CHAPTER XX

STRENGTH OF PLAIN CONCRETE

The strength of plain concrete, that is, of concrete without steel reinforcement, is governed primarily by

(1) The quality of the cement.

(2) The texture of the aggregate.*

(3) The quantity of cement in a unit volume of concrete.

(4) The density† of the concrete.

The percentage of cement and the density of the concrete, which are of special importance to the user in determining the proportions of materials, may be expressed more explicitly as follows:

(1) With the same aggregate the strongest concrete is that containing the largest percentage of cement in a given volume of concrete, the strength varying nearly in proportion to this percentage.

(2) With the same percentage of cement but different arrangement of the aggregates, the strongest concrete is usually that in which the aggregate is proportioned so as to give a concrete of the greatest density, that is with the smallest percentage of voids. In many cases relative densities nearly correspond to relative weights.

Although these laws have been long recognized in a general way, having been partially proved by experiments of Mr. John Grant as early as 1871, but few attempts have been made to apply them practically in the comparison of strengths of different mixtures of concrete.

The authors have evolved a formula (see p. 356) from which, knowing the exact quantities of the raw materials entering into a concrete of a certain strength, it is possible to estimate the approximate strength of any other concrete mixed in different proportions of the same materials, under similar conditions of manufacture, storage, age, and methods of testing.

The compressive fiber strength of concrete, which is an essential factor in the design of reinforced concrete, is proportional to the strength of concrete in direct compression.

The table of tests of beams on page 376 covers so wide a range of proportions that it may be employed for comparing the transverse strength of different mixtures.

*The word aggregate is defined on page 1.

†The meaning of density is illustrated on pages 172 and 173.

Further information relating to the strength of concrete made from different materials and under various conditions is presented under separate headings in this chapter. The methods of making concrete specimens for testing are outlined on page 395.

COMPRESSIVE STRENGTH OF CONCRETE

The actual strength of concrete in compression, because of the limited capacity of testing machines, can be determined only by experiments upon comparatively small specimens from 4 to 12 inches square. The results from tests of such specimens are probably slightly lower than the actual strength of concrete in practice, carefully mixed and laid, because of the difficulty in obtaining homogeneous specimens. Experiments by the authors show that the strength of the same mixture tends to increase with the size of the specimen even if the relative dimensions remain constant. Of course carelessness or inexperience will produce irregular work in either actual or experimental construction.

The experimental strength of concrete is not always a criterion for fixing the proportions of mixture, in fact most concrete must be made stronger than the theoretical loading would require. A lean concrete, for example, although it may gain sufficient strength before the load is applied, may not be sufficiently strong at a short period to permit the removal of the molds or the ordinary wear during building, or for many purposes the lean concrete may be too porous. Often a lean Portland cement concrete may thus present no special advantage over a richer natural cement concrete. (See Chapter IV.)

Comparative Strength of Concretes of Different Proportions. The formula for strength of mortar derived by Mr. R. Feret and presented on page 141, as Mr. Feret himself states,* is not applicable to concrete. Our formula for concrete mixtures is therefore presented as a practical working formula of sufficient accuracy to compare the compressive strength of mixtures of the same materials in different proportions. Starting with the principles laid down in the two fundamental laws stated at the commencement of the chapter, it is evolved by trial by the method given on page 357, to fit the average results of a large number of tests made in this country and Europe.

Let

P = unit compressive strength of concrete.

c = absolute volume† of cement in a unit volume of concrete.

*Chimie Appliquée, p. 522.

†Method of determining densities and absolute volumes are described on page 135.

s = absolute volume of sand in a unit volume of concrete.
g = absolute volume of stone in a unit volume of concrete.
M = a coefficient, constant for all proportions of the same material mixed and stored under similar conditions, but varying with the texture of the coarse aggregate and the age of the specimen.

Then

$$P = M\left(\frac{c}{1 + c - (s + g)} - 0.1\right) \qquad (1)$$

The absolute volumes, as indicated on page 138, are really ratios of the actual volume of the concrete, representing the actual mass or total volume of solid particles in a unit volume of concrete. Since ratios are independent of the unit selected, the absolute units are the same for any system of measurement, and by changing the value of M the formula is adapted to English or Metric System. For example, if P expressed in terms of kilos grams per square centimeter requires a value of $M = 880$, P in pounds per square inch will require a value of $M = 880 \times 14.2^* = 12\ 500$. It follows that knowing for a given age the value of M and the strength of a concrete composed of known percentages of materials, it is possible to estimate the compressive strength at the same age of any other concrete of exactly known composition made under like conditions from similar materials, but differently proportioned.

A very slight variation in the values of the terms will so largely influence the result that the formula is only useful, on the one hand, where the specific gravities of the materials and the weights entering into a unit volume of concrete are determined so accurately that the absolute volumes can be calculated, and, on the other hand, for comparison of the strength of different mixtures of concrete under assumed average conditions. For the latter purpose the specific gravity of cement may be taken at 3.1 and of sand at 2.65, the weight of a barrel of cement as 376 pounds, the weight of the dry sand contained in a cubic foot of moist sand as 89 pounds, and the percentage of voids in the stone as 46%. In computations, values of absolute volumes must be carried to three places of decimals.

Now let

P' = compressive strength in pounds per square inch.
c_b = barrels of cement contained in a cubic yard of the concrete.
s_c = cubic yards of sand contained in a cubic yard of concrete.
g_c = cubic yards of stone contained in a cubic yard of concrete.
M' = a coefficient adapted to pounds per square inch.

Then assuming solid cement with no voids to weigh 193 lb. per cu. ft. and the solid particles of sand 165 lb. per cu. ft. formula (1) becomes,

$$P' = M'\left\{\frac{c_b \frac{376}{193}}{27 + \frac{376}{193}c_b - 27\left(\frac{89}{165}s_c + 0.54g_c\right)} - 0.1\right\}$$

$$P' = M'\left\{\frac{c_b}{13.85 + c_b - 7.48\,(s_c + g_c)} - 0.1\right\} \qquad (2)$$

This formula, as stated above, is only adapted for average comparative determinations, or where the conditions exactly correspond to those assumed. It may be adapted to other sand and stone by altering the coefficients of s_c and g_c. The table on page 360 is based upon these formulas (1) and (2).

Formula (1) on page 356 is based upon the actual strength of concrete, as determined by tests of Mr. E. Candlot in France and those of several other authorities at the Watertown Arsenal, U. S. A. To illustrate its

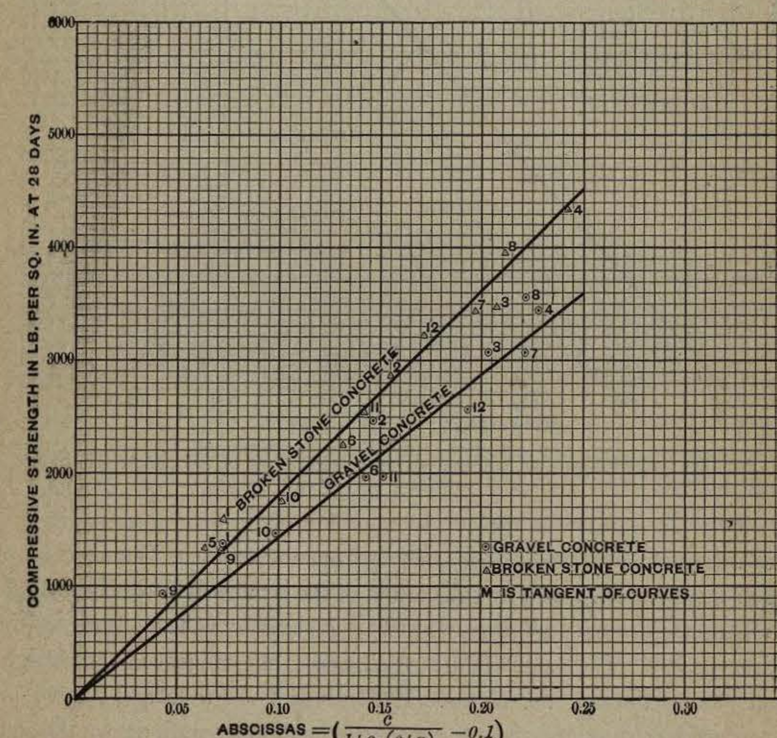

FIG. 116.—Comparison of Authors' Formula with Tests of E. Candlot. (*See p.* 358.)

agreement with actual experiments, tests of Mr. Candlot upon broken stone and gravel concrete 28 days old, quoted in full on page 367, are plotted on the diagram, Fig. 116, page 357, and Mr. George A. Kimball's tests made at the Watertown Arsenal on specimens 6 months old in Fig. 117.

The accuracy of the formula is shown by the nearness of the points on

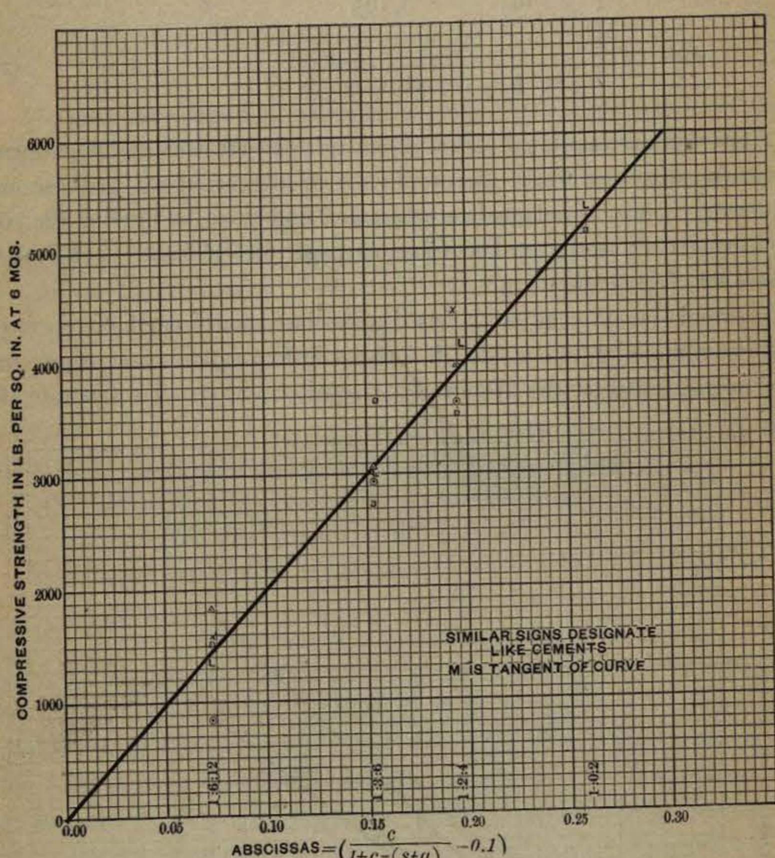

FIG. 117.—Comparison of Authors' Formula with Tests of George A. Kimball. (*See p.* 358.)

each diagram to straight lines starting from the origin. The abscissa of each point is determined by calculation of the term in brackets in formula (1), and the ordinate is the actual breaking strength of the specimen at the given period. The value of M in each case is the tangent of the straight line drawn through the points. If Mr. Candlot's tests are plotted on cross-section paper and smooth curves of growth in strength drawn through

them, it will be found that the new values taken from such curves, which partially eliminate inequalities in the breaking, approach even more nearly to the straight lines.

After a study of the strength of concrete at different periods, the authors suggest the following values for M at different ages. The values for broken stone concrete are based upon stone ranging in size from 2 to 2½ inch down to ¼ to ½ inch. For broken stone of finer size the values will be slightly lower. The composition of the concrete does not affect the value of M, since the term of the formula in large brackets is itself dependent upon the proportions of the mixture and the density of the concrete. The values of M are directly proportional to relative strengths at different ages.

Value of Coefficient M for Compressive Strength in Pounds per Square Inch.

Age.	Coefficient M for broken stone concrete	Ratio of growth based on age at one month
7 days	9 500	0.76
1 month	12 500	1.00
3 months	15 600	1.25
6 months	16 900	1.35
1 year	18 000	1.44

The ratios, which are taken from the curve on page 375, are based on the assumption that growth in strength of concrete, mixed under similar conditions and of similar consistency, is the same for all proportions of like materials. This, as stated on page 374, is not strictly true, but is sufficiently accurate for practical purposes.

Table of Compressive Strength. The strength of concrete mixed in various proportions, given in the table on page 360, is based upon a strength with proportions 1 : 3 : 6, that is, one barrel cement to 11.4 cubic feet sand to 22.8 cubic feet stone, of 1950 lb. per square inch at the age of one month, this value being selected as the average of tests by different experimenters. It corresponds to a value of M of 12 500. Using 1950 lb. per square inch for 1 : 3 : 6 as the starting point, the strengths for other mixtures are calculated from formula (1) page 356, the absolute units for the different proportions being deduced from the average quantities of cement, sand, and stone, contained in a unit volume of concrete. The values employed are similar to those in the table on page 231, except that it was necessary to carry them to three places of decimals. The strength at the age of six months is based on the growth in strength given on the curve on page 375. The assumption, which corresponds to average conditions, is made that a cubic foot of moist bank sand contains 89 lb. of dry grains having a specific gravity of 2.65, and that the specific gravity of the cement is 3.1. The stone is assumed equal in quality to sound, hard limestone, ranging in size from ¼ inch to 2 inches. Stone of ½ inch maximum size may give strength about 20% lower. Specimens mixed of very wet consistency show lower strength especially at early periods. Cold weather retards strength. Prisms test lower than cubes.

The values in the table may be readily transformed to safe working strength by dividing by the proper factor of safety.

Relative Compressive Strength of Portland Cement Concrete of Different Proportions.

Based on Cube Specimens and Medium Consistency.

(*See important foot-notes, also p.* 359.)

Proportions.			Age, one month. Voids in Broken Stone or Gravel.					Age, six months. Voids in Broken Stone or Gravel.				
Cement.	Sand.	Stone.	*50 % lb. per sq. in.	**†45% lb. per sq. in.**	‡40% lb. per sq. in.	§30% lb. per sq. in.	§20% lb. per sq. in.	*50% lb. per sq. in.	**†45% lb. per sq. in.**	‡40% lb. per sq. in.	§30% lb. per sq. in.	§20% lb. per sq. in.
1	1½	2	2880	2860	2840	2800	2760	3890	3870	3840	3780	3730
1	1½	3	2780	2750	2720	2670	2610	3750	3710	3680	3600	3530
1	1½	4	2680	2650	2610	2540	2460	3620	3570	3520	3430	3330
1	2	3	2560	2540	2510	2460	2410	3460	3420	3390	3320	3250
1	2	4	2480	2440	2410	2350	2290	3340	3300	3250	3170	3090
1	2	5	2400	2350	2310	2230	2170	3230	3180	3120	3010	2930
1	2	6	2320	2260	2230	2140	2060	3130	3060	3010	2890	2780
1	2½	3	2370	2340	2320	2270	2230	3200	3160	3130	3070	3020
1	2½	4	2290	2260	2230	2180	2110	3090	3050	3010	2940	2850
1	2½	5	2210	2180	2130	2070	2000	2980	2940	2880	2790	2700
1	2½	6	2140	2100	2060	1980	1910	2890	2830	2780	2670	2570
1	3	4	2120	2090	2060	2020	1970	2860	2830	2780	2720	2660
1	3	5	2060	2030	1990	1930	1870	2780	2740	2690	2610	2530
1	3	6	1990	1950	1910	1840	1770	2680	2630	2580	2480	2390
1	3	8	1860	1810	1770	1680	1600	2510	2440	2390	2280	2160
1	4	6	1710	1680	1650	1590	1530	2310	2270	2220	2140	2070
1	4	7	1660	1620	1590	1530	1460	2240	2190	2150	2060	1980
1	4	8	1610	1570	1530	1460	1400	2170	2120	2070	1970	1880
1	4	10	1510	1460	1420	1340	1260	2040	1980	1920	1810	1700
1	5	10	1310	1270	1230	1160	1090	1770	1720	1660	1570	1470
1	6	12	1060	1020	980	910	840	1430	1380	1320	1230	1140

NOTE.—Proportions are based on a barrel of 3.8 cu. ft. Values are for average ultimate strength, which must be divided by a factor of safety for working loads. Quality of materials and methods of mixing may affect the strength by 25% in either direction, while the relative values for different proportions are not materially changed.

*Use 50% columns for broken stone screened to uniform size.

†Use 45% **columns for average conditions** and for broken stone with dust screened out.

‡Use 40% columns for gravel or mixed stone and gravel.

§Use these columns for graded mixtures.

In the table the stone with the smaller percentage of voids gives the lower strength. This is due to the proportioning by volume. To illustrate, a cubic foot of stone measured loose with 40% voids contains more solid material than stone with 50% voids, and hence makes a greater bulk of concrete with the same proportions by volume. This is further illustrated in the table on page 234. Consequently, there is less cement in a unit volume of the concrete when the stone has 40 per cent voids; and while the density is slightly greater, it is not enough greater to counterbalance the decrease in the percentage of cement. If the proportions had been altered so as to use less sand with the stone having 40 per cent voids, the concrete would have been stronger, with the same amount of cement per cubic yard of concrete, because of the greater density.

From this it must not be inferred that the aggregate with the largest percentage of voids is best to use. As indicated above, it requires more cement to a given volume of concrete, and the concrete is apt to be slightly less dense than with an aggregate having fewer voids, so that the latter is usually the more economical even although it is sometimes slightly inferior in strength. In the example in the preceding paragraph, with Portland cement at $2 per barrel, the concrete with stone having 50% voids would require 0.11 bbl. more cement per cubic yard than the concrete with stone having 40% voids, and would therefore cost 22 cents higher per cubic yard.

The following table is presented to indicate in round numbers the probable

Approximate Average Crushing Strength of Concrete

PROPORTIONS BY VOLUME.	MEDIUM CONSISTENCY. Cubes.		WET CONSISTENCY. Cubes.		WET CONSISTENCY. 8 by 16 inch Cylinders	
	30 days. lb. per sq. in.	6 mos. lb. per sq. in.	30 days. lb. per sq. in.	6 mos. lb. per sq. in.	30 days. lb. per sq. in.	6 mos. lb. per sq. in.
1 : 1½ : 3	2800	3700	2600	4100	2300	3600
1 : 2 : 4	2500	3300	1900	3100	1700	2700
1 : 2½ : 5	2200	2900	1700	2700	1500	2400
1 : 3 : 6	1900	2600	1500	2400	1300	2100
1 : 4 : 8	1500	2100	1000	1600	900	1400

Proportions are based on the unit measure of one barrel (4 bags) cement assumed as 3.8 cu. ft.

The first column of strength values is taken from the table on the opposite page; the cylinders at one month are arranged as averages of a large number of tests in various laboratories made during the years 1904 to 1908; the ratio of strength of cubes to cylinders is based upon the St. Louis tests (p. 370) and the growth of strength of wet consistency upon tests by the authors (p. 384). The ultimate strength of long columns is probably from 90 to 95 per cent of the strength of cylinders (p. 370.)

strength of different mixtures of concrete under working conditions. As stated on the opposite page, so many conditions affect the strength that such data can be presented only as extremely rough approximations.

Variation in Weight of Concrete of Different Proportions. The weights of specimens of similar concrete are of interest in comparing the relative strength of different mixtures or of different specimens of the same mixture. Of twelve pairs of duplicate cubes which the authors had tested in 1903 at the Watertown Arsenal and the Massachusetts Institute of Technology, the heavier specimen, except in one case, was found to be the stronger.

The following table of tests selected from tests of concrete and mortar cubes made by Mr. James E. Howard* at the Watertown Arsenal illus-

Weights of Portland Cement Concrete of Different Proportions.

Age four months. Watertown Arsenal. (*See p.* 362.)

Item	Proportions by Volume: Cement	Sand	Broken Stone†	Weight per cu. ft. lb.	Compressive strength per sq. in. lb.	Item	Proportions by Volume: Cement	Sand	Broken Stone†	Weight per cu. ft. lb.	Compressive strength per sq. in. lb.
1	1	1	0	136.5	4370	11	1	5	10	140.2	797
2	1	2	0	134.2	2506	12	1	6	12	138.2	738
3	1	3	0	133.8	1812						
4	1	4	0	120.9	830	13	1	2	2	140.3	1768
5	1	5	0	119.3	532	14	1	2	3	145.2	1911
6	1	6	0	116.9	169	15	1	2	4	149.1	2147
7	1	7	0	111.5	118	16	1	2	5	150.9	2452
						17	1	2	6	151.2	2124
8	1	2	4	150.7	2178	18	1	2	7	146.4	1650
9	1	3	6	146.9	1815	19	1	2	8	142.4	1295
10	1	4	8	143.2	1135						

trates the comparative variation in weight and strength of concrete mixed in varying proportions:

Compressive Tests of Plain Concrete. The tests on pages 363, 367, and 366 (Fig. 119), are selected from among the best series of concrete experiments on record in America and Europe, so that the reader may form a general idea of the results obtained by expert experimenters. For practical comparisons of strength of different mixtures, reference should be made to the more complete table on page 360. The variation in strength of concretes mixed in the same proportions is due not only to the difference in the materials, but also to the different methods of making the tests, and to the fact that in many cases the unit of measurement

*Tests of Metals, U. S. A., 1899, pp. 788–795.

†Items (8) to (12), 2½ inch screened broken trap, and items (13) to (19), 1½ inch screened broken trap.

Strength of Concrete in Compression from Various Authorities. Age, one month.

In pounds per square inch. (*See p.* 362)

Authority.	Size of Specimen.	1:0	1:1	1:2	1:3	1:4	1:0:2	1:0:5	1:1:3	1:1½:4	1:2:2	1:2:3	1:2:4	1:2:5	1:2:7	1:2½:5	1:3:2	1:3:3	1:3:4	1:3:5	1:3:6	1:3:7½	1:4:3	1:4:5	1:4:8	1:5:10	1:6:12
Portland Cement Concrete.																											
[1]Hawley & Krahl *a*	6"																				1795						
[2]Henby *b*	3" x 3½" x 11"												2800	2500							1900				1150		
[3]Humphrey *c*	6"																				1620						
[4]Luegers													2560								1990				1710	1350	1140
[5]Von Mauk	8"	4350	3560	3200	2560	1630					2060	1570					1570	1420	920				850				
[6]Dyckerhoff *d*	4"							1000				2800		2430						1660				1290			
[7]G. A. Kimball *e*	12"						3846						2399								2164						994
[8]Taylor & Thompson *f*	12"															3255											
[9]Woodb'y & Leight'n *g*	12"																			1623							
[10]Watertown Arsenal *h*	12"	5500	4400						4917											2042							
[11]Webster	6"																				1763						
Natural Cement Concrete.																											
[12]Clarke & Son	12"									1449					746							991					
Cinder Concrete with Portland Cement.																											
[13]Watertown Arsenal	12"								1466			1098	904	769													
[14]Henby *i*	3½" x 3½" x 11"												1000	750							550	450					

[1]Engineering News, June 7, 1900, p. 375.
[2]Journal Association Engineering Societies, Sept., 1900, p. 145.
[3]Transactions American Society of Civil Engineers, Vol. XLVIII, p. 561.
[4]Lueger's Lexicon, Vol. II, p. 295.
[5]Portland Zement, Berlin, p. 92.
[6]Portland Zement, Berlin, p. 90.
[7]Tests of Metals, U. S. A., 1899, p. 740.
[8]Tests at Watertown Arsenal, 1903.
[9]Tests at Watertown Arsenal, 1896.
[10]Tests of Metals, U. S. A., 1898, p. 650.
[11]Tests at Philadelphia, Penn., 1903.
[12]Tests of Metals, U. S. A., 1901, p. 611.
[13]Tests of Metals, U. S. A., 1898, p. 569.
[14]Journal Association of Engineering Societies, Sept., 1900, p. 145.

*a*Drift sand, blue limestone.
*b*½" to 2" limestone.
*c*Gravel concrete.
*d*Stone 0.2" to 1.2" diameter.
*e*Average of 5 cements. Conglomerate stone.
*f*Newburyport sand ¼" to 1" trap.
*g*Broken trap.
*h*1" trap.
*i*Cinders unscreened.

used in proportioning is indefinite, and, as discussed on page 218, similar nominal proportions may apply to quite different actual mixtures. Notwithstanding these opportunities for variation, however, it is noticeable that the results reached by different parties really show less percentage

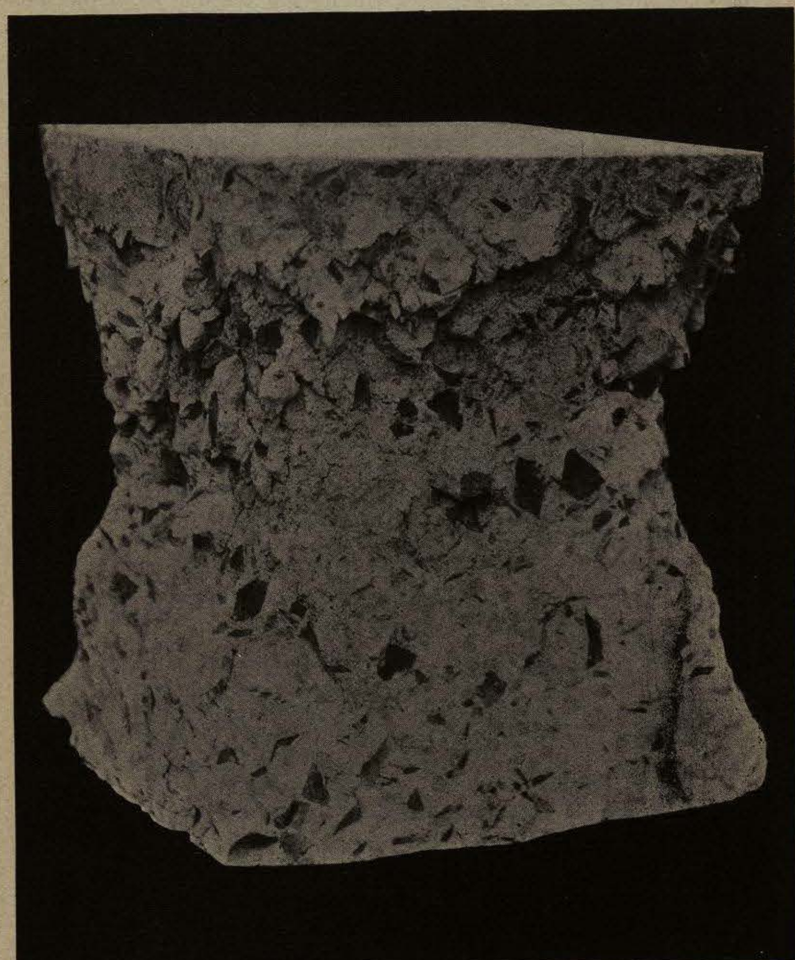

Fig.118. Twelve-inch Concrete Cube after Crushing in Emery Testing Machine at Watertown Arsenal. *(See p. 365.)*

variation than is expected in the tensile tests of neat cements and sand mortars in different laboratories even with the same brand of cement.

In the table on page 363 of data from various authorities, only tests at the age of one month are recorded. Strength of the specimens at longer

and shorter periods may be estimated by referring to the curve in Fig. 122, page 375.

The appearance of a concrete cube after crushing, showing the manner in which the sides flake off, leaving a double pyramid, and the shearing of the particles of stone, is illustrated in Fig. 118. The specimen is one of a series tested for the authors at the Watertown Arsenal, U. S. A.

Kimball's Tests. A series of experiments upon 12-inch cubes made by Mr. George A. Kimball,* Chief Engineer of the Boston Elevated Railway Company, and tested at the Watertown Arsenal, although included in the above table, covers so wide a range in time and proportions that more complete values are worth quoting and are presented in the curves on page 366. Mr. Kimball also determined the elastic properties of these specimens, and tested some of the specimens with a concentrated load, as referred to on page 368. He states that the stone used was conglomerate from Roxbury, Mass., containing 49.5 per cent. voids. Its analysis was as follows:

Passing 2½-inch ring	100.0%
" 2-inch "	95.2%
" 1-inch "	18.5%
" ½-inch "	0.5%

The sand and cement were made into a mortar of about the consistency of damp sand, and then spread upon the stone, which previously had been drenched with water. After ramming with iron rammers and tamping bars, the water barely flushed to the surface of the 1:0:2 and 1:2:4 mixture, while the surface of the 1:3:6 and the 1:6:12 mixtures appeared merely moist, so that the concrete was what ordinarily would be termed dry. The average quantity of water used with the different mixtures in addition to the water for wetting the stone is expressed in percentages of the weight of the cement and of the cement plus sand as follows:

Percentages of Water Employed in Kimball's Tests.

	In terms of weight of cement.	In terms of weight of cement plus sand.†
Mixture 1:0:2	20.9%	20.9%
" 1:2:4	30.3%	10.7%
" 1:3:6	39.3%	10.5%
" 1:6:12	71.1%	8.6%

These percentages *do not* include the water used in wetting the stone.

The specimens were made in cold weather, and therefore set slowly.

*Tests of Metals, U. S. A., 1899, p. 717.

†Approximate.