is greater with rich than with lean concrete, but on the other hand, tests of specimens made at the Watertown Arsenal indicate the reverse. The difference is slight in both cases, however, and it may be assumed for practical purposes that the rate of growth is approximately the same whatever the proportions. A wet consistency of the concrete produces lower strength, especially at early periods, and a larger percentage of growth than is indicated in the diagram. (See page 383.)

The curve does not apply to concretes of Natural cement mortar. 12-inch cubes of concrete in various proportions made from Akron Star cement tested at the Watertown Arsenal for William Wirt Clarke & Son* show an average ratio of increase in strength between one month and one year of 1.96. With this series of specimens the average strength at the age of one year was no greater than at seven months, but this is probably an exceptional case, since, for instance, tests by Capt. William M. Black on Natural cement concrete show a slower and continual growth, with an equally large ultimate strength.

TRANSVERSE STRENGTH OF CONCRETE

The strength of a beam of plain concrete is limited by the tensile strength of the concrete at the place of greatest strain, which, with vertical loading, is its lowest surface. The value of this transverse "fiber" strength or modulus of rupture is of less importance than the crushing strength, because, on account of the brittleness of concrete in tension, that is, its liability to crack from shrinkage or sudden loading, it is seldom safe, and usually is not economical, to construct beams or girders without metal reinforcement. Most formulas for reinforced design disregard the tensile strength of the concrete. In certain computations, however, the tensile strength must be considered. Since concrete beams can be broken with less powerful and less expensive apparatus than crushing specimens, this form of specimen is often convenient for comparing the relative strength of different mixtures or different materials, and while the ratios thus obtained will not exactly coincide with those for crushing strength, they will be sufficiently close for many purposes.

Fuller's Beam Tests. The table† on page 376 gives the results of a comprehensive series of tests of 6 by 6 by 72-inch beams made by Mr. William B. Fuller at Little Falls, N. J. Although different materials than those used by Mr. Fuller will of course show slightly different strength, the table is sufficiently representative of average conditions to permit its use for comparisons of different proportions, and, with a proper

*Tests of Metals, U. S. A., 1901, p. 609.

†Especially prepared for this treatise by Mr. Fuller.

factor of safety, as a working guide to the safe transverse strength of concrete.

The proportions are given by weight but can be transformed to volume measure by referring to the footnote. The various columns present valuable data on weights and volumes and voids.

The curves in Fig. 123 are plotted from the results in the table, and illustrate also the proportions corresponding to maximum strength for a given per cent. of cement.

Tests by other authorities are mentioned under Strength of Beams in References, Chapter XXXI.

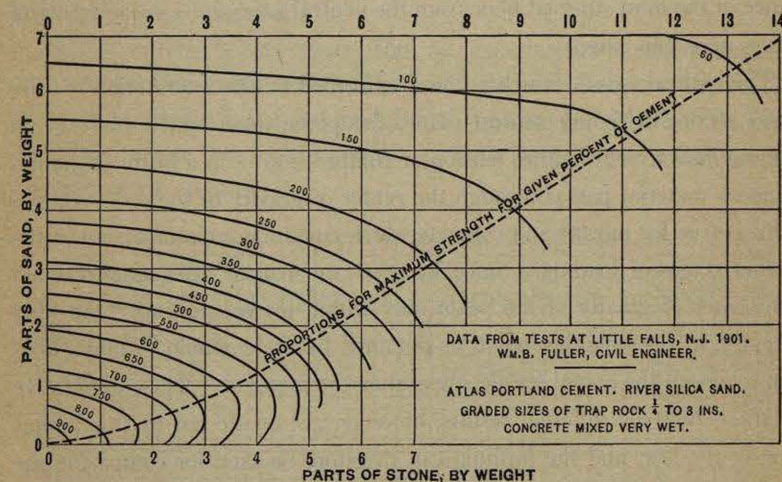

FIG. 123. Curves showing strength of beams in pounds per square inch for various proportions by weight of sand and stone to one part Portland cement. Age 34 days

Formula for Transverse or Bending Stress in Plain Concrete. The common formulas for representing the longitudinal forces of compression and tension upon a beam are usually expressed with the following notation:

Let

f = intensity of stress at any point in the beam.

M = bending moment.

I = moment of inertia about its neutral axis of section containing the point under consideration.

y = distance of the point from the neutral axis.

b = breadth of beam.

h = height of beam.

Then $f = \frac{My}{I}$ (5) also, $M = \frac{fI}{y}$ (6)

For rectangular sections, $I = \frac{bh^3}{12}$ and up to the elastic limit for beams of homogeneous material (but not for reinforced beams), $y = \frac{1}{2} h$.

Hence for rectangular beams of homogeneous material,

$$f = \frac{6M}{bh^2} \qquad (7) \qquad \text{also, } M = \frac{1}{6} f b h^2 \qquad (8)$$

In considering the strength of a beam, since the stress is greatest at one or the other of the surfaces, y is generally understood to represent the distance of the most strained fiber from the neutral axis, and f the intensity of stress upon this fiber.

The neutral axis — which is the line formed by the intersection of any cross section with the neutral plane, the plane upon which there is no longitudinal stress of either tension or compression — in a beam of homogeneous material passes through the center of gravity of the cross section. This is true for mortar and concrete which contain no reinforcement in the earlier stages of loading. Since, however, the neutral axis passes through the center of gravity of the beam only within the elastic limit,* the fiber stress, f, at the breaking point, as obtained by the common formula, does not represent the actual tensile stress upon the material. The comparative relations between different results, however, are unaffected by this limitation of the law, and the formula can therefore be used for comparing the strength of beams composed of similar material. For example, while the stresses at the instant of breaking, that is, the moduli of rupture, as figured by the formula, are not strictly correct either for 8 or 10 inch beams, they are nearly *proportional* to the actual stresses, so that the strength of plain concrete beams of different dimensions may be compared by means of the formula without appreciable error.

For convenience in designing, a table is given in Chapter XXI for bending moments caused by uniformly distributed loads and for loads concentrated at different points. Also, in the same chapter, the moments of inertia, I, for various sections are tabulated. These tables are applicable for the most part to both plain and reinforced beams.

* Although concrete and mortar have no true elastic limit the general principles apply to beams of these materials.

Relation of Transverse to Compressive Strength of Concrete. There is no fixed relation between the tensile fiber stress of concrete beams and the crushing strength of specimens made from the same material under identical conditions. The growth of strength is different in the two classes of tests, and although the general laws of increase in strength due to increasing the percentage of cement and the density appear to hold in both cases, the authors' formula given on page 356 for compressive strength is not applicable to transverse tests.

Experiments by the authors comparing 8-inch cubes and 8-inch beams of 1 : 2½ : 5 concrete give a ratio of crushing strength to modulus of rupture at one and two months of 6 : 1.

Mr. A. Fairlie Bruce† states from his experiments on the strength of concrete bars and arch ribs that he found the ratio between the crushing strength of the arch and the modulus of rupture of the bars to be about 6 : 1 for concrete two to four weeks old, then increasing to about 10 : 1 at the age of six months.

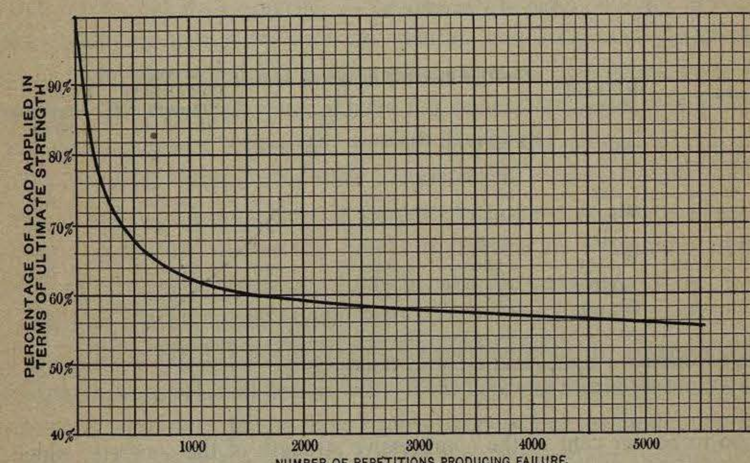

FIG. 124.—Fatigue of Neat Cement under Compression. (*See p.* 381.)

THE FATIGUE OF CEMENT

The action of cement under repeated stresses has been slightly investigated by Prof. J. L. Van Ornum* at Washington University. The experiments were made upon 2-inch neat Portland cement cubes four weeks old. The results of tests on 92 blocks are shown in the diagram in Fig. 124. The effect upon concrete of repeated applications of a load is discussed in the following chapter.

† *Engineering Record*, Oct. 31, 1903, p. 533.
* Transactions American Society of Civil Engineers, Vol. LI, p. 443.

STRENGTH OF CONCRETE IN SHEAR

The actual strength of concrete* in direct shear is much greater than was formerly supposed because in many of the earlier tests this was confused with diagonal tension which, as indicated in the following chapter, may be dangerous in a beam even when the vertical shear is small. Owing to the difficulty in eliminating in experiments the effect of bearing action, diagonal tension and beam stresses in general, it is not easy to devise a form of test specimen and a manner of testing which will determine satisfactorily the resistance of concrete to direct shear. In tests made at the Massachusetts Institute of Technology under the direction of Prof. Charles M. Spofford in 1904 and 1905, the final failure of the specimens appeared to be by true shear. These tests gave a shearing strength ranging in general

Shearing Strength of Concrete

By Prof. Charles M. Spofford.

Massachusetts Institute of Technology. (See p. 382)

Age of Concrete 24 to 32 days.

Mixture.	Method of Storing.	Shearing Strength lb. per sq. inch.			Average Compressive Strength in lb. per sq. inch.	Ratio of Compression to Shear.
		Maximum.	Minimum.	Average.		
1 : 2 : 4	Air	1630	960	1310	2070	0.63
1 : 2 : 4	Water	2090	1180	1650	2620	0.63
1 : 3 : 5	Air	1590	890	1240	1310	0.95
1 : 3 : 5	Water	1380	840	1120	1360	0.82
1 : 3 : 6	Air	1450	950	1180	950	1.24
1 : 3 : 6	Water	1200	1040	1120	1270	0.88
Average Ratio for 1 : 2 : 4 and 1 : 3 : 5 Concrete						0.76

from 60 to 80 per cent of the compressive strength of the concrete, which agrees substantially with experiments made by Prof. Arthur N. Talbot† in 1906.

This direct shear must not be confounded with shear in a beam involving diagonal tension where the concrete may break with a shearing stress 10% of the crushing strength.

At the Institute three grades of concrete were used, and the specimens were stored both in air and water. The test specimens were cylinders 5 inches in diameter by 18 inches long, and in testing, the end thirds of the

* Shearing tests of mortar, by Mr. Feret, are recorded on page 136.
† University of Illinois, Bulletin No. 8, 1906.

cylinders were held rigidly by cast iron yokes, the pressure being applied through a cast iron half cylinder bearing, fitting between the two yokes, so as to shear the concrete across two planes. To compare the compressive strength of the concrete with the shearing strength, six extra cylinders of the same dimensions were crushed. The following table gives the relation between the shearing and crushing tests.

From the experiments made at the University of Illinois, referred to, the conclusion was drawn that the resistance to shear is dependent upon the strength of the stone as well as upon the strength of the mortar, and for the richer mixture the strength of the stone probably exerts the greater influence.

EFFECT OF THE CONSISTENCY UPON THE STRENGTH

The general result of experiments and practice tends to show that the strongest concrete can be secured with a mixture containing only sufficient water to produce a film of mortar upon the surface after very hard ramming in thin layers, but with a wetter "quaking" mixture the ultimate strength will be nearly as high as with the dry mixture, and because of the greater ease in laying and obtaining a homogeneous mass, it is generally to be preferred. An excess of water injures the cement by decomposing parts of it before it has had opportunity to set. The actual strength of concrete is often of less importance than other considerations. If, as in many classes of structures, there is an excess of strength, cheapness in placing, the appearance of the surface, or the proper imbedding of reinforcing metal, may be of primary importance. In such cases the quantity of water must be suited to the attendant conditions.

The curves in Fig. 125 are plotted from experiments by the authors* upon the strength, density†, and permeability of the concrete mixed with different percentages of water. In the three curves the points of maximum density, strength and water-tightness all lie not far from the medium quaking consistency, although for maximum water-tightness a still softer consistency appears to be slightly more efficient.

These tests further indicate that (1) the consistency which will produce the densest concrete will result in the greatest ultimate strength provided an excess of water is not employed; (2) dry mixtures attain highest strength at short periods, but mixtures of quaking consistency approach the dryer specimens after longer setting; (3) very wet mixtures, especially of lean proportions, may be chemically injured, by the excess of water.

* Proceedings of American Society for Testing Materials, Vol. VI, 1906, p. 358.
† See p. 1 for definition and p. 138 for method of determining density.

Effect of "Laitance." Whenever concrete is laid under water, the water is likely to be clouded by what appear to be particles of cement floating up from the mass which is being laid. This whitish substance is generally termed "laitance." A similar formation occurs on the surface

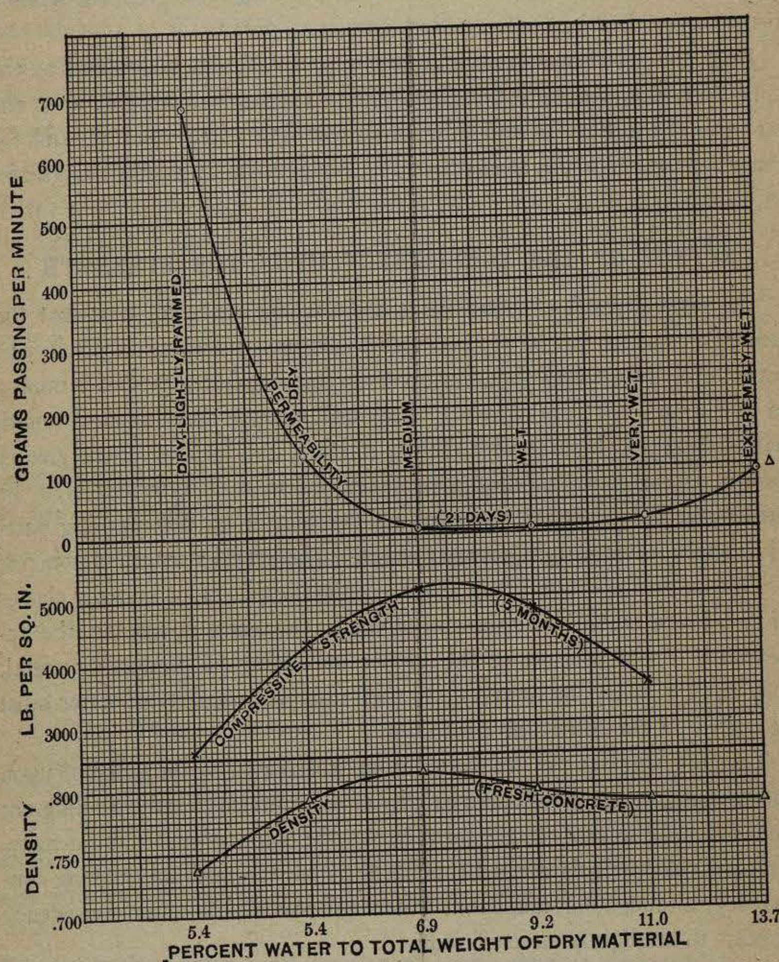

FIG. 125.—Comparative Permeability, Strength and Density of 1 : 2¼ : 1½ Concrete, mixed with Different Percentages of Water, By Taylor and Thompson. (*See p.* 383.)

of concrete laid with a large excess of water. In certain cases, we have found as much as ⅛ inch rising from a layer of 1 : 2½ : 5 concrete less than five inches thick.

Chemical and microscopical analyses, which Mr. Clifford Richardson has very kindly made for us, show that this laitance has nearly the same chemical composition,* except for a large loss on ignition, as normal Portland cements, but consists largely of amorphous material of an isotropic nature,—that is to say, it does not affect polarized light, and has almost no setting properties.

It is evident, therefore, that when concrete or mortar is laid under water, or with a large excess of water, a portion of the cement is rendered incapable of setting, and the strength of the mass is consequently reduced in proportion to this loss. The conclusion is naturally reached that for concrete laid under water, or in locations where a large excess of water is required in mixing, a higher percentage of cement than usual, about one-sixth more, should be employed.

A lean mixture has been found to be more seriously injured by an excess of water than a rich one, probably because the water has a greater opportunity to penetrate the mass, and therefore to dissolve the cement.

GRAVEL VS. BROKEN STONE CONCRETE

Comparative tests of broken stone and gravel concretes, in the same proportions by volume, show almost invariably that concrete made from hard broken stone, such as trap, or hard limestone, gives higher compressive strength than concrete made from gravel. This appears to be the rule not only when the materials are mixed by measured volumes, regardless of the percentages of voids, but also when the broken stone and gravel are each screened to substantially the same sizes.

The relative values of gravel and broken stone concrete in the table which follows are based on the comprehensive series of a comparative test made by Mr. Candlot in France and tabulated on page 367.

Comparative Strength of Broken Stone and Gravel Concrete.

From Candlot's Experiments

Age.	Ratio of strength of broken stone concrete to gravel concrete. With equal voids	Broken stone 47.4% voids. Gravel, 40% voids.
7 days	1.30	1.33
1 month	1.26	1.19
6 "	1.18	1.20
1 year	1.12	1.09

Each ratio gives the extra strength of broken stone over gravel concrete of similar age. For example, if a concrete containing gravel having

* See page 302.

40 % voids tests 2 000 lb. per sq. inch at the age of six months, a concrete in similar proportions by volume containing broken stone with 47.4% voids should, according to Candlot's experiments, test 1.20 times greater or 2 400 lb. per sq. inch.

The last column is averaged directly from Candlot's table, and may be taken as applicable to average conditions. It is noticeable that the gravel concrete approaches the broken stone concrete as its age increases. Since in many cases the ultimate strength of concrete is determined by the strength of its coarse aggregate, it follows that at, say, the age of a few months a gravel concrete may reach or surpass the strength of a broken stone concrete having a coarse aggregate of soft stone of low strength.

Although the claim is frequently made that gravel concrete is stronger than broken stone concrete, the authors have failed to find substantial proof of this. On the other hand, various records, among them a number of tests at the Watertown Arsenal,* as well as the tests tabulated on page 388, tend to show the probable accuracy of Candlot's tests.

Another argument in favor of broken stone concrete lies in the fact that gravel is often covered with a film of dirt, difficult to remove, which lowers the strength. In experiments for the East Boston Tunnel† by Mr. Howard A. Carson, Chief Engineer, concrete beams made with washed gravel were about one-third stronger than beams made with gravel coated with a thin film of dirt.

Advocates of gravel concrete, among them Mr. R. Feret,‡ assert that as the rounded stones slip more readily into place, it is easier to make with them a compact mass. Loose rounded stones also contain a smaller percentage of voids than angular, but this is at least partly offset by the fact shown by the experiments of the authors, tabulated on page 171, that broken stone compresses more on ramming.

Although the weight of evidence apparently favors broken stone concrete, it by no means follows that broken stone always should be used to the exclusion of gravel. In many instances, the ultimate strength of the concrete is of minor importance because the proportions of the concrete are determined by other considerations. Often, where strength is the criterion, but gravel is cheaper than broken stone, an additional percentage of cement may be economical. Moreover, the ultimate strength of gravel concrete is undoubtedly greater than that of concrete made with a poor quality of broken stone. With fixed proportions, as discussed on page 15,

* Tests of Metals, U. S. A., 1898, pp. 649 to 654.
† Boston Transit Commission, 7th Annual Report, 1901, p. 39.
‡ Chimie Appliquée, p. 533.

gravel is cheaper for the contractor than broken stone, because a given loose volume makes a larger quantity of concrete.

As indicated on page 388, in mixtures of like proportions by volume, the gravel concrete will have less cement in a cubic yard of concrete than a broken stone concrete unless the stone is well graded. Under ordinary conditions to attain concretes of nearly equal strength, with gravel and with broken stone, the sand should be proportioned in each according to the volume and dimensions of the voids in the stone,* and the quantity of cement per unit volume of compacted concrete should be the same in each. The gravel concrete thus will be apt to be the denser, and this will tend to overcome the slight difference in strength due to the varying character of the surfaces of the particles of the gravel and broken stone.

Sometimes it is advantageous to mix a small percentage of gravel with broken stone.

In comprehensive tests at the U. S. Government Laboratories, St. Louis,† upon concrete beams, cylinders and cubes of different aggregates, a granite concrete was about 10 per cent stronger than a gravel concrete made of exceptionally clean hard gravel pebbles, while the gravel concrete showed a strength about 10 per cent greater than that attained by a limestone concrete.

Tests made by Messrs. William B. Fuller and Sanford E. Thompson‡ at Jerome Park Reservoir, New York City, in 1905, upon the density and strength of concrete with different aggregates are illustrated in the curves in Fig. 126.§ Because of the greater density, the proportions by volume being the same, the specimens made with gravel and sand contained, in the set concrete, a slightly larger percentage of cement, so that the strength of the gravel concrete is slightly higher than if allowance had been made for this. The relatively low strength of the concrete with broken stone and screenings may be due in part to the character of the screenings, since tests by other experimenters have sometimes given exceptionally high strength when screenings were used.

The following conclusion was drawn with reference to the relative strength of broken stone and gravel concrete.

A concrete with an angular coarse aggregate, such as broken stone, is stronger than one with a rounded coarse aggregate, like gravel, and the

* This can be better accomplished by trial mixtures, thoroughly compacted, of the dry aggregate, or, still better, of small batches of concrete, than by water measurements of the voids. The proportions of the aggregates giving the smallest bulk of concrete to a given weight of the mixture of aggregates will be the best. Also, see Chapter XI on Proportioning.
† U. S. Geological Survey, Bulletin No. 344, 1908.
‡ Transactions American Society of Civil Engineers, Vol. LIX, p. 67, 1907
§ *Engineering News*, May 30, 1907, p. 599.

same sand and cement—although the rounded aggregate produces greater density—thus indicating a stronger adhesion of cement to broken stone than to gravel. However, if the sand is also angular, like screenings, but

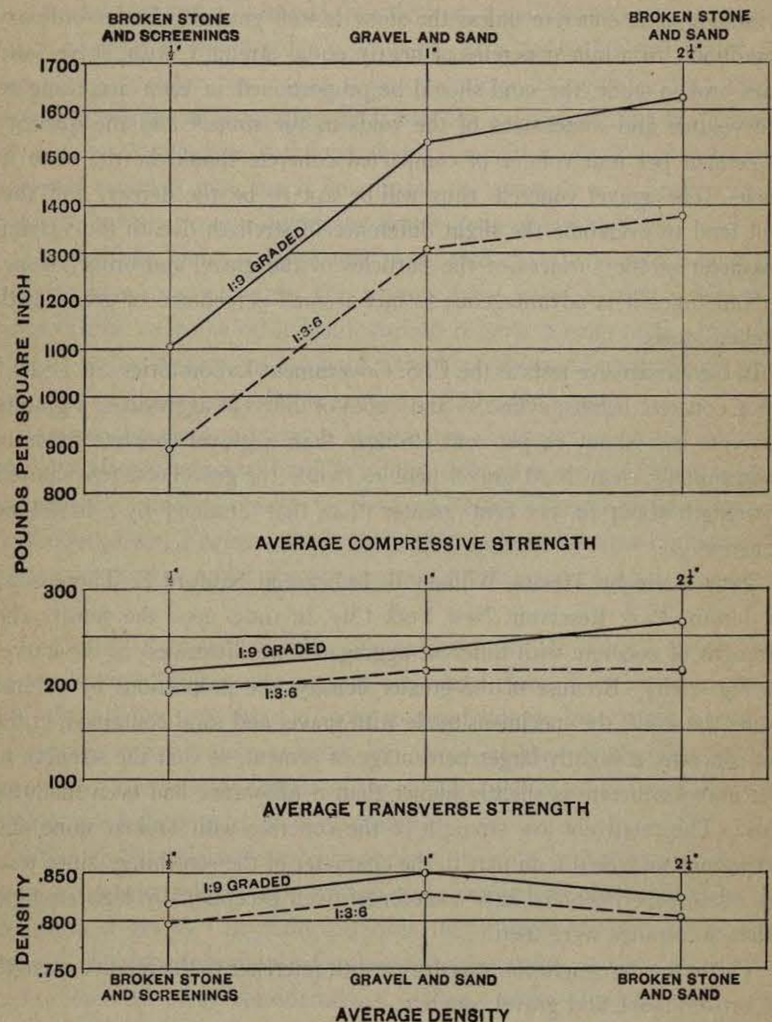

FIG. 126.—Comparative Density and Strength of Concrete made with Different Aggregates. By Fuller and Thompson. (*See p.* 387)

with its grains of the same sizes as the sand, the concrete with rounded coarse and fine aggregate is the stronger, probably because of its greater density.

EFFECT OF THE SIZE OF STONE OR GRAVEL UPON THE STRENGTH OF CONCRETE

The dimensions of the largest particles of stone and gravel which may be used in a concrete are often limited by practical considerations of mixing and placing. For ordinary work it is often specified that the stone shall pass through a 2-inch, or, more often, through a $2\frac{1}{2}$-inch ring. For ordinary mass concrete of wet consistency the limit may be placed as high as 3

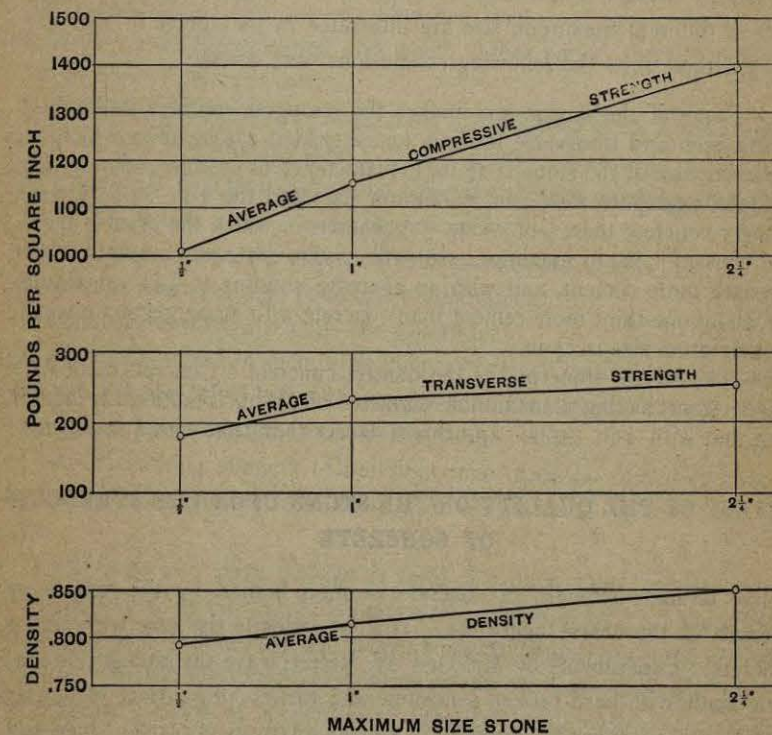

FIG. 127.—Comparative Density and Strength of Concrete made from Broken Stone of Different Maximum Sizes. Proportions 1:3:6. Age, 140 Days. (*See p.* 390.)

inches. In some cases, however, the stone must be small enough to pack readily around reinforcing metal, while in walls whose surface is to be picked or washed as described on page 289, a better appearance will result with stones under, say, one inch diameter, although the strength of concrete appears generally to increase with the size of the largest particles of stone in the mixture. This is illustrated with the gravel and the finer trap in experiments by Mr. Howard* at the Watertown Arsenal upon 12-inch

* Test on Metals, U. S. A., 1898, p. 654.