The specimen recommended for crushing tests by the Joint Committee on Concrete and Reinforced Concrete, and used at the U. S. Government Laboratories at St. Louis, is a cylinder 8 inches diameter by 16 inches long.

For reinforced concrete beams the Committee recommended 8 by 11 inches by 13 feet long, testing this on a 12-foot span.

Beams for testing the transverse strength of concrete are usually made from 6 to 12 inches square. The smaller size is satisfactory provided the mixture is a fairly wet one so that the corners and surfaces of the molds can be filled. For specimens 6 inches square a convenient length is 6 feet, to be broken on a 60-inch span. The halves of the specimens may be afterwards broken to average with the full beam test or to compare the strength at different periods. Experiments prove that the ultimate fiber stress in the half beams will be practically, as well as theoretically, the same as that in the whole beams.

Specimens for crushing must be faced with some material which will transmit the strain to all points in the surfaces. At the Watertown Arsenal plaster of Paris or neat cement is employed. After spreading the surface with a coat of plaster or cement, a block of polished steel is placed upon it, and it is allowed to set. Before crushing, the surface is tested with a straight-edge, and any irregularities are smoothed off with its sharp edge.

Specimens for Rough Tests. If the quality of sand is questioned and a laboratory is not available, a rough test may be made by mixing up a block of mortar or concrete, using the same aggregates mixed in the same proportion and to the same consistency that is to be employed in the work and examining the specimens from day to day. If kept in a warm room under a moist cloth, the mortar or concrete should harden after 24 hours so as to resist the pressure of the thumb and at the end of a week in the air it should be hard and sound.

Method of Quartering. To obtain an average sample from a pile of sand, gravel, or stone, the method of quartering is useful. Shovelfuls of the material are taken from the various parts of the pile, mixed together and spread in a circle. The circle is quartered, as one would quarter a pie, two of the opposite quarters are shoveled away from the rest, thoroughly mixed, spread, and quartered as before. The operation is repeated until the quantity is reduced to that required for the sample.

CHAPTER XXI

REINFORCED CONCRETE DESIGN

Reinforced concrete is concrete in which steel or other metal is imbedded to increase its strength. Although it has been employed generally as a building material for only a few years, the laws governing the effective combination of concrete and steel are now sufficiently well established to enable the engineer to design a structure with assurance of permanent strength and durability.

Occasional failures have occurred in reinforced concrete construction through neglect of essential principles. The causes have been (1) poor design, particularly in the details which do not occur in steel design; (2) poor materials, especially poor sand; (3) misplacement of reinforcement; and (4) too early removal of forms. These are all readily preventable causes under careful engineering and superintendence. Some of the more important points to guard against are outlined in Chapter II, page 28a.

Until recently there has been considerable divergence in the theory of beam design and of column design. Authoritative reports were brought out in Europe in 1907 and 1908. In America, the Joint Committee on Concrete and Reinforced Concrete presented its first Progress Report early in 1909. This Joint Committee is composed of members selected from the American Society of Civil Engineers, the American Society for Testing Materials, the American Railway Engineering and Maintenance of Way Association, and the Association of American Portland Cement Manufacturers, and therefore represents the highest authority in the United States. Its recommendations have tended to standardize general practice.

In this chapter the recommendations on design of this American Joint Committee have been followed, not only because of their general acceptance as a standard, but because they agree with the views of the authors and represent the most satisfactory rules thus far formulated. This has necessitated no changes in the methods of analysis given in the first edition, since the theory of stress there presented has since been generally adopted.

Results of recent tests have made possible a more complete treatment of the details of design, and extensive study and investigation have led to the addition of simple working formulas and practical recommendations.

In general, only brief discussions together with the rules and principal formulas for design are given in the text, the analytical treatment of each subject being transferred to the Appendix or printed in footnotes for the use of readers interested in the theory.

In the following pages, then, are discussed:

In other parts of the treatise are discussed various special types of reinforced concrete construction and details of design, including:

The notation adopted in the formulas is the Standard Notation as adopted by the Joint Committee . . . 529

GENERAL PRINCIPLES OF REINFORCED BEAMS

A concrete beam, when reinforced with iron or steel rods properly placed, develops a capacity for carrying loads several times greater than its carrying capacity when without reinforcement. It is evident that the location of the reinforcement in the beam must conform to the principles of mechanics so that the concrete shall be strengthened in its weakest part. Hence, since concrete is comparatively weak in its resistance to pull, reinforcing metal should be placed where it will aid the concrete in carrying tension. In a beam or slab the metal should be as near to the surface on the tension side of the beam as is consistent with properly imbedding it and providing a sufficient thickness of concrete to protect it from rust and fire.

Since concrete is a brittle material and steel a comparatively ductile one, it might be expected that the stretching of the tension surface of a beam would result in the formation of cracks on the under surface of the concrete, and that all the pull would be imposed upon the steel. Tests by Prof. Frederick E. Turneaure* and others have shown that cracks in the concrete are actually produced by the tension and that the tension load is thus transferred to the metal. However, while these cracks reduce the strength of the concrete, they are so minute, being at first invisible to the naked eye, and so distributed over the section, that the reinforcing metal, as shown by tests, is protected by the concrete from corrosion even up to the point of the elastic limit of the steel.†

Not only must the steel be correctly located, but it is essential to have the proper quantity of metal in the beam. It is obvious that if the cross-section of the metal is too large as compared with the area of the concrete in compression, the beam, in case of failure, will give way by compression in the concrete, whereas, if the area of the metal is too small, weakness will show itself as soon as the metal has reached its yield point, which is usually not far from one-half the actual breaking strength of the steel. The area of the reinforcing metal in rectangular beams and slabs is apt to vary according to the conditions from about ½% to 1½% of the area of the cross-section of the reinforced beam above the steel. For example, a beam 10 inches wide and 11 inches deep with steel one inch above its bottom surface (100 square inches net area) requires, according to circumstances, from ½ square inch to 1½ square inches section of steel. In any given design this area of reinforcement should be determined from the character of the member and the strength and elasticity of the concrete and the steel. More than 1% of steel is not usually economical in a rectangular beam unless the concrete is allowed to be stressed beyond the high pressure of 750 pounds per square inch.

In designing a beam composed of concrete with steel imbedded in it, the bending moment produced by the superimposed load,—which is termed the live load,—plus the weight of the beam itself, the dead load, must be no greater than the moment of resistance of the beam (*i.e.*, the moment of the internal resisting forces of the strength of the concrete and steel) divided by a proper factor of safety.

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* Proceedings American Society for Testing Materials, 1904.

† See page 410.

That which differentiates the study of a reinforced concrete beam from that of a beam composed of a single homogeneous material is the determination of the pull, which is borne by the steel alone, and of the compression, sustained entirely by the concrete. The problem is rendered the more complex because the strength and elasticity of concrete vary through a wide range according to the nature of its ingredients and their proportions. Current practice, borne out by experiments made at various American universities, indicates that beams may be designed on the assumption that the concrete in the upper part of the beam resists all the compression and the steel in the bottom of the beam takes all of the pull. This is always on the safe side, since the concrete assists the steel in tension to a slight degree. The theories of the distribution of the stresses in reinforced concrete, which are based on the elasticity of the concrete and the steel, are sufficiently accurate for the practical purposes of design. Before giving formulas and tables to be used in the design of reinforced beams, the principles governing the assumption of the distribution of stresses and the properties of the materials will be considered.

A Plane Section Before and After Bending. While experiments at the Massachusetts Institute of Technology indicate that the law of plane sections before and after loading does not apply exactly to reinforced concrete beams, nevertheless, it is sufficiently accurate for practical purposes to assume it correct, viz: that if a plane section is taken through a beam before loading, after loading, this section, even though inclined to its original position by the bending due to the load, remains a plane section. From this it follows, as in the common theory of beams, that the stretching or shortening per unit of length of any fiber which cuts the section considered may be assumed as proportional to the distance of this fiber from the neutral axis of the section.

MODULUS OF ELASTICITY OF STEEL

The modulus of elasticity of steel varies from 28 000 000 pounds per square inch to 31 000 000 pounds per square inch; 30 000 000 is customarily taken as an average value, and is the value adopted in this treatise.

All Steel, irrespective of its Ultimate Strength, Elastic Limit or Chemical Composition, has Substantially the Same Modulus of Elasticity. It follows therefore from the principles of elasticity that the stretch under a given pull is independent of the character of the steel.

MODULUS OF ELASTICITY OF CONCRETE

The modulus of elasticity is an important item in reinforced concrete design and is discussed at length in the pages which follow. **For practical design it is recommended that the ratio of the modulus of elasticity of steel to that of concrete be taken at 15,** corresponding to a concrete modulus of 2 000 000.

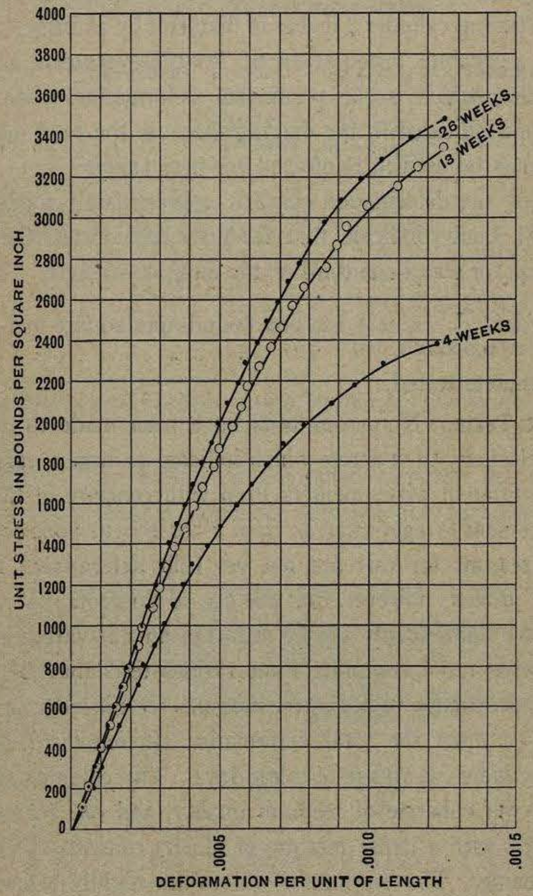

FIG. 129. Stress Deformation Diagram, Limestone Concrete Cylinders of Medium Consistency and Extra Good Quality.* (*See p.* 404).

Determination of Modulus of Elasticity. The modulus of elasticity, *E*, may be taken as the quotient of the stress per unit of area divided by the deformation (that is, the elongation or the shortening) in a unit length. In

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* Bulletin No. 344, U. S. Geological Survey, p. 33.

customary English units where the modulus is in pounds per square inch,

$$E = \frac{\text{stress per square inch}}{\text{deformation per linear inch}}$$

It is determined in the laboratory by measuring the deformation for the loads successively applied and plotting them as shown in Fig. 129. The curves in the diagram represent the deformations, at different stages of the loading, for a typical cylinder 8 inches in diameter by 16 inches high of extra strong 1 : 2 : 4 concrete, tested at the St. Louis Government Laboratory in 1907. The set, which is the permanent deformation when the load is released, is not indicated in the diagram because the total deformation is that which must be used in reinforced concrete analysis.

The form of the deformation curve is approximately a parabola,* but the tests at St. Louis† indicate that for first-class concrete the modulus is nearly constant for about one-third of the ultimate strength. The modulus at this point is $\frac{800}{0.00025}$, or 3 200 000 pounds per square inch, in the four weeks old concrete tested.

Results of Tests. Numerous tests have been made to determine the modulus of elasticity of concrete which indicate as large a range in results obtained by different experimenters, even with concrete of the same proportions of cement to aggregate, as from 1 500 000 to 5 000 000 per square inch. The reasons for this are not yet fully determined; it has been conclusively proved, however, that the age of concrete, its richness and its density have undoubtedly a large influence on this variation.

The following table, compiled from various tests, may be of value as suggesting approximate values of the modulus for different proportions of concrete based upon the total deformation at one-third the crushing strength of cylinders at an age of thirty days. Two columns are given, one for ordinary wet concrete of medium quality, and one for concrete very carefully made with a dense mixture of mushy consistency and kept wet during hardening. The "ordinary" values are slightly below those which should be expected in practice on construction work.

The modulus of elasticity of concrete probably bears a definite relation to its ultimate strength, but the factors which enter into this relation probably will never be determined exactly. Plotting the results of a large number of tests made at the Watertown Arsenal, at the Government Laboratory at St. Louis, and at many of the colleges, indicates an approximate ratio of 1300 between the modulus of elasticity and the ultimate strength.

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* See discussion by Prof. Talbot in University of Illinois Bulletin, No. 10, Feb. 1, 1907, p. 21.

† Bulletin No. 344, U. S. Geological Survey, pp. 36–53.

Kimball's Tests. The moduli at different loads from tests of Mr. George A. Kimball made at the Watertown Arsenal upon 12-inch cubes are given in table below. The moduli are computed with the set deducted from the deformation, so that the values are slightly higher than would be obtained from total deformation.

Moduli of Elasticity of Concrete of Different Proportions. Approximate Average Values. (See p. 404.)

	PROPORTIONS.	ORDINARY WET CONCRETE.		EXCEPTIONALLY STRONG CONCRETE.	
		Crushing Strength at 30 days. lb. per sq. in.	Modulus of Elasticity lb. per sq. in.	Crushing Strength at 30 days. lb. per sq. in.	Modulus of Elasticity lb. per sq. in.
Broken stone or gravel concrete	1 : 1½ : 3	2300	2 500 000	2800	3 600 000
	1 : 2 : 4	1700	2 000 000	2500	3 200 000
	1 : 2½ : 5	1500	1 800 000	2200	2 800 000
	1 : 3 : 6	1300	1 600 000	1900	2 500 000
	1 : 4 : 8	900	1 300 000	1500	2 000 000
	1 : 2 : 5	700	900 000	1000	1 300 000

NOTE—A modulus of 2 000 000, corresponding to a ratio of 15, is recommended for general use.

Elastic Properties of Broken Stone Concrete 12-inch Cubes.

Portland cement,* bank sand and broken conglomerate stone.

BY GEORGE A. KIMBALL at Watertown Arsenal. (*See p.* 405.)

COMPOSITION			Age	MODULUS OF ELASTICITY BETWEEN LOADS PER SQUARE INCH OF			Compressive strength per sq. in. lb.
Cement	Sand	Broken Stone		100 and 600 lb.	100 and 1 000 lb.	1 000 and 2 000 lb.	
1	2	4	7 days	2 593 000	2 054 000	1 351 000	1 730
1	2	4	1 mo.	2 662 000	2 445 000	1 462 000	2 567
1	2	4	3 mos.	3 671 000	3 170 000	2 158 000	2 975
1	2	4	6 mos.	3 646 000	3 567 000	2 582 000	3 989
1							
1	3	6	7 days	1 869 000	1 530 000		1 511
1	3	6	1 mo.	2 438 000	2 135 000	1 219 000	2 260
1	3	6	3 mos.	2 976 000	2 656 000	1 805 000	2 741
1	3	6	6 mos.	3 608 000	3 503 000	1 868 000	3 068
1							
1	6	12	1 mo.	1 376 000			1 146
1	6	12	3 mos.	1 642 000	1 364 000		1 359
1	6	12	6 mos.	1 820 000	1 522 000		1 592

Various other tests of modulus of elasticity may be found in Tests of Metals, U. S. A., during the years 1898 to 1907.

Tests of Mortar Prisms. Elastic properties of prisms of neat Portland cement and cement mortar, from tests made by Mr. Howard* at the Watertown Arsenal, are presented in the following table:

Elastic Properties of Cement and Mortar Prisms 6 by 6 by 18 inches.

Watertown Arsenal. (*See p.* 406.)

Brand of Cement	COMPOSITION		Age	MODULUS OF ELASTICITY BETWEEN LOADS PER SQUARE INCH OF			Permanent sets after loads per square inch of			Compressive strength per square inch.
	Cement	Sand	Days	100 and 600 lb.	100 and 1 000 lb.	1 000 and 2 000 lb.	600 Inch	1 000 Inch	2 000 Inch	lb.
Alpha	Neat	0	7	7 143 000	5 000 000	8 333 000	0.	0.	0.	4 783
			7	4 167 000	3 600 000	3 448 000	0.	0.	.0002	5 000
Alpha	1	1	15	3 125 000	2 812 000	2 326 000	−.0002	−.0002	.0007	3 846
			36	2 381 000	2 500 000	2 941 000	0.	.0002	.0012	4 763
			36	2 632 000	2 727 000	3 030 000	.0001	.0002	.0010	4 948
Alpha	1	2	15	1 724 000	1 475 000		.0005	.0023		1 376
			36	2 273 000	2 195 000	1 538 000	.0001	.0006	.0040	2 184
			38	2 778 000	2 812 000	2 325 000	0.	.0004	.0020	2 755

Gaged length, 10 inches.

Modulus of Elasticity in Beams vs. Columns. The modulus of elasticity in beams as determined by measurements and computations by Professor Talbot is approximately the same or possibly slightly lower than in columns.

Effect of Consistency of Concrete upon the Modulus of Elasticity. An excess of water in the concrete not only decreases the strength (see page 382), but also affects the deformation curve so as to show a more variable modulus near the beginning of the test. The moduli of concrete of different consistencies and at different ages are shown in the tables from tests of the authors on following page,

Relation of Stress Deformation Curve to the Theory of Beams. The theory of beams is worked out under the assumption that a section plane before bending remains plane after bending so that the deformation or stretch at any point in the compressive portion of the beam is proportional to the distance of this point from the neutral axis. According to this assumption the distribution of stresses is also proportional to the distance from the neutral axis so long as the moment of elasticity is constant. This distribution may be then represented by a straight line as shown in Fig. 131, p. 417. When, however, the modulus of elasticity changes Hook's law—that stress is proportional to deformation—is no longer applicable, since the intensity of stress is no longer proportional to the distance from the neutral axis but changes according to the relation of the moduli of elasticity at different loadings, and the line representing the distribution becomes a curve.*

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*Tests of Metals, U. S. A., 1898.

Modulus of Elasticity of Concrete of Different Consistencies.† *Proportions by Volume* $1, : 2\frac{1}{4} : 4\frac{1}{2}$

BY TAYLOR AND THOMPSON. (*See p.* 406.)

Approximate age in months.	DRY.		MEDIUM.		VERY WET.	
	Compressive strength. Pounds per sq. in.	Modulus at $\frac{1}{3}$ ultimate strength. Pounds per sq. in.	Compressive strength. Pounds per sq. in.	Modulus at $\frac{1}{3}$ ultimate strength. Pounds per sq. in	Compressive strength. Pounds per sq. in.	Modulus at $\frac{1}{3}$ ultimate strength. Pounds per sq. in.
1	4370	4 050 000	3360	4 500 000	2110	2 100 000
2	5430	4 050 000	3940	4 550 000	2770	3 400 000
6	5170	5 255 000	5170	3 760 000	3350	2 880 000
17	5510	3 920 000	4720	3 750 000	2430	2 080 000

Since the modulus is nearly constant within the working limits the authors have adopted the straight line theory of distribution of stress as simplest and most practical.‡

Formerly the parabolic distribution of pressure in concrete above the neutral axis was used in preference to the straight line theory because it corresponds somewhat more nearly to actual test. The two theories, however, require practically identical percentages of steel and the only difference is in the determination of the unit stress in the concrete. When using the parabola theory, about 15% lower compressive stress in the concrete must be used than when figuring by the straight line theory to obtain similar results. For example, 650 pounds per square inch safe compression by the straight line theory corresponds to about 565 pounds per square inch by the parabola theory.

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565×1,15 = 650

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*A comprehensive analytical discussion of the effect of a varying modulus of elasticity upon the pressure in a beam under different loadings is presented by Prof. Talbot in Journal Western Society of Engineers, Aug. 1904.

†"The Consistency of Concrete," by Sanford E. Thompson, American Society for Testing Materials. Vol. VI, 1906.

‡It is also recommended by the Joint Committee, 1909.

Value to Use for the Ratio of Elasticity in Compression. For beam and slab design and also for column design, tests indicate that a practical value of 15 for the ratio of the moduli of steel to concrete corresponding to a concrete modulus, E_c = 2 000 000, best satisfies the conditions for ordinary 1 : 2 : 4 concrete, and without serious error may be used for all classes of concrete, and is therefore recommended for general use.* For calculations relative to deflections where the tensile strength of the concrete, taken into account, a value of 8 to 12 may be used as should properly be giving results corresponding more nearly to actual conditions. The value of 15 has been adopted in the American, British, German and Austrian rules up to 1909. The French rules for 1907 authorize a range from 8 to 15, according to conditions.

A lower modulus of elasticity for concrete (that is, a higher ratio) should be used in determining the location of the neutral axis in beam design than the values obtained at working loads in compression tests, to compensate for the neglect, in the ordinary formulas, of the effect of tension in the concrete. The use of a high ratio is generally on the safe side also, since it lowers the apparent location of the neutral axis and increases the amount of steel required. These reasons explain the selection of a ratio of 15, which is a higher value than is obtained in compression tests. On the other hand, when the modulus is to be used to determine the deflection of a beam, a lower ratio (i. e., a higher modulus) should be used to make up for the omission of the tensile stress unless this is allowed for in the formulas.

In column design, while the use of a low ratio is most conservative, a high ratio (i. e., a low modulus) corresponds more nearly to actual conditions, because if there is a weak spot in the column or unusual loading, the steel will be brought into action to an amount indicated by the lower modulus.

The ratio of modulus of elasticity within working limits in beams figured by the parabola and by the straight line methods is indicated by Prof. Talbot's studies† to be in the ratio of about 13 to 12.

Modulus of Elasticity in Tension. But few tensile tests of concrete have been made, but these indicate‡ that the elastic modulus in tension is probably the same as the modulus in compression.

ELONGATION OR STRETCH IN CONCRETE

According to tests of Professor Turneaure, already mentioned, reinforced concrete under a pull, as in the lower portion of a beam, will usually stretch

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* It is thus recommended by the Joint Committee, 1909.

† University of Illinois, Bulletin No. 4, April 18, 1906.

‡ Prof. W. Kendrick Hatt, Journal Association Engineering Societies, June 1904, p. 32.

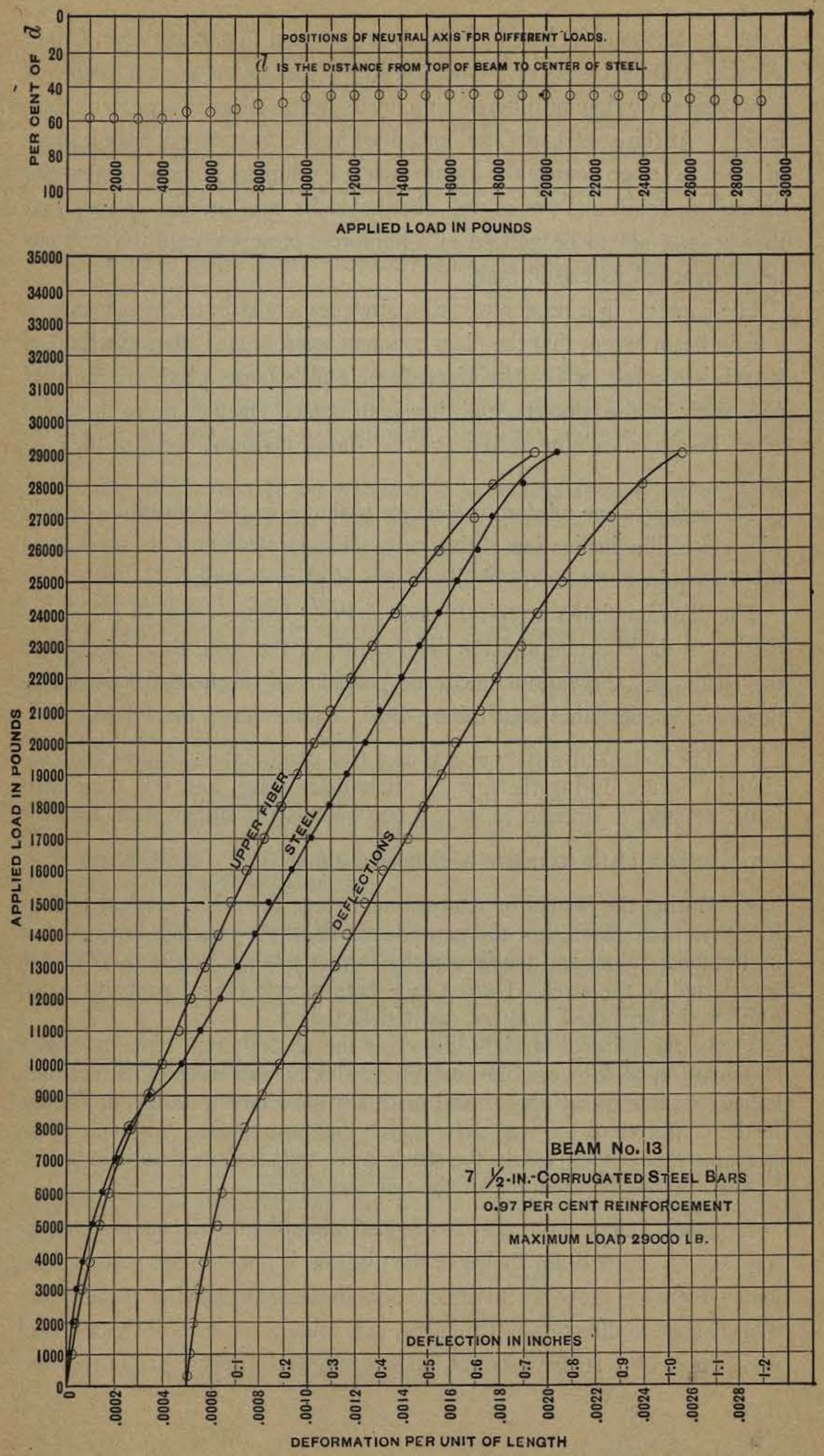

Fig. 130. Typical Deformation and Deflection Curves of a Reinforced Beam By Prof. A. N. Talbot. (*See p.* 410.)