete should be exceeded. Tables for determining the dimensions and loading of rectangular beams are given on pages 509 to 511, and the methods of practical computation and details of design are illustrated in Example 6, page 469. T-beams are treated on page 423.

The selection of bending moments to use in design of continuous beams is treated on p. 439.

DESIGN OF SLABS

A slab, as far as the computation is concerned, is a rectangular beam and the depth and percentage of steel are therefore obtained by the formulas just given.

Since the bending moment is figured for a width of slab equal to one foot, b in formulas (1) and (2) becomes 12 inches and the formulas change (using notation on page 418) to

$$d = 0.29C\,\sqrt{M} \quad (9)$$
$$A_s = 12\,pd \quad (10)$$

For stress recommended by the Joint Committee, 1909

f_c = 650 pounds per square inch, f_s = 16 000 pounds per square inch and n = 15, substituting corresponding value for C from Table 10, page 519, the above formulas become

$$d = 0.028\,\sqrt{M} \quad (11)$$
$$A_s = 0.092d \quad (12)$$

The use of these formulas is illustrated in Example 6, page 469.

Slabs which are continuous over the supports, such as those in a floor or in a buttressed retaining wall, must be designed with provision for the negative moment at the supports. For uniformly loaded spans continuous over 2 or more intermediate supports, a moment $M = \frac{1}{12}\,wl^2$ may be used both in the centers of the spans and also at the supports, while for end spans a moment $M = \frac{1}{10}\,wl^2$ is necessary.

In practice to provide for the moments over the supports some designers bend up all the bars near the $\frac{1}{4}$ point, but a better way, in order to be sure that no point in tension is unprovided with steel, is to bend up one-half, two-thirds or three-quarters of the bars and run them over the supports allowing the remainder to continue at the bottom of the slab. To provide the rest of the steel at the support, the bars in the adjoining span can be carried back over the support. Where the bars are so long as to extend over several spans, they can be arranged to break joints at different places, and thus accomplish this object.

The bend in the bars should be near the $\frac{1}{4}$ points in the span, and usually at an angle of about 30 degrees with the horizontal. Too sharp an angle may tend to crack the slab, while, on the other hand, they must be brought to the top of the slab far enough from the support to properly provide for the negative moment.